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G01S 7/35

(52) UK CL (Edition O)
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US 4201986 A **US 3813599 A**

(58) Field of Search
Online databases: WPI, INSPEC, CLAIMS, JAPIO

(71) Applicant(s)
Commonwealth of Australia

(Incorporated in Australia)

GEC Patent Department, Waterhouse Lane,
CHELMSFORD, Essex, CM1 2QX, United Kingdom

(72) Inventor(s)
Brent Summers

(74) Agent and/or Address for Service
G Cockayne
GEC Patent Department, Waterhouse Lane,
CHELMSFORD, Essex, CM1 2QX, United Kingdom

(54) Signal processing apparatus and method

(57) By employing waveforms having a waveform repetition interval and bandwidth related such that the product of the waveform repetition interval and the bandwidth is an integer the performance and efficiency of both signal processing apparatus and methods can be improved, particularly where the waveform comprises a plurality of identical sweeps. Such as arrangement enables non-linear systems to have very good out-of-band emission characteristics. A Tukey window function (a convolution of $\cos^n x$ and a step function) may be applied to the frequency spectrum in order to remove the discontinuities at the start and finish of each dwell.

At least one drawing originally filed was informal and the print reproduced here is taken from a later filed formal copy.

GB 2 291 301 A

Fig.1.

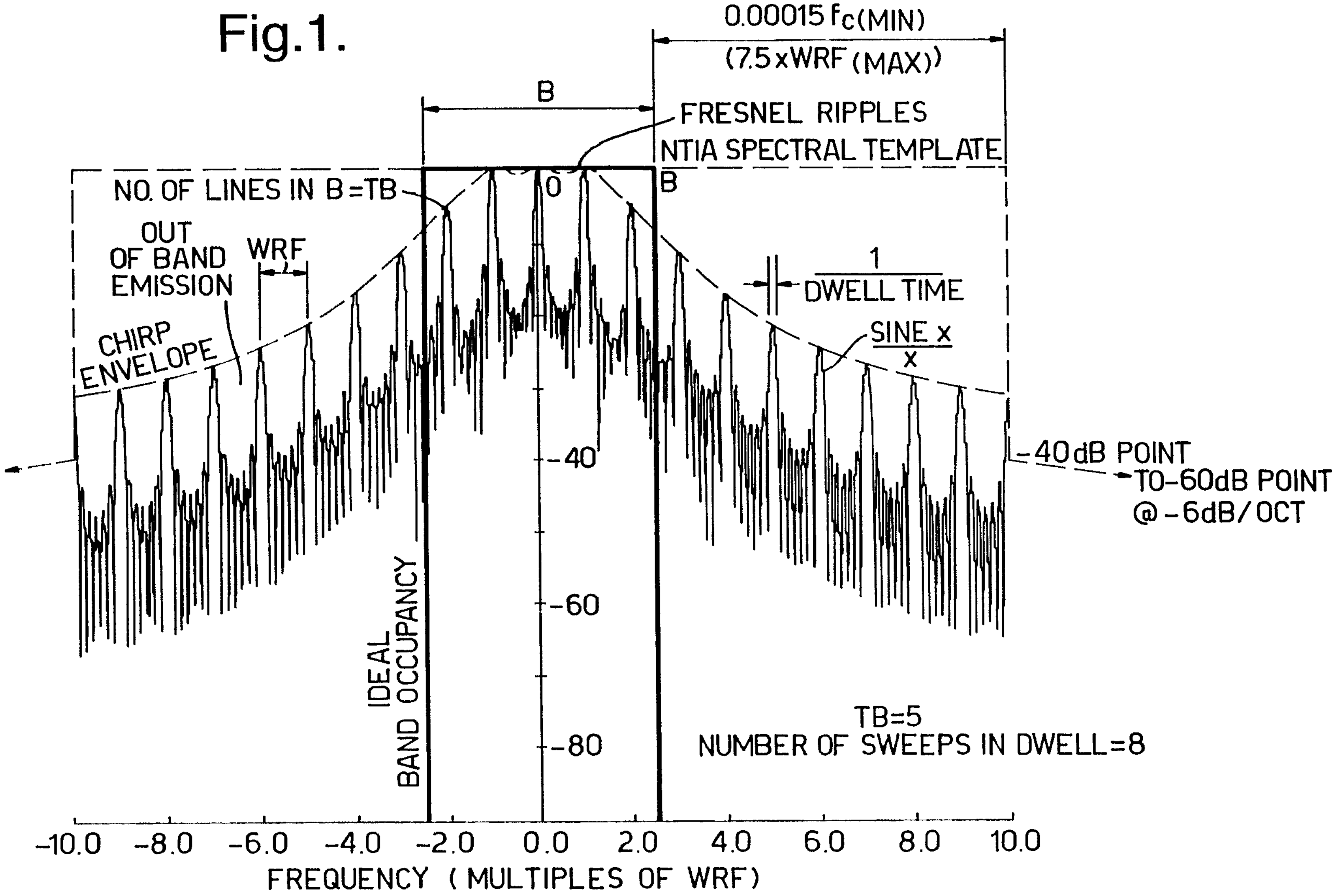

0.00015 fc(MIN)
(7.5xWRF (MAX))
B
FRESNEL RIPPLES
NTIA SPECTRAL TEMPLATE
NO. OF LINES IN B=TB
0
B
OUT OF BAND EMISSION
WRF
CHIRP ENVELOPE
1/DWELL TIME
SINE x/x
-40dB POINT
TO-60dB POINT
@-6dB/OCT
-40
-60
-80
IDEAL BAND OCCUPANCY
TB=5
NUMBER OF SWEEPS IN DWELL=8
-10.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0
FREQUENCY (MULTIPLES OF WRF)

Fig.2.

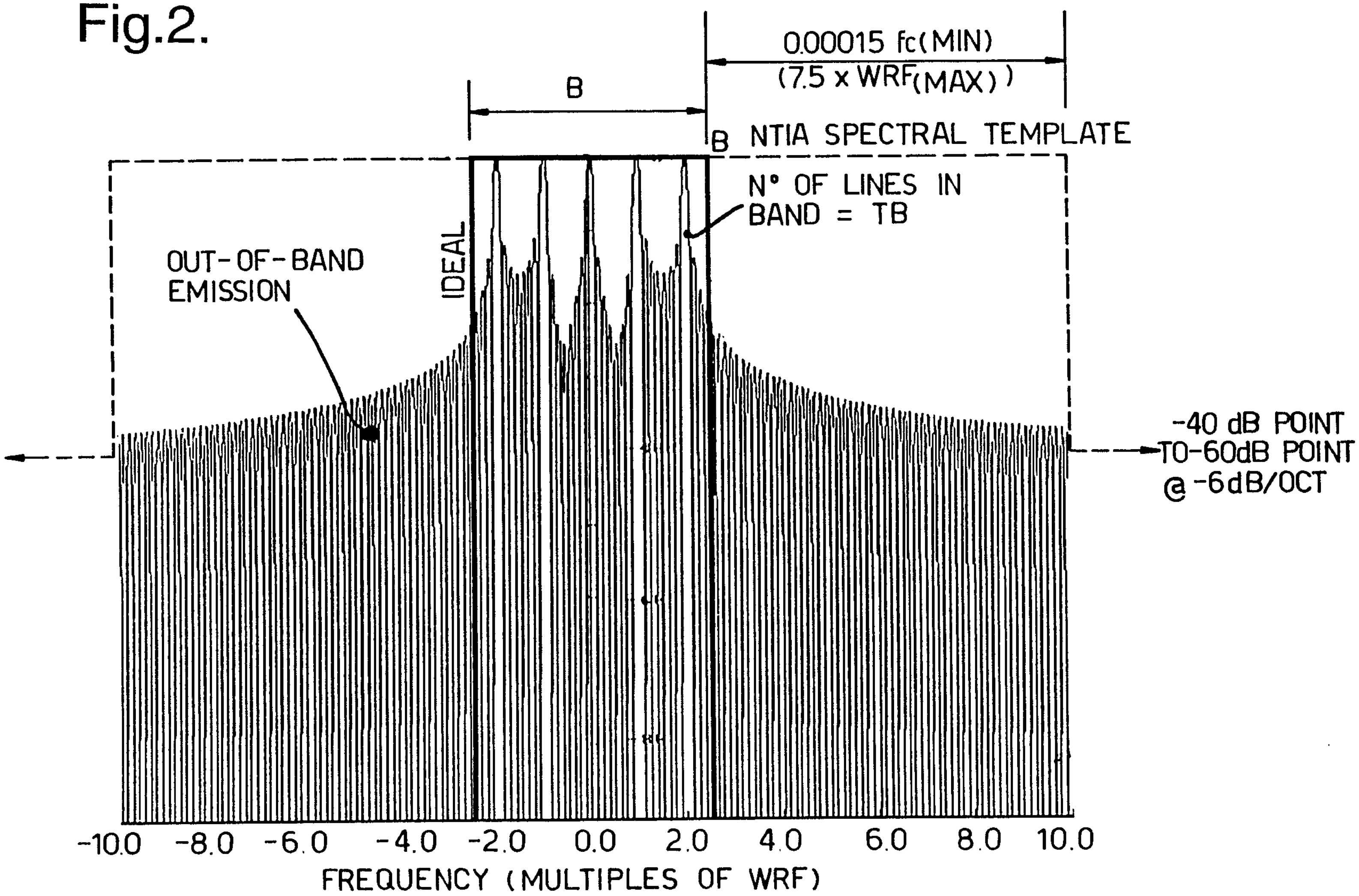

0.00015 fc(MIN)
(7.5 x WRF(MAX))
B
B
NTIA SPECTRAL TEMPLATE
N° OF LINES IN BAND = TB
IDEAL
OUT-OF-BAND EMISSION
-40 dB POINT TO-60dB POINT @-6dB/OCT
-10.0
-8.0
-6.0
-4.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0
FREQUENCY (MULTIPLES OF WRF)

Fig.3A.

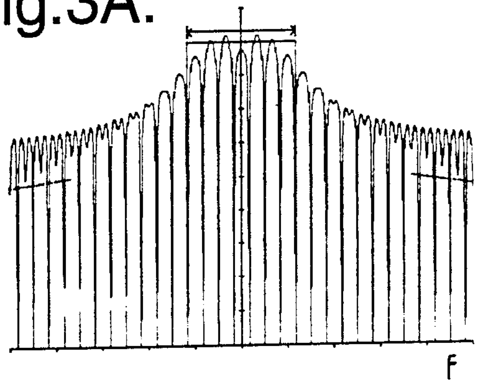
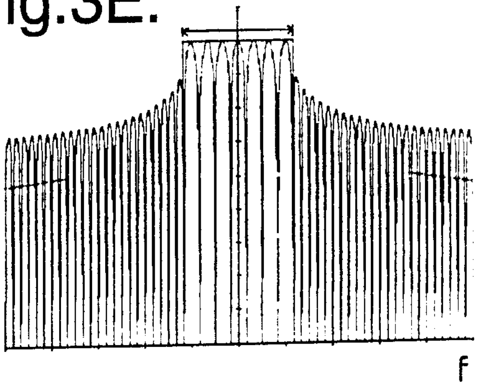


Fig.3E.



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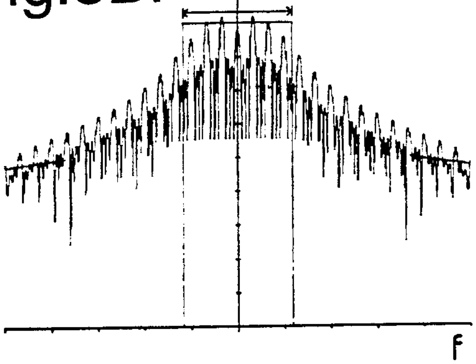

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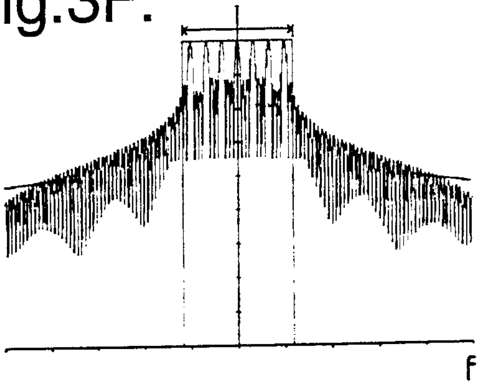

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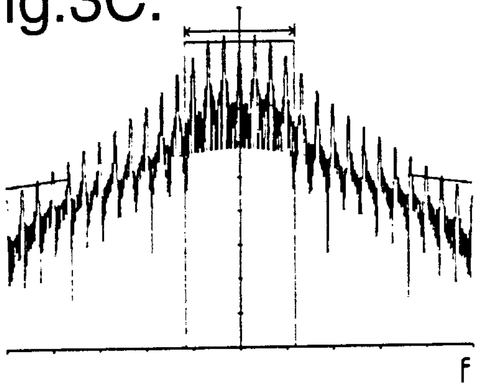
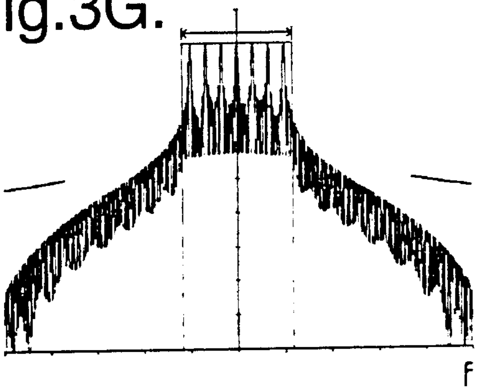


Fig.3G.



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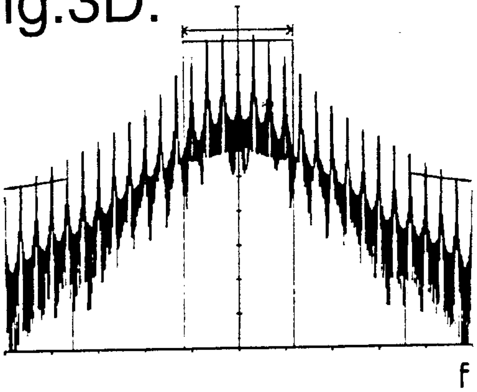

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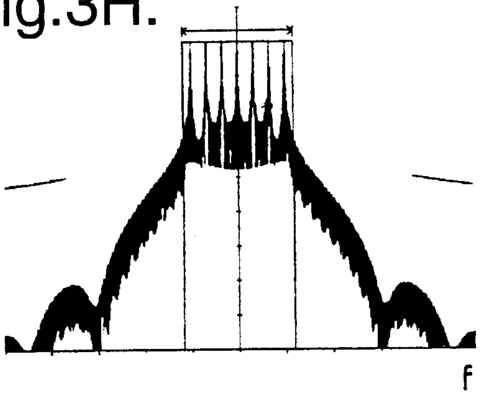
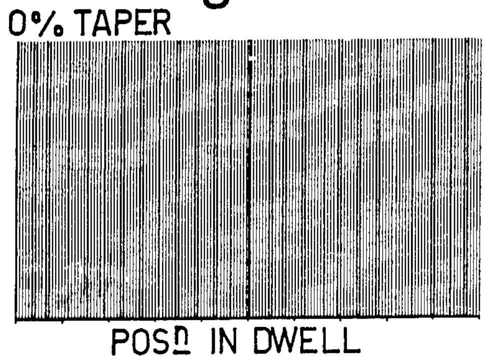


Fig.4A.



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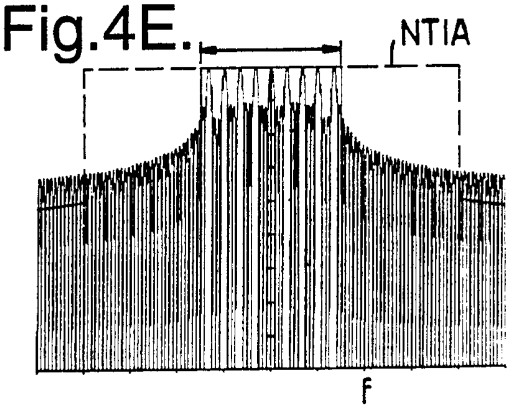

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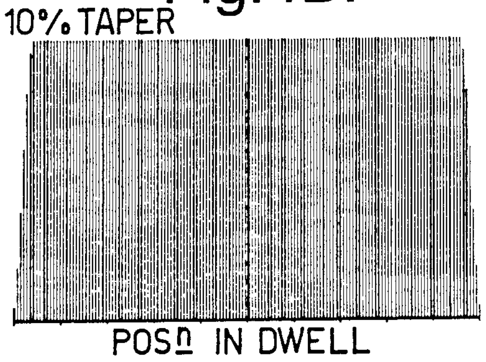

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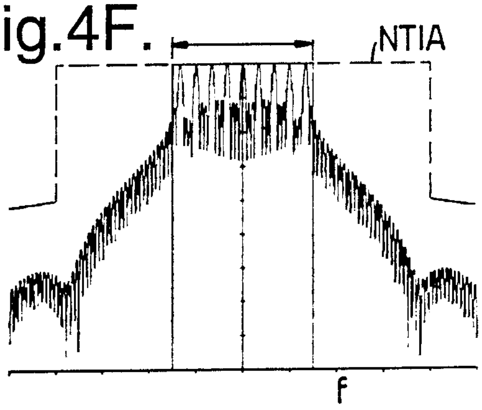

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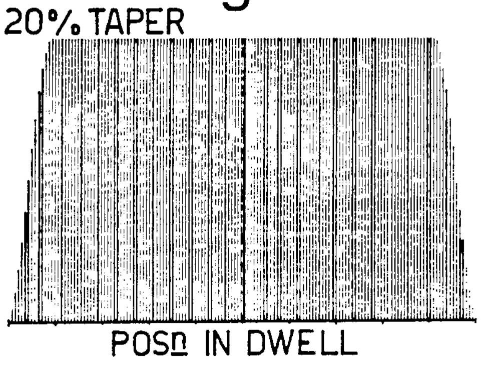

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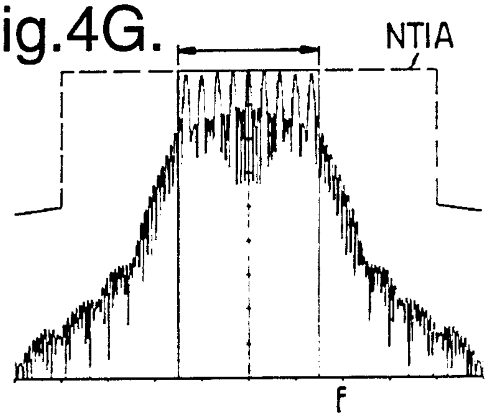

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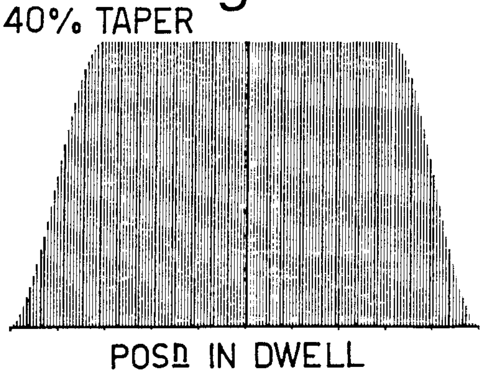

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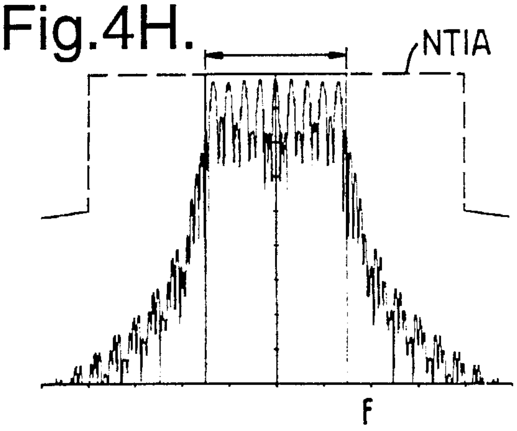

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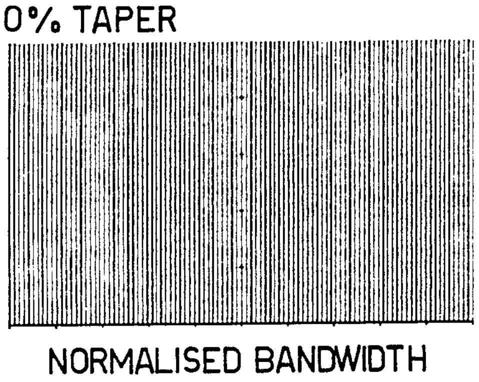

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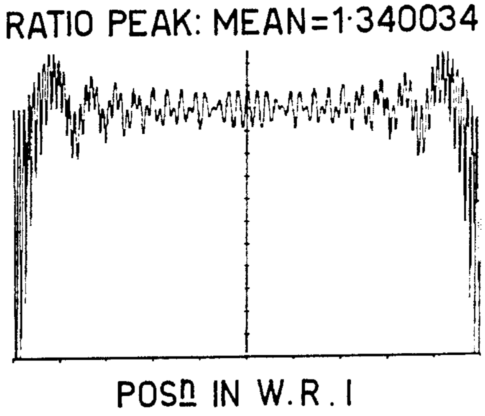

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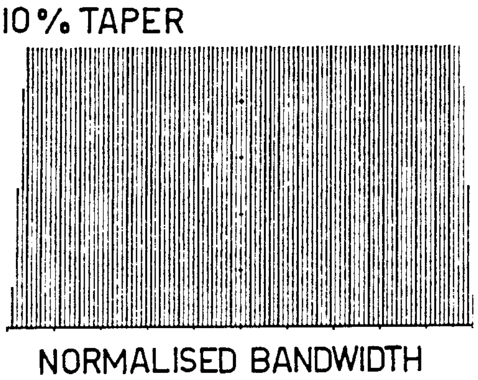

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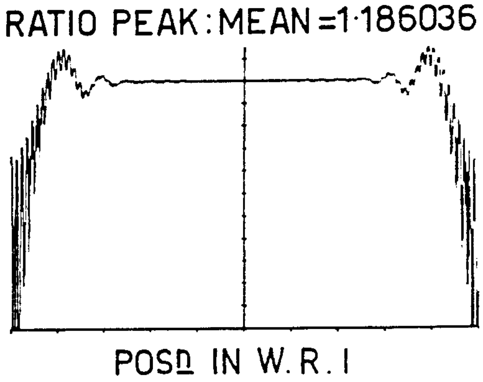

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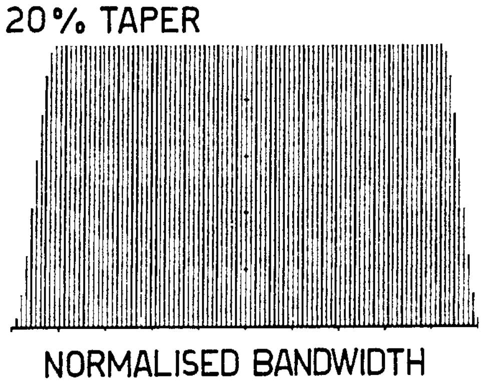

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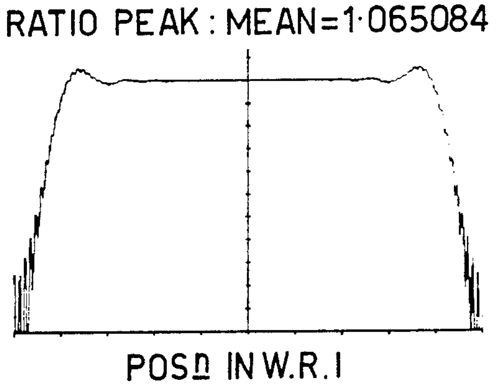

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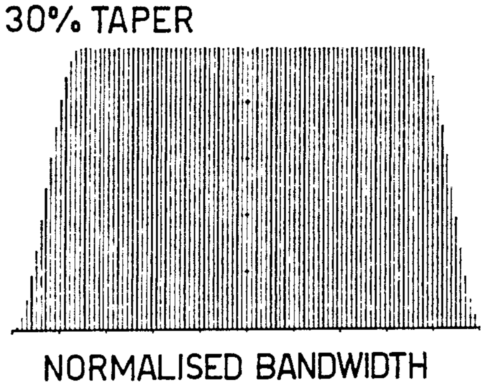
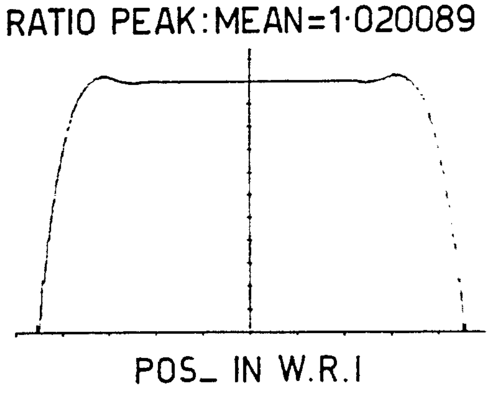


Fig.5H.



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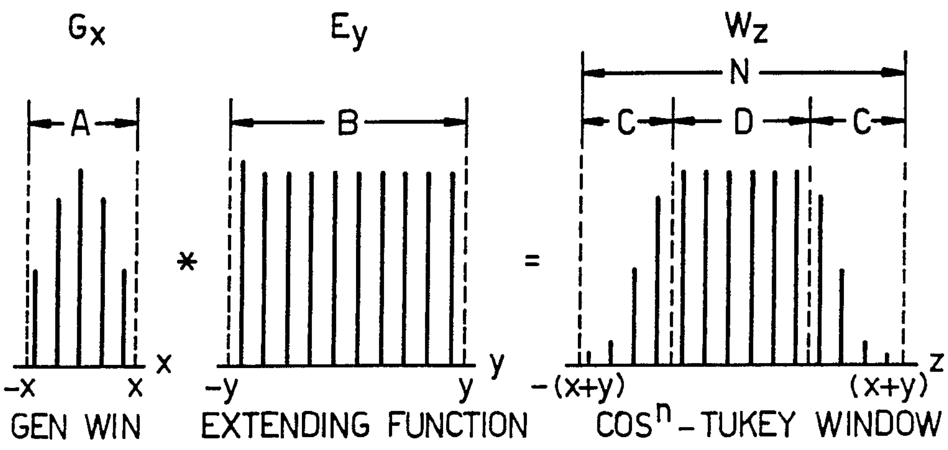


$$W_z = \frac{1}{Cg} \cdot \sum_{x=-X}^{X} G_x^n \; E_{z-x} \quad : z = -(x+y), \ldots, (x+y)$$

AND,

$$G_x^n = \cos^n \left(\frac{x\pi}{1+k} \right) \quad : x = -X, \ldots, X \quad n \geqslant 0$$

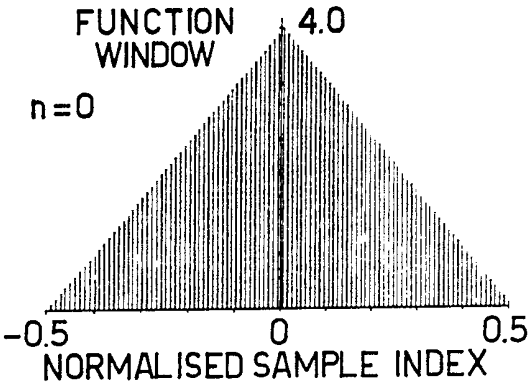
$$E_y = 1 \quad : \quad y = -Y, \ldots, Y$$

$$C_g = \frac{1}{N} \cdot \sum_{z=-(x+y)}^{(x+y)} W_z$$

THEN,

FOR N ODD	FOR N EVEN
$\propto = \frac{2k}{N}$: KE $\{0, 1, \ldots, M\}$	$\propto = \frac{2k}{N}$: KE $\{0, 1, \ldots, (M-1)\}$
$0 \leqslant \propto \leqslant (N-1)/N, M = (N-1)/2$	$0 \leqslant \propto \leqslant (N-2)/N, M = N/2$
$A = 1+k$	$A = 1+$
$B = 2M-k+1$	$B = 2M-$
$C = k$	$C = k$
$D = 2(M-k)+1$	$D = 2(M-k)$
$X = k/2$	$X = k/2$
$Y = (2M-k)/2$	$Y = (2M-k-1)/2$

Fig.7A.



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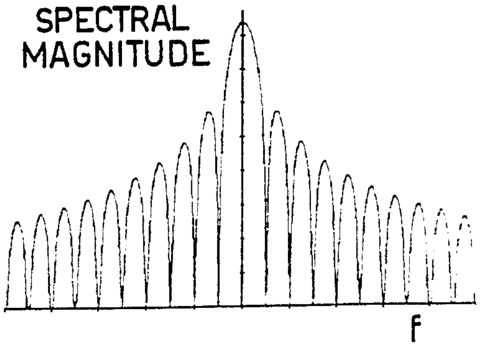
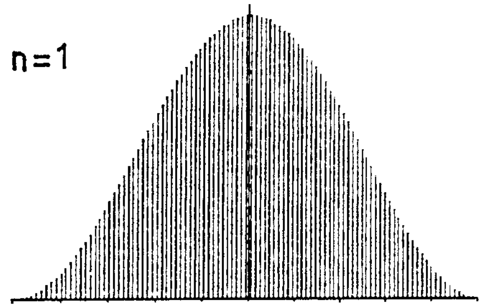


Fig.7B.



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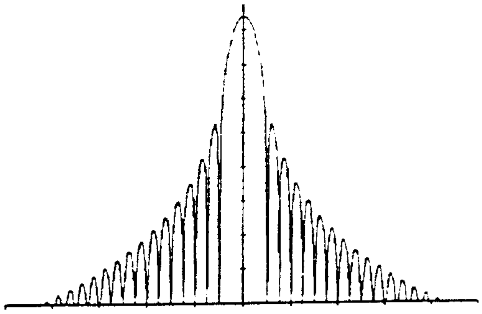


Fig.7C.

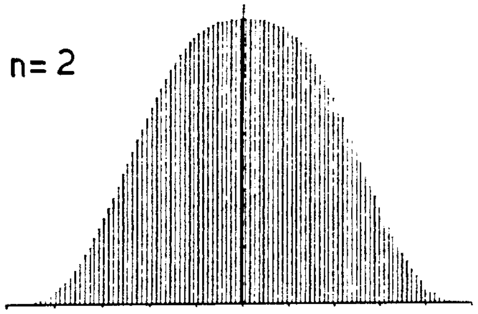


Fig.7G.

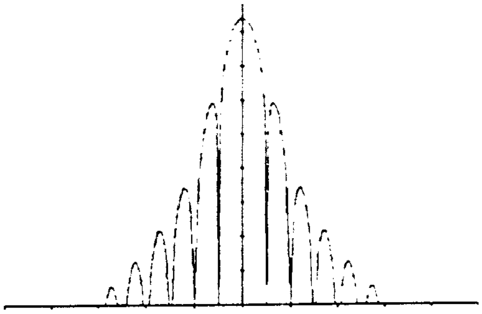


Fig.7D.

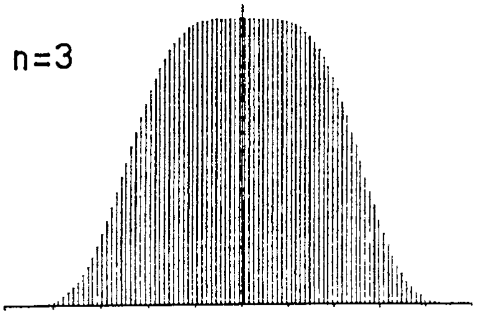


Fig.7H.

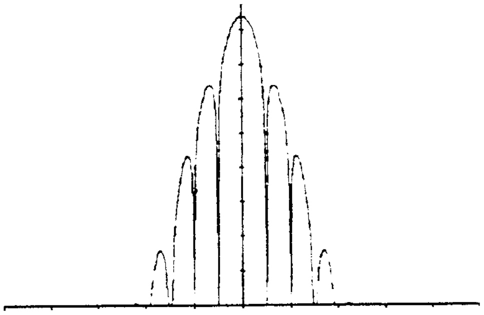
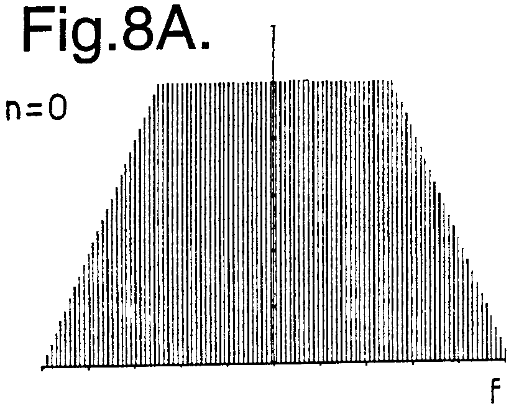


Fig.8A.



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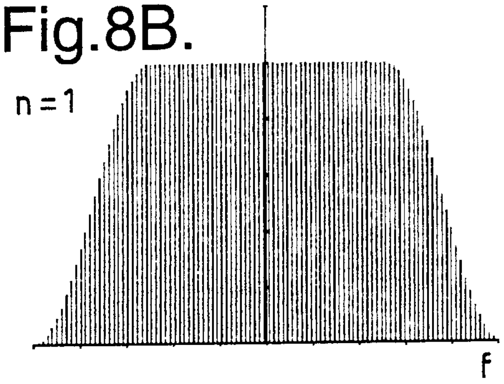

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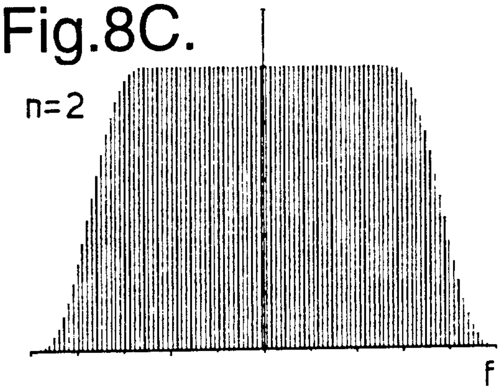

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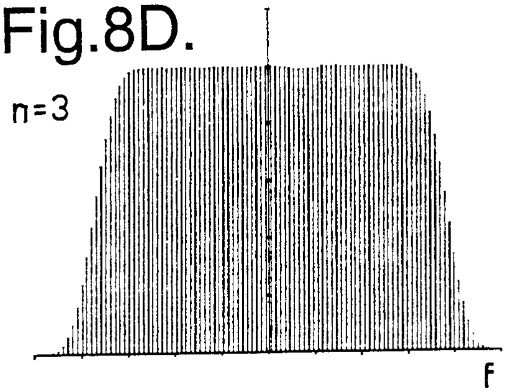

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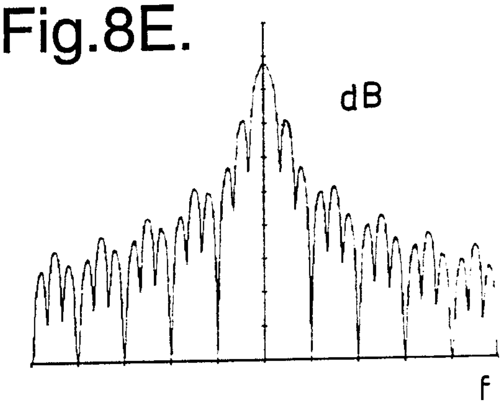

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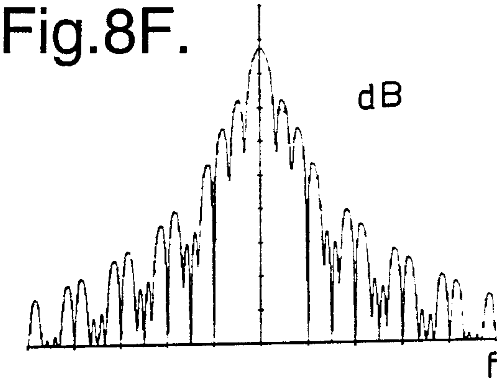

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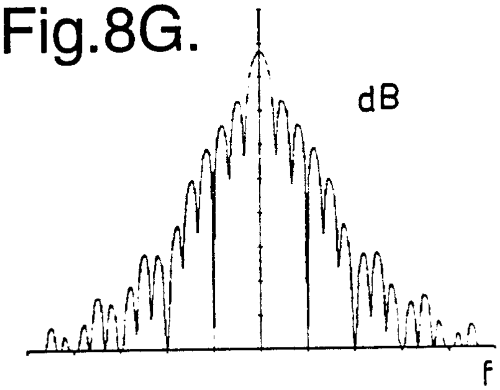

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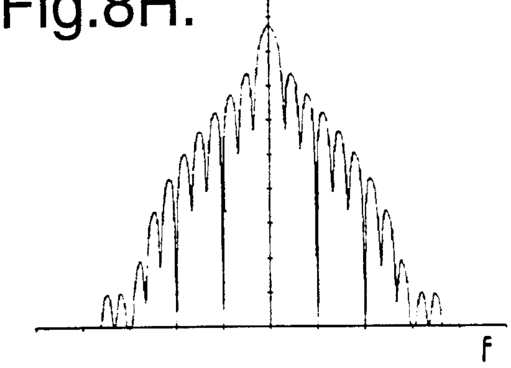
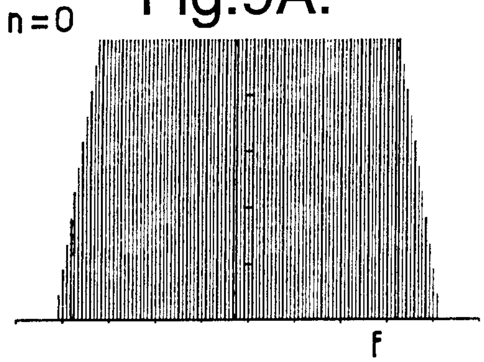


Fig.9A.



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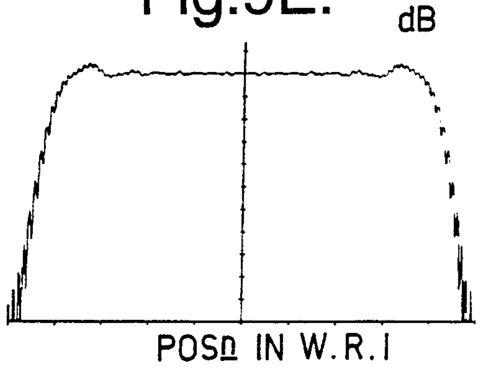

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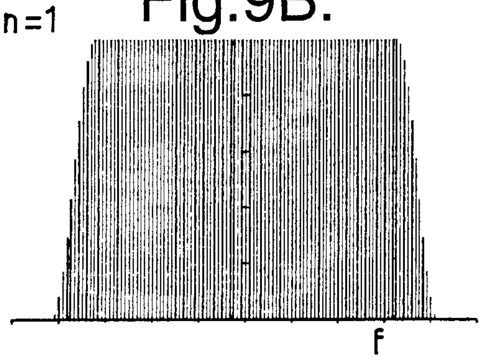

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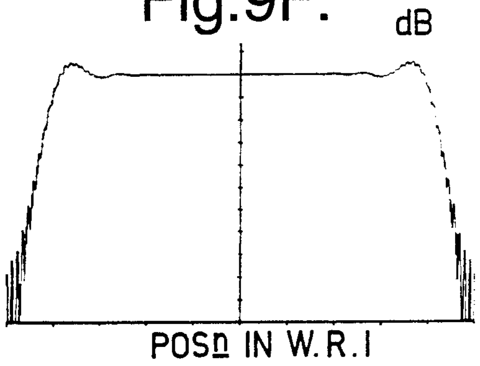

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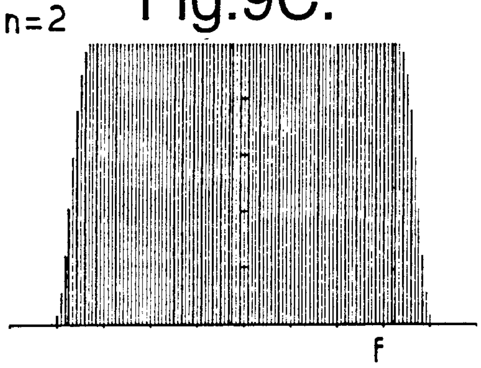

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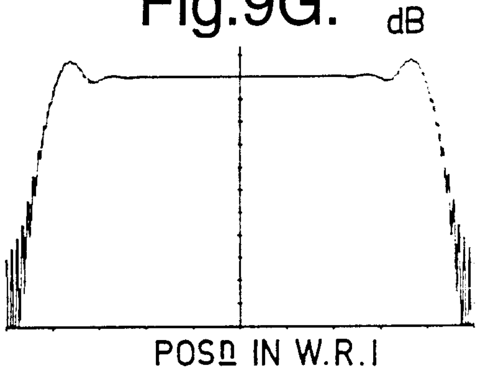

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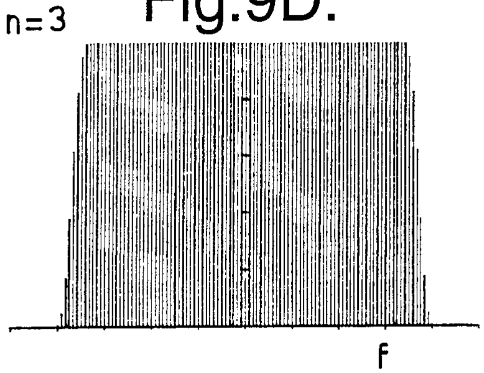

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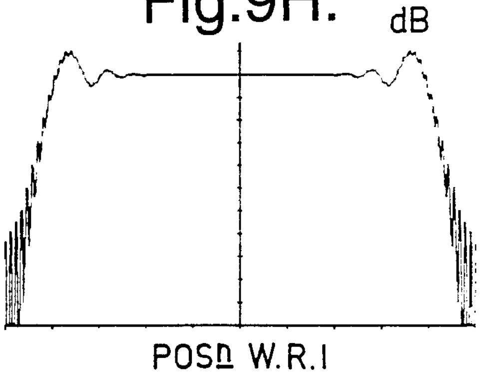
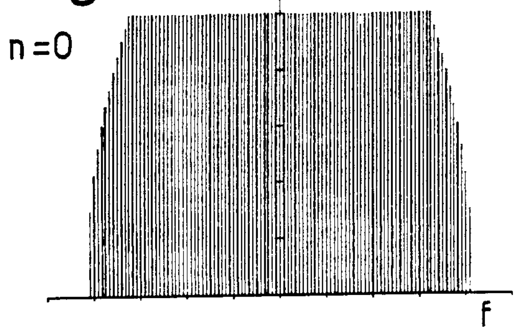


Fig.10A.



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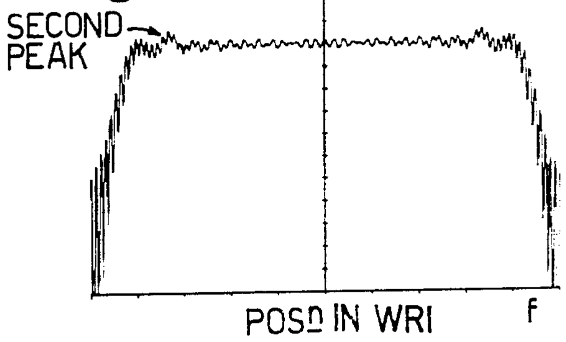

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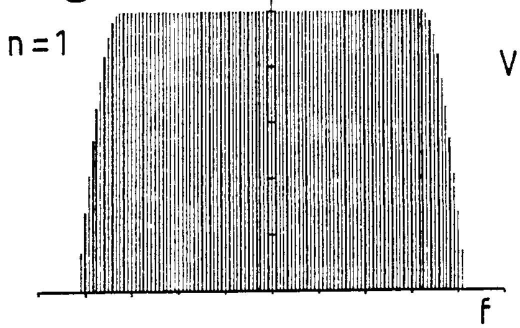

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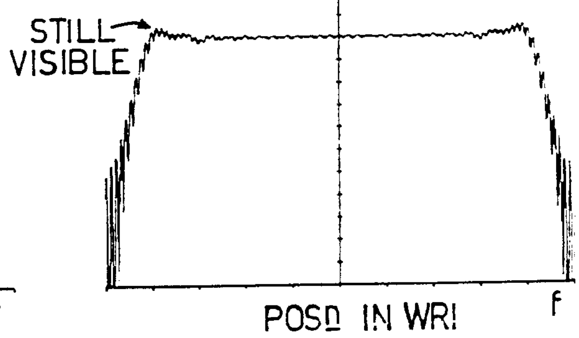

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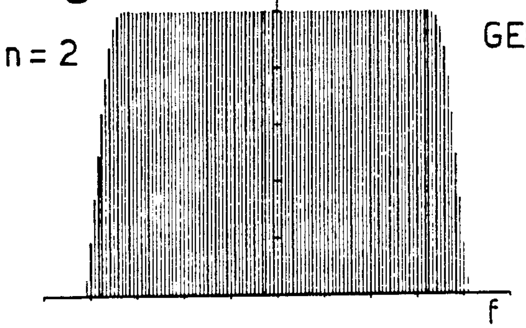

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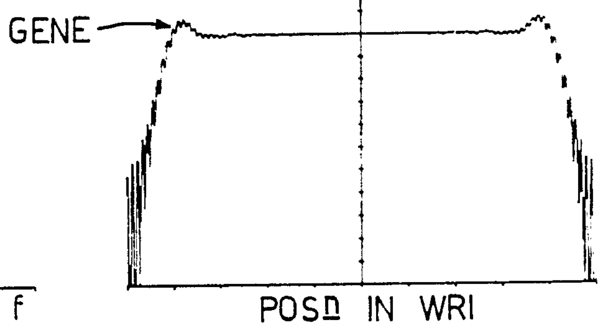

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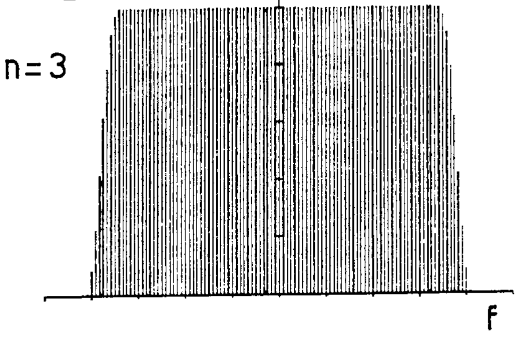

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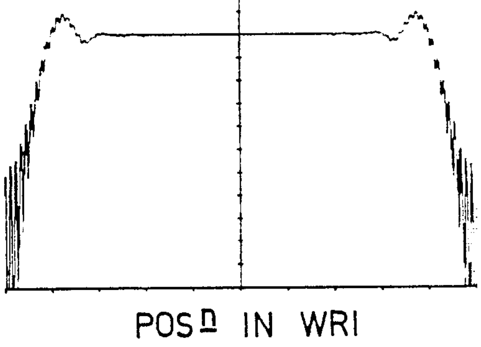


Fig.11A.

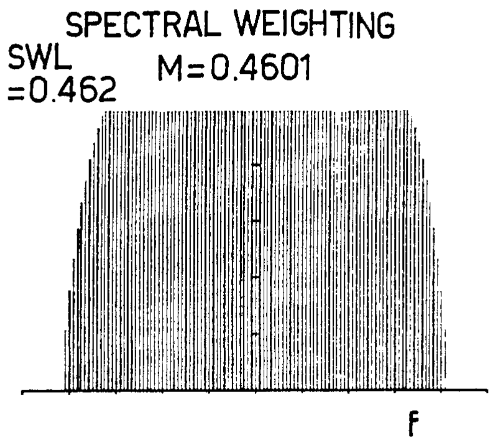


Fig.11B.

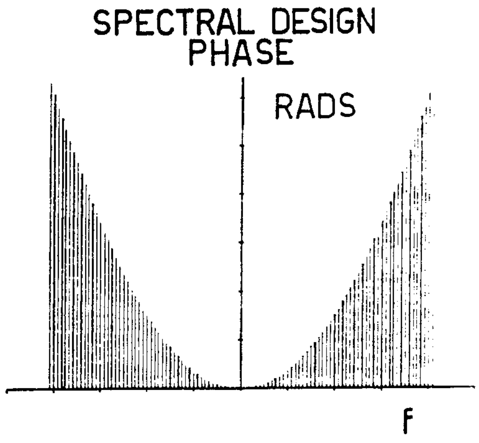


Fig.11C.

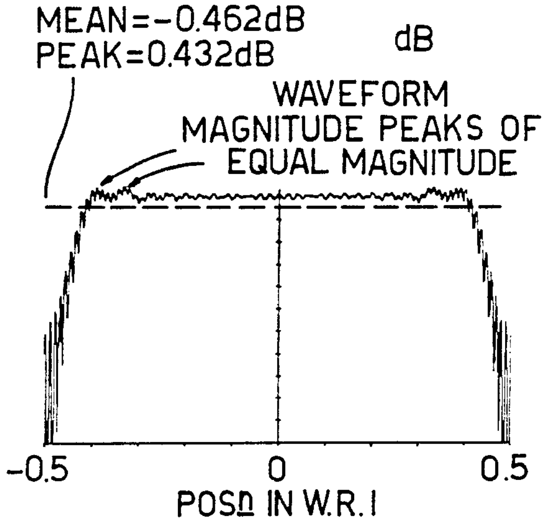


Fig.11D.

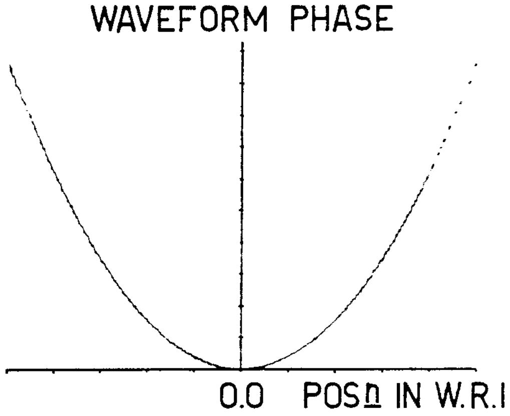


Fig.11E.

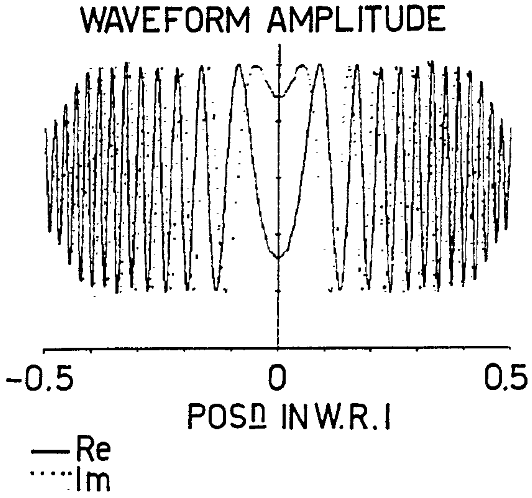


Fig.11F.

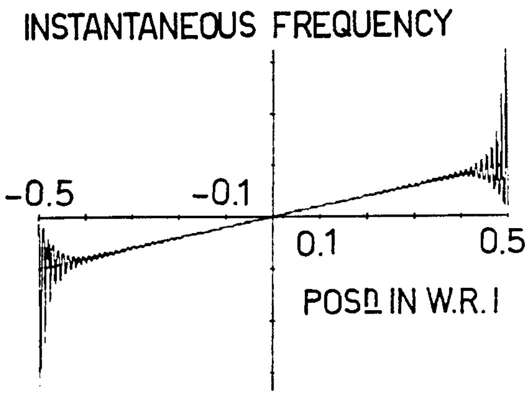


Fig.11G.

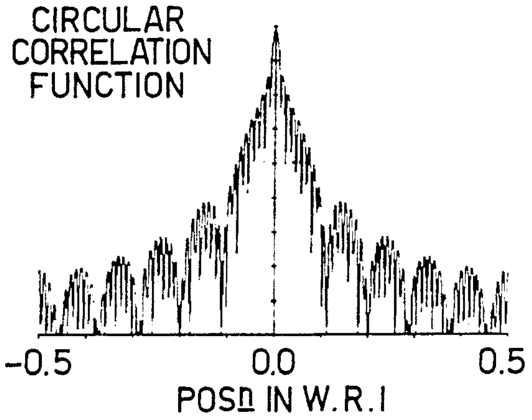


Fig.11H.

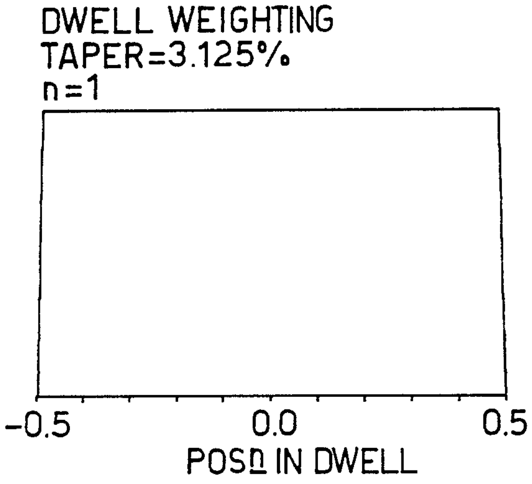


Fig.12.

		dB		
TB	n_{opt}	Peak	Mean	P/M
21	0.76	1.116	-0.669	1.758
23	0.56	1.176	-0.607	1.783
25	0.31	1.189	-0.555	1.744
27	-0.05	0.808	-0.696	1.504
29	0.48	0.942	-0.645	1.587
31	0.65	1.004	-0.600	1.604
33	0.96	0.666	-0.714	1.380
35	0.65	0.676	-0.669	1.345
37	0.19	0.724	-0.631	1.355
39	0.48	0.515	-0.726	1.241
41	0.36	0.563	-0.687	1.250
43	0.06	0.609	-0.653	1.262
45	0.26	0.630	-0.621	1.252
47	0.72	0.417	-0.700	1.117
49	0.58	0.453	-0.669	1.122
51	0.40	0.476	-0.641	1.117
53	0.96	0.311	-0.710	1.022
55	0.90	0.350	-0.682	1.033
57	0.80	0.374	-0.656	1.031
59	1.28	0.270	-0.718	0.989
61	1.12	0.273	-0.693	0.967
63	1.06	0.300	-0.669	0.969
65	1.02	0.318	-0.647	0.965
67	1.40	0.257	-0.702	0.959
69	1.32	0.255	-0.680	0.935
71	1.26	0.257	-0.659	0.916
73	1.62	0.230	-0.709	0.940
75	1.58	0.239	-0.688	0.928
77	1.50	0.246	-0.669	0.916
79	1.86	0.223	-0.715	0.938
81	1.76	0.225	-0.696	0.921
83	1.70	0.227	-0.678	0.905
85	1.66	0.227	-0.661	0.888
87	2.02	0.217	-0.703	0.920
89	1.98	0.216	-0.685	0.902
91	1.90	0.221	-0.669	0.890
93	2.16	0.213	-0.708	0.922
95	2.14	0.216	-0.692	0.909
97	2.14	0.217	-0.677	0.894
99	1.92	0.210	-0.713	0.923

Fig.14.

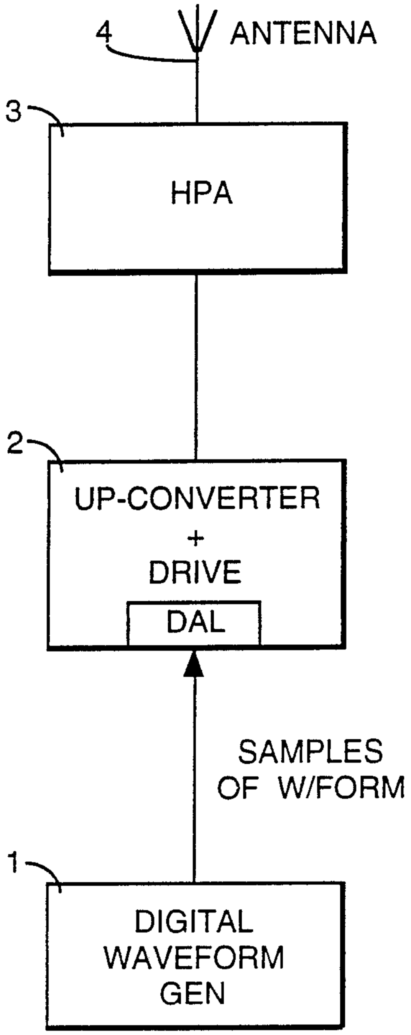
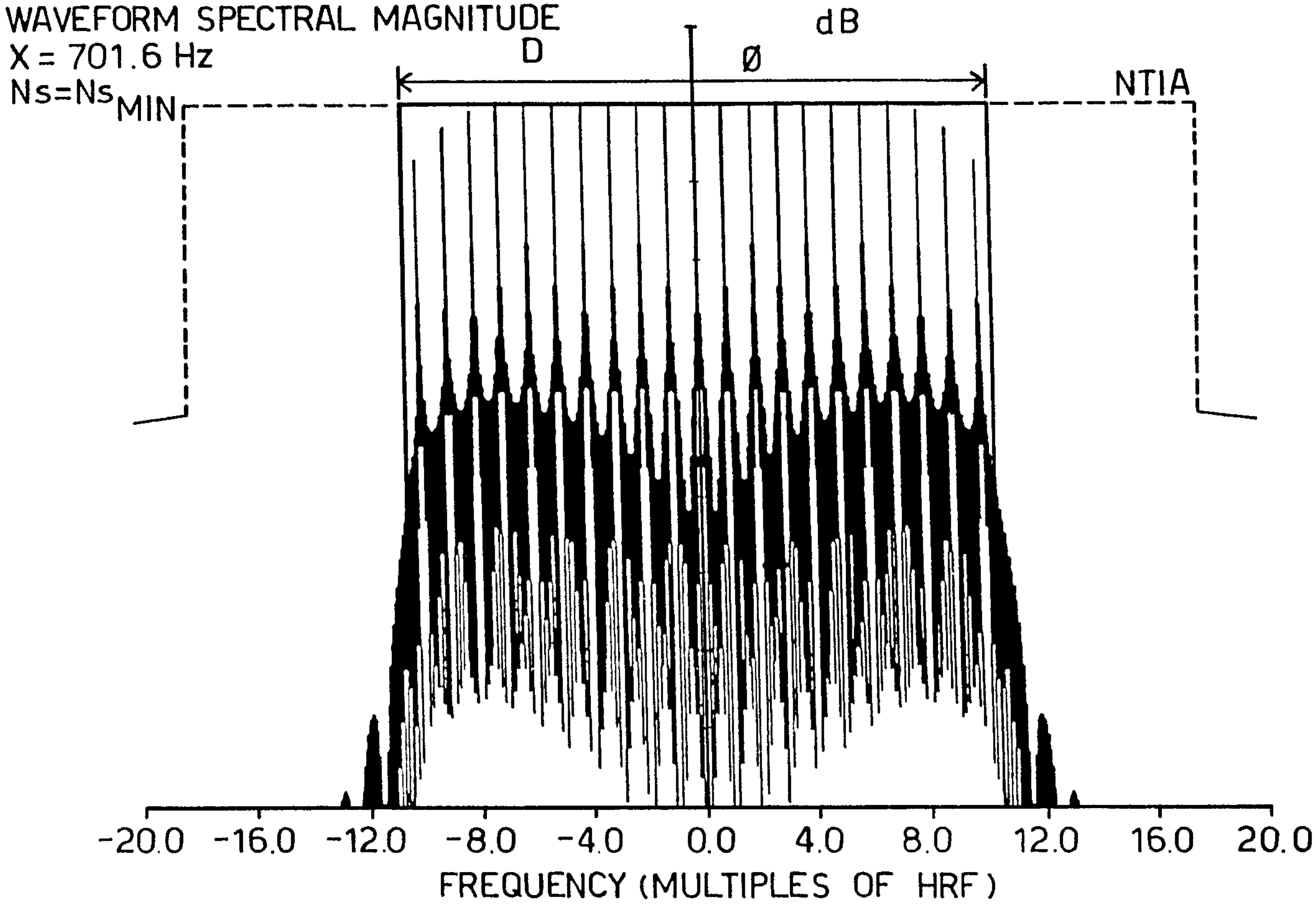


Fig.13.


WAVEFORM SPECTRAL MAGNITUDE
X = 701.6 Hz
Ns=Ns$_{MIN}$
dB
D
0
NTIA
-20.0
-16.0
-12.0
-8.0
-4.0
0.0
4.0
8.0
12.0
16.0
20.0
FREQUENCY (MULTIPLES OF HRF)

SIGNAL PROCESSING APPARATUS AND METHOD

This invention relates to a signal processing apparatus and method and in particular to apparatus and methods for use in rf transmission systems operating with Frequency modulated continuous wave (FMCW) type waveforms.

The invention enables a waveform, for use in a radar system for example, to be generated which is optimized for peak power limited transmission systems by minimising the peak to mean ratio whilst simultaneously providing exceptionally good out-of-band emission characteristics and conventional radar waveform properties. Consequently, the method is very suitable for use with non-linear transmission systems, for example those employing a high power amplifier in compression which are required to comply with tight out-of-band spectral emission specifications.

In a conventional radar transmit sub-system FMCW waveform generation can be accomplished by either direct rf synthesis or baseband synthesis followed by up conversion. Either technique can be implemented in analogue or digital technology. The disadvantages of rf synthesis are two fold. Firstly, the waveform flexibility is limited by either the precision in the analogue components, for analogue systems, or the computational rate for digital systems. Secondly, the dynamic range and/or linearity of such implementations are frequently technology limited.

The disadvantage of baseband synthesis followed by up-conversion is that the precision generated in a baseband synthesized waveform is in practice difficult to preserve in the up-convertion and amplification stages because analogue mixers are by definition non-linear devices and for reasons of efficiency high power amplifiers often operate with some compression.

The problem therefore is how to preserve the fidelity of a precision generated waveform through a system with a non-linear transfer function, when, as is well known, passage of a bandlimited signal through a non-linearity results in out-of-band emission and in-band distortion. Out-of-band emission wastes power and causes interference with other users of the band and often international regulations governing the extent of this exist while in-band distortion results in a degradation or loss of the waveforms properties.

Fundamentally, an ideal transmitted waveform must be simultaneously power efficient, bandwidth efficient and preserve the integrity of the general waveforms natural properties. To explain these subtle ideals a little further:

A maximally 'power efficient' waveform is one in which the ratio of the peak to mean temporal magnitude envelope is unity.

A 'bandwidth efficient' waveform is one in which all of the power transmitted is uniformly 'contained' within the sweep/receiver bandwidth. Power transmitted out-of-band may not only interfere with other users but, from the radar point of view, is a system loss - since it serves no useful purpose whatsoever in the process of target detection.

Preservation of a waveforms natural properties is taken to mean minimisation of in-band distortion for both amplitude and phase. In this way, a waveform which has very good correlation properties is not compromised by the transmission path. It is often the case that a waveform with, say, very low time-sidelobes in correlation is very sensitive to phase distortions or bandwidth mismatch.

This invention provides a signal processing method and apparatus for producing and employing waveforms coming closer to this ideal than has previously been possible.

In a first embodiment this invention provides signal processing apparatus employing a waveform having a waveform repetition interval and bandwidth related such that the product of the waveform repetition interval and the bandwidth is an integer.

Preferably the waveform comprises a plurality of identical sweeps, each sweep having a waveform repetition interval and bandwidth such that the product of the waveform repetition interval and the bandwidth is an integer and may further comprise a series of dwells each comprising a plurality of identical sweeps.

Advantageously a window function can be applied at the start and finish of each dwell in order to remove the boundary discontinuity at the start and finish of the dwell. A suitable window function is a Tukey window function.

Advantageously the frequency spectrum of the waveform can be weighted, preferably by applying a window function such a Tukey window function to it.

In a second embodiment this invention provides a window function comprising a $\cos^n x$ function in which n is non zero and positive convolved with an extending function having a constant non-zero value across a single continuous range and a value of zero elsewhere.

In this case a window function Wz can be used defined by the equation;

where

$$W_z = \frac{1}{Cg} \cdot \sum_{\alpha=-X}^{X} \overset{n}{G_x} \, E_{z-x} \; : \; z = -(x+y),\ldots,(x+y)$$

$$\overset{n}{G_x} = \cos^n\left(\frac{X\pi}{1+k}\right) \; : \; x = -X,\ldots,X \;\; n \geq O$$

$$Ey = 1 \qquad ; y = -Y,\ldots,Y$$

$$Cg = \frac{1}{N} \sum_{z=-(X+Y)}^{(X+Y)} W_z$$

Preferably the window function is the square root of the result of the convolution. In this case a window function Wz can be used defined by the equation;

where

$$W_z = \sqrt{\frac{1}{Cg} \cdot \sum_{\alpha=-X}^{X} \overset{n}{\underset{x}{G}} \, E_{z-x}} \quad :z=-(x+y),\ldots,(x+y)$$

$$\overset{n}{\underset{x}{G}} = \cos^n\left(\frac{X\pi}{1+k}\right) \quad : x = -X,\ldots,X \; n \geq O$$

$$Ey = 1 \qquad ;y = -Y,\ldots,Y$$

$$Cg = \frac{1}{N} \sum_{z=-(X+Y)}^{(X+Y)} W_z$$

A particularly advantageous arrangement of signal processing apparatus provided by the first embodiment of the invention is to weight the waveform frequency spectrum in such signal processing apparatus with a window function provided by the second embodiment of the invention. When this is done the value of the index n in the window function can be selected to minimise the peak to mean ratio of the waveform, the preferred way of doing this being to select the value of the index n such that the first and second Fresnel peaks in the waveform envelope magnitude function have equal magnitude.

In a third embodiment this invention provides a method of generating a waveform by defining the waveform repetition interval (T) and bandwidth (B) such that T.B. = an integer. Preferably the waveform is generated by then evaluating the equation

$$g(t) = (1\sqrt{TB}). \sum_{n=-M}^{M} G_n . e^{\, j(an2 \,+\, bn \,+\, c)}$$

$$\textit{where} \quad M = (TB-1)/2$$

$$\left\{ G_n = 1 \right\}_{n=-M}^{M}$$

Advantageously the variables a, b and c can be defined as;

$$a = \Pi/TB$$

$$b = 2\Pi(t/T)$$

and

$$c = \frac{\Pi}{4}\left[-1^{\frac{(TB+1)}{2}} + 2TB + 1\right] \textit{ for phase continuity (TB odd case only)}$$

In a fourth embodiment this invention provides a signal processing method employing a waveform having a waveform repetition interval and bandwidth related such that the product of the waveform repetition interval and the bandwidth is an integer.

Preferably the waveform comprises a plurality of identical sweeps, each sweep having a waveform repetition interval and bandwidth such that the product of the waveform repetition interval and the bandwidth is an integer and may further comprise a series of dwells each comprising a plurality of identical sweeps.

Advantageously a window function can be applied at the start and finish of each dwell to remove the boundary discontinuity at the start and finish of the dwell. A suitable window function is a Tukey window function.

Advantageously the frequency spectrum of the waveform can be weighted, preferably by applying a window function such a Tukey window function to it.

It is particularly advantageous for a signal processing method according to the fourth embodiment of the invention to weight the waveform frequency spectrum with a window function provided by the second embodiment of the invention.

Apparatus and methods employing the invention will now be described by way of example only with reference to the accompanying diagrammatic figures in which;

Figure 1 shows the frequency spectrum of a typical conventionally generated waveform;

Figure 2 shows the frequency spectrum of a waveform produced employing the invention;

Figures 3A to 3D show the frequency spectra of waveforms produced conventionally and employing Tukey windows;

Figures 3E to 3H respectively show the frequency spectra of waveforms corresponding to those in Figures 3A to 3D respectively and produced employing the invention and Tukey windows;

Figures 4A to 4D are explanatory diagrams showing Tukey dwell weighting functions; and

Figures 4E to 4H respectively show the waveform spectral occupancies produced by the Tukey dwell weighting functions of Figures 4A to 4D respectively.

Figures 5A to 5D are explanatory diagrams showing Tukey spectral weighting functions having different values of taper;

Figures 5E to 5H respectively show the waveform envelopes corresponding to the Tukey functions of Figures 5A to 5D respectively;

Figure 6 shows a full definition of the new window function according to the invention;

Figures 7A to 7D show examples of the new window function for different values of n and having 100% taper;

Figures 7E to 7H respectively show the spectral magnitudes corresponding to the windows shown in Figures 7A to 7D respectively;

Figures 8A to 8D show the new window function for different values of n with 50% taper;

Figures 8E to 8H respectively show the spectral magnitudes corresponding to the window functions shown in Figures 8A to 8D respectively;

Figures 9A to 9D show the $\cos^n$Tukey spectral weighting function with 20% taper for different values of n;

Figures 9E to 9H respectively show the waveform envelope magnitude functions corresponding to the spectral weighting functions of Figures 9A to 9D respectively;

Figures 10A to 10D show the square root $\cos^n$Tukey spectral weighting function for 20% taper different values of n;

Figures 10E to 10H respectively show the waveform envelope magnitude functions corresponding to the spectral weighting functions of Figures 10A to 10D respectively;

Figures 11A to 11H show different waveform characteristics for an optimised square root $\cos^n$Tukey spectral related waveform;

Figure 12 is a table of some optimum values of index n for various waveform parameters.

Figure 13 shows the spectral occupancy of an optimised waveform; and

Figure 14 shows signal processing apparatus for use in a radar system employing the invention.

In conventional waveform generation over a time interval (T) a bandwidth (B) is swept in a linear manner, i.e. $f = f_0 + Bt/T$.

Since frequency can be defined as the rate of change of phase with respect to time, i.e. $f = d\phi/dt$ the phase ϕ of a time-domain signal g'(t) is obtained by integration.

Hence

$$g'(t) = A(t).e^{j2\Pi \left(\int_0^t d\phi/dt \; dt\right)}$$

$$g'(t) = A(t).e^{j2\Pi(f\,t + (TB/2)\,.\,(t/T)2)} \; for \; t < T/2$$

where, A (t) is some amplitude scaling factor (usually 1).

It is a popular misconception that the spectral occupancy of a linear frequency modulated waveform (LFM), commonly known as a chirp, contains just those frequencies of the swept bandwidth. It does not. Such a signal is a time-limited signal and it is a physical fact that 'if a signal is time-limited it cannot be simultaneously bandlimited and vice versa'.

Consequently, the digital representation of a chirp suffers from aliasing unless oversampled by a considerable amount.

A radar or sonar often uses a waveform termed a "dwell" and comprising a number of sweeps. In such a case the dwell waveform g"(t) (also time limited) is obtained by replicating the time-limited signal g'(t) at intervals of T

$$g''(t) = g'(t) * \sum_{k=o}^{Ns-1} \delta(t-kT)$$

where * represents convolution

Ns = Number of sweeps in a dwell

T = waveform repetition interval (WRI)

Such a signal (depending on the values of T and B) may or may not be continuous, i.e. it may not or may posses discontinuities in either amplitude or phase or their derivatives at the WRI boundaries.

In the optimum case, the waveform is continuous and as a result the frequency spectrum or spectral magnitude has a line structure with a line spacing dependent on the dwell time rather than the WRI. However in the strict sense this is still not bandlimited as there are an infinite number of these lines. Hence the problem in digital representation. An example of such a frequency spectrum is shown in Figure 1.

In Figure 1 it can be seen that the spectral occupancy of the waveform extends far beyond that of the design bandwidth B. It can be further seen that the spectral line peaks in-band are of different magnitude due to the Fresnel ripples. This also is undesirable because it increases the peak to mean ratio of the waveform, making it less power efficient.

In the present invention the method of waveform definition and generation is completely different from that of the conventional approach hereinbefore described, and is described below as a series of steps. Although the greatest benefits can be obtained by use of all the steps it must be emphasised that some benefit can be obtained by using only some or even only one of them in isolation from the others.

Step 1

Define a relationship between the waveform parameters T and B, i.e.

$$T \times B = \text{integer}$$

In this way, the waveform is defined as true 'periodic bandlimited waveform' containing no discontinuities at the WRI boundaries. Such signals are entirely defined by a line spectrum comprising a finite set of lines rather than an infinite set. Consequently, the waveform can be defined not only in the instantaneous frequency-domain (as is the case of the conventional chirp) but also in the normal frequency domain as comprising a finite set of unit spectral lines of 1/WRI spacing, that is spaced at the waveform repetition frequency, quadratically phased. As a result the waveform can be exactly represented by digital samples and the waveform can be precisely defined in the frequency domain even though it is time limited. In order to exactly represent a conventional (non-bandlimited) chirp waveform digitally it would theoretically be necessary to have an infinite sampling rate. In practice a sampling rate high enough to give acceptable results is used but a trade off between accuracy of representation and sampling rate must always be made.

The number of spectral lines in the design bandwidth is T.B.

This is the only way of defining a waveform that is perfectly bandlimited and this provides great advantages in baseband synthesis because it allows the synthesised waveform to be an exact representation of the desired waveform.

Step 2

The basic waveform is obtained in the time-domain by means of a inverse finite Fourier series (or polynomial). That is to say:

$$g(t) = (1\sqrt{TB}).\sum_{n=-M}^{M} G_n.e^{j(an2 + bn + c)}$$

$$\text{where} \quad M = (TB-1)/2$$

$$a = \Pi/TB$$

$$b = 2\Pi(t/T)$$

$$c = \frac{\Pi}{4}\,[-1\frac{^{TB+1}}{2} + 2TB + 1] \textit{ for phase continuity (TB odd case only)}$$

$$\left\{ G_n = 1 \right\}_{n=-M}^{M}$$

Since the above equation is expressed in terms of the continuous variable 't' it is valid for all values of time.

Therefore a dwell waveform g" (t) can be evaluated directly without the need to replicate a WRI waveform g'(t) as in the conventional case. Although this could be done if there was some practical advantage in doing so.

Furthermore, since the function is exactly bandlimited the approach lends itself to digital synthesis.

Other values for the variables a, b and c can be selected to allow other phase-coded waveforms to be generated by this method if desired.

Waveforms generated in this way can be 100% power efficient when they are not amplitude modulated since they can have a peak to mean ratio of 1 and all of the transmitted power can be contained within a set design bandwidth. It is possible to trade off bandwidth against power efficiency because as the bandwidth is reduced the power efficiency will drop and vice-versa since power transmitted outside the designed bandwidth is wasted in a radar or sonar system.

Step 3

Digital baseband samples of the waveform are simply obtained by evaluating the above equation at discrete values in time i.e. replace the continuous variable 't' with k .

So long as the sampling frequency $(1/\tau)$ is greater than the design bandwidth B, Nyquist's rule is satisfied and the samples will be an exact representation, in the sampling theorem sense, of the waveform. In contrast samples taken at the same rate of a conventional time-domain generated chirp can never be an exact representation due to the infinite member of lines in the frequency spectrum.

In radar and sonar systems the received and transmitted signals are correlated in order to identify where the received signal has been returned from, the fact that the digital samples of the waveform produced according to the invention are an exact representation of the waveform improves the temporal robustness of the waveforms when correlated.

Figure 2 illustrates the basic spectral occupancy characteristics of the polynomal waveform evaluated from the above equation for TB = 5 and N_s = 8.

When compared with the conventional case of Figure 1 it can be seen that the number of lines is finite as expected and the out-of-band emission is entirely determined by the sidelobes of the sinc due to the finite dwell. Furthermore the in-band lines are of uniform magnitude.

Unlike the conventional waveform the number of sweeps in a dwell directly affects the out-of-band emission levels since the width of the sinc is inversely proportional to the dwell-time.

Consequently, for a given sidelobe roll-off rate, a doubling in Ns will double the roll-off rate. This is a very important feature since most practical radar or sonar waveforms comprise a relatively large number of sweeps per dwell (typically 256) and as a result will automatically have a very high sidelobe roll of rate, reducing out of band emissions.

Both Figure 1 and 2 illustrate a typical spectral template for out-of-band emission levels to allow simple comparison between the conventional and inventive waveforms.

Step 4

Since the out-of-band emission level is entirely determined by the finite dwell-time, further improvements in out-of-band suppression can be achieved by application of a window function across the dwell.

However, in order to minimise the weighting loss the essential requirement is to minimise the discontinuity at the start and end of the dwell. This can be accomplished by means of a Tukey window function, that is a window function having a raised cosine start and end taper of small percentage taper. Weighting loss is defined as;

Weighting loss (db) =

$$10 \log \frac{1}{N} \sum_{k=0}^{N-1} W_k$$

Figures 3A to 3D illustrate the effect of application of a 3,125% taper Tukey window on a conventional waveform for $N_s = 2, 4, 8$ and 16 respectively while figures 3E to 3H respectively show the corresponding polynomial waveforms according to the invention. In the case of the conventional waveforms it will be noted that the peaks of the out of band lines do not decay with increasing Ns. Whereas in the case of the new waveforms the increased decay rate is dramatic. Thus the invention gives a very considerable improvement in out-of-band suppression for minimal weighting loss, (.14db).

Since the raised cosine end tapers of the Tukey window place the discontinuity into the second derivative the decay rate is .18dB/octave increasing by .6db/octave for each doubling of Ns.

Whilst increasing the percentage taper also improves the situation this is considered undesirable since the weighting loss will become more significant. The effects of increasing percentage taper are shown in Figures 4. Figures 4A to 4D show the dwell weighting functions for Tukey windows with 0%, 10%, 20% and 40% taper respectively while Figures 4E to 4H respectively show the corresponding waveform spectral occupancies for the inventive waveform with $N_s = 4$.

Step 5

Having improved the out-of-band emission levels by dwell weighting a polynomal waveform consideration is now given to simultaneously improving the peak to mean ratio.

In the time-domain the magnitude of g(t) contains the Fresnel ripples - as illustrated by

Figures 5E to 5H.

These ripples (in excess of 3db relative to the mean) for a peak power limited system are a problem since to avoid saturation (or clipping) it is necessary to reduce the effective transmitted power.

Consequently, it is very desirable to minimise this ratio by some means. It has been found that the application of spectral tapering has this effect.

Figures 5 illustrate the effect of application of a Tukey windows of 0, 10, 20 and 30% taper on the design spectral magnitude {Gn} on the time-domain amplitude envelope. Figures 5A to 5D show the Tukey spectral weighing functions and Figures 5E to 5H the corresponding time domain waveform envelope magnitude functions for Tukey windows of 0, 10, 20 and 30% taper respectively. It can be seen that as the % taper is increased there is a corresponding decrease in the peak-to-mean ratio.

Step 6

A further extension of this idea is to change the raised cosine shape of the end taper so as to minimise the effect of the taper on the in-band signal.

In order to do this a new window function has been invented dubbed $\cos^n$ - Tukey.

This window function is generated by convolving a $\cos^n x$ function with what is termed an 'extending function' and a full definition of the new window function is given in Figure 6.

The ratio of the number of elements in the generating window to extending function determines the % taper whereas the index 'n' determines the shape of the taper.

It will be realised that the $\cos^n$-Tukey window function can be used generally in any application where window functions are employed, but as will be explained it is particularly advantageous in conjunction with waveforms produced by the above method.

Figures 7A to 7D illustrate the $\cos^n$-Tukey window with 100% taper for n = 0, 1, 2 and 3 respectively while Figures 7E to 7H respectively show the corresponding spectral magnitudes.

It will be noted that when n = 0 a triangular window results.

When n = 1 a raised consine window results. And for every integer increase in 'n' the sidelobe roll-off rate increases by .6dB/octave.

Figures 8A to 8H illustrate the same scenario as in Figures 7 for the $\cos^n$-Tukey window with 50% taper.

Figures 9A to 9D show the $\cos^n$Tukey spectral weighting function with 20% taper for n=0, 1, 2 and 3 respectively while Figures 9E to 9H respectively show the corresponding waveform envelope magnitude functions.

Application of $\cos^n$Tukey window to the design spectral magnitude for a fixed 20% taper and n = 0, 1, 2 and 3 is shown in Figures 9. It can be seen that the shape of the window affects the peak-to-mean ratio as well as the sidelobe roll of rate.

A further step in this process is to take the square root of the window to produce a window function dubbed the square root $\cos^n$ Tukey window function. The effect of this is shown in Figures 10. Figures 10A to 10D show the square root $\cos^n$ Tukey spectral weighting function with 20% taper for n = 0, 1, 2 and 3 respectively while figures 10E to 10H respectively show the corresponding waveform envelope magnitude functions.

This has the effect of leaving more energy in-band but it also reveals a feature which is further exploited. If Figure 10E is examined it can be seen that a second peak emerges which in this case is larger than the first. In Figure 10F when n=1 this second peak is still visible but is now smaller than the first.

From this observation is concluded a very important point:

Minimum peak-to-mean ratio occurs when the size of the first and second amplitude envelope peak are equal. In other words the energy in the first peak which dominates the peak-to-mean ratio is equal split between two peaks. For the case illustrated in Figures 10 this implies that there is an optimum value of 'n' which exists somewhere between 0 and 1.

It turns out that it exists at n = 0.4601. Figures 11 illustrate the complete waveform characteristics for this case.

With reference to Figures 11;

11A Design spectral magnitude with 20% square root cos$^{n = 0.4601}$ - Tukey Window.

11B Design quadratic phase.

11C Amplitude envlope of waveform in the time-domain illustrating equal peaks.

11D Corresponding time-domain phase of waveform - notice also quadratic.

11E Real and Imaginary components of waveform illustrating generated chirp.

11F Corresponding instantaneous frequency response.

11G Circular correlation function of waveform

- Note that since it was a window applied to the spectral magnitude in waveform design the circular correlation function is the transform of window. Hence .18dB/oct are time sidelobe roll off rate.

11H Dwell weighting function 3.125% Tukey.

The optimum value of the index 'n' to minimise peak to mean ratio is a function of TB and the percentage spectral taper and can be computed beforehand so that it can be used as a look-up table or could be calculated in real time for each desired waveform. Figure 12 is a table of some optimal values of 'n' found by computer optimisation for TB's in the range 20 to 100 for the case of 30% spectral taper.

Figure 13 illustrates the spectral occupany of an optimized waveform for TB=21, Ns = 64. It can be seen that in comparison to the conventional chirp illustrated in Figure 1 there is a dramatic improvement in the out-of-band emission levels. Minimum peak-to-mean (i.e. <0.9dB) provides an optimal compromise between bandwidth efficiency and power efficiency and minimises the effect of non-linearities.

Figure 14 shows signal processing apparatus for use in a radar system employing waveforms produced using the techniques described above. The waveforms are produced by a digital waveform generator 1 and supplied as a series of baseband samples spaced to form an exact replica of the waveform to an upconverter and drive 2. The r.f. signal from the upconverter 2 is supplied to a high power amplifier 3 which is connected to an antenna 4 for transmission.

In practical apparatus of this type the upconverter 2 and amplifier 3 will be non-linear and the use of the inventive waveform definition techniques described above allow waveforms minimising the effects of these non-linearities to be produced.

CLAIMS

1. Signal processing apparatus employing a waveform having a waveform repetition interval and bandwidth related such that the product of the waveform repetition interval and the bandwidth is an integer.

2. Signal processing apparatus as claimed in claim 1 in which the waveform comprises a plurality of identical sweeps, each sweep having a waveform repetition interval and bandwidth such that the product of the waveform repetition interval and the bandwidth is an integer.

3. Signal processing apparatus as claimed in Claim 2 in which the waveform comprises a series of dwells, each comprising a plurality of identical sweeps.

4. Signal processing apparatus as claimed in claim 3 in which a window function is applied at the start and finish of each dwell.

5. Signal processing apparatus as claimed in claim 4 in which the window function is a Tukey window function.

6. Signal processing apparatus as claimed in any preceding claim in which the frequency spectrum of the waveform is weighted.

7. Signal processing apparatus as claimed in claim 6 in which the waveform frequency spectrum is weighted by applying a window function to it.

8. Signal processing apparatus as claimed in claim 7 in which the window function used is a Tukey window function.

9. A window function comprising a $\cos^n x$ function in which n is non zero and positive convolved with an extending function having a constant non-zero value across a single continuous range and a value of zero elsewhere.

10. A window function as claimed in claim 9 in which n is 1.

11. A window function as claimed in claim 9 or 10 where the window function Wz is defined by the equation;

$$W_z = \frac{1}{Cg} \cdot \sum_{x=-X}^{X} G_x^n E_{z-x} \quad :z=-(x+y),\ldots,(x+y)$$

where

$$G_x^n = \cos^n\left(\frac{x\pi}{1+k}\right) \quad : x = -X,\ldots,X \;\; n \geq O$$

$$Ey = 1 \qquad ;y = -Y,\ldots,Y$$

$$Cg = \frac{1}{N} \sum_{z=-(X+Y)}^{(X+Y)} W_z$$

12. A window function as claimed in claim 9 or claim 10 in which the square root of the

result of the convolution is used as the window function.

13. A window function as claimed in claim 12 where the window function Wz is defined by the equation;

$$W_z = \sqrt{\frac{1}{Cg} \cdot \sum_{\alpha=-X}^{X} \overset{n}{G}_x E_{z-x}} \quad :z=-(x+y),\ldots,(x+y)$$

where

$$\overset{n}{G}_x = \cos^n\left(\frac{x\pi}{1+k}\right) \quad : x = -X,\ldots,X \; n \geq O$$

$$Ey = 1 \quad ;y = -Y,\ldots,Y$$

$$Cg = \frac{1}{N} \sum_{z=-(X+Y)}^{(X+Y)} W_z$$

14. Signal processing apparatus employing a window function as claimed in any one of claims 9 to 13.

15. Signal processing apparatus as claimed in claim 7 in which the waveform frequency spectrum is weighted by a window function as claimed in any one of claims 9, 10 or 12.

16. Signal processing apparatus as claimed in claim 7 in which the waveform frequency spectrum is weighted by a window function as claimed in claim 11 or claim 13.

17. Signal processing apparatus as claimed in claim 16 which employs a waveform spectrally weighted by the application of a window function in which the value of the index n is selected to minimise the peak to mean ratio of the waveform.

18. Signal processing apparatus as claimed in claim 17 in which the value of the index n is selected to make the first and second Fresnel peaks have equal magnitude.

19. A method of generating a waveform by defining the waveform repetition interval (T) and bandwidth (B) such that T.B. = an integer.

20. A method of generating a waveform as claimed in claim 19 in which the waveform is generated by evaluating the equation.

$$g(t) = (1\sqrt{TB}).\sum_{n=-M}^{M} G_n.e^{j(an2 + bn + c)}$$

$$where\quad M = (TB-1)/2$$

$$\left\{G_n=1\right\}_{n=-M}^{M}$$

21. A method of generating a waveform as claimed in claim 20 in which

$$a = \Pi/TB$$

$$b = 2\Pi(t/T)$$

and

$$c = \frac{\Pi}{4}\left[-1^{\frac{TB+1}{2}} + 2TB + 1\right] \textit{ for phase continuity (TB odd case only)}$$

22. A signal processing apparatus employing a waveform generated by the method of any one of claims 19 to 21.

23. A signal processing method employing a waveform generated by the method of any one of claims 19 to 21.

24. A signal processing method employing a waveform having a waveform repetition interval and bandwidth related such that the product of the waveform repetition interval and the bandwidth is an integer.

25. A signal processing method as claimed in claim 24 in which the waveform comprises a plurality of identical sweeps, each sweep having a waveform repetition interval and bandwidth such that the product of the waveform repetition interval and the bandwidth is an integer.

26. A signal processing method as claimed in claim 25 in which the waveform comprises a series of dwells each comprising a plurality of identical sweeps.

27. A signal processing method as claimed in claim 26 in which a window function is

applied at the start and finish of each dwell.

28. A signal processing method as claimed in claim 27 in which the window function is a Tukey window function.

29. A signal processing method as claimed in any one of claims 24 to 28 preceding claim in which the frequency spectrum of the waveform is weighted.

30. A signal processing method as claimed in claim 29 in which the waveform frequency spectrum is weighted by applying a window function to it.

31. A signal processing method as claimed in claim 30 in which the frequency spectrum is weighted by a Tukey window function.

32. A signal processing method employing a window function as claimed in any one of claims 9 to 13.

33. A signal processing method as claimed in claim 30 in which the waveform frequency spectrum is weighted by a window function as claimed in any one of claims 9, 10 or 12.

34. A signal processing method as claimed in claim 30 in which the waveform frequency spectrum is weighted by a window function as claimed in claim 11 or claim 13.

35. A signal processing method as claimed in claim 34 which employs a waveform

spectrally weighted by the application of a window function in which the value of the index n is selected to minimise the peak to mean ratio of the waveform.

36. A signal processing method as claimed in claim 35 in which the value of the index n is selected to make the first and second Fresnel peaks have equal magnitude.

Examiner's report to the Comptroller under Section 17
(The Search report)

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Relevant Technical Fields

(i) UK Cl (Ed.) -

(ii) Int Cl (Ed.) -

Databases (see below)
(i) UK Patent Office collections of GB, EP, WO and US patent specifications.

(ii) ONLINE: WPI, INSPEC, CLAIMS, JAPIO

Search Examiner
DR E PLUMMER

Date of completion of Search
12 JANUARY 1995

Documents considered relevant following a search in respect of Claims :-
1, 19, 24 AT LEAST

Categories of documents

X: Document indicating lack of novelty or of inventive step.

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A: Document indicating technological background and/or state of the art.

P: Document published on or after the declared priority date but before the filing date of the present application.

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Category	Identity of document and relevant passages		Relevant to claim(s)
X	US 4201986	(DUCROCQ) see column 5 lines 14, 37 and 38	1, 19, 24 at least
X	US 3813599	(CAMPBELL) see column 3 lines 39 to 60	1, 19, 24 at least

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