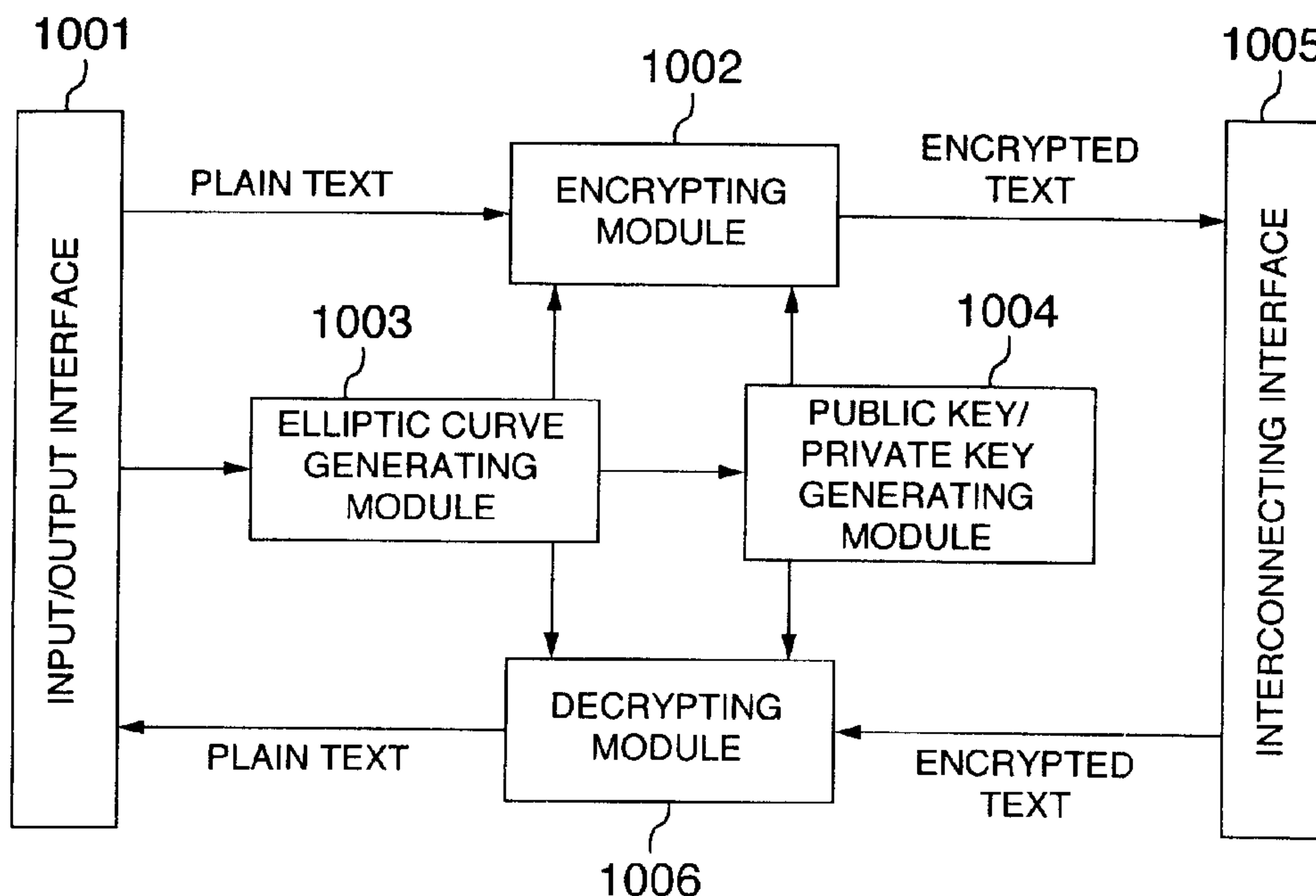




(22) Date de dépôt/Filing Date: 1999/12/20
 (41) Mise à la disp. pub./Open to Public Insp.: 2000/06/22
 (45) Date de délivrance/Issue Date: 2004/02/17
 (30) Priorité/Priority: 1998/12/22 (10-364277) JP

(51) Cl.Int.⁷/Int.Cl.⁷ H04L 9/28, G11B 23/00, H04L 9/30
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(54) Titre : METHODE ET APPAREIL POUR LA CRYPTOGRAPHIE A L'AIDE D'UNE EQUATION ELLIPTIQUE ET MILIEU D'ENREGISTREMENT CONNEXE
 (54) Title: METHOD AND APPARATUS FOR ELLIPTIC CURVE CRYPTOGRAPHY AND RECORDING MEDIUM THEREFOR



(57) Abrégé/Abstract:

A method and an apparatus capable of realizing at a high speed an elliptic curve cryptography in a finite field of characteristic 2, in which the elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ ($b \neq 0$) and an elliptic curve cryptography method which can protect private key information against leaking from deviation information of processing time to thereby defend a cipher text against a timing attack and a differential power analysis attack are provided. To this end, an arithmetic process for executing scalar multiplication arithmetic $d(x, y)$ a constant number of times per bit of the private key d is adopted. Further, for the scalar multiplication $d(x, y)$, a random number k is generated upon transformation of the affine coordinates (x, y) to the projective coordinates for thereby effectuating the transformation $(x, y) \rightarrow [kx, ky, k]$ or alternatively $(x, y) \rightarrow [k^2x, k^3y, k]$. Thus, object for the arithmetic is varied by the random number (k) .

ABSTRACT OF THE DISCLOSURE

A method and an apparatus capable of realizing at a high speed an elliptic curve cryptography in a finite field of characteristic 2, in which the elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ ($b \neq 0$) and an elliptic curve cryptography method which can protect private key information against leaking from deviation information of processing time to thereby defend a cipher text against a timing attack and a differential power analysis attack are provided. To this end, an arithmetic process for executing scalar multiplication arithmetic $d(x, y)$ a constant number of times per bit of the private key d is adopted. Further, for the scalar multiplication $d(x, y)$, a random number k is generated upon transformation of the affine coordinates (x, y) to the projective coordinates for thereby effectuating the transformation $(x, y) \rightarrow [kx, ky, k]$ or alternatively $(x, y) \rightarrow [k^2x, k^3y, k]$. Thus, object for the arithmetic is varied by the random number (k) .

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METHOD AND APPARATUS FOR ELLIPTIC CURVE CRYPTOGRAPHY
AND RECORDING MEDIUM THEREFOR

BACKGROUND OF THE INVENTION

The present invention relates generally to a technique for ensuring security in a computer network. More particularly, the present invention is concerned
5 with a method of realizing an elliptic curve cryptography (encryption/decryption), an apparatus for carrying out the method and a recording medium for storing the same in the form of a program executable with a computer.

The elliptic curve cryptography (encryption/
10 decryption) is one of the public key cryptology algorithms invented by V. Miller and N. Koblitz independently. As the postulation for the public-key cryptograph technology imposed from the viewpoint of security, discovery of a private key on the basis of the
15 counterpart public key laid open to the general public must be made impossible in practice. On the other hand, the public key cryptosystem requires intrinsically a lot of time for encryption and decryption when compared with the private key cryptosystem. Thus, in the present state
20 of the art, there exists a great demand for a high-speed processing technique for enabling encryption and decryption in the public key cryptosystem. Under the circumstances, as the public key cryptograph technique which can satisfy both requirements for the security and
25 the high-speed processing susceptibility which are, so to

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say, contradictory to each other, the elliptic curve cryptography which has more competence for dealing with the above problem than the RSA (Rivest, Shamir & Adleman) cryptography and the ElGamal cryptography both known
5 heretofore is now attracting attention.

The elliptic curve cryptograph can be represented by the standard form of an elliptic curve in a finite prime field, i.e., $y^2 = x^3 + ax + b$ ($4a^3 + 27b^2 \neq 0$) or alternatively by the standard form of an elliptic
10 curve in a finite field of characteristic 2 (which may also be referred to as the extension field of "2"), i.e., $y^2 + xy = x^3 + ax^2 + b$ ($b \neq 0$). By adding a point at infinity to the points on such curve, an Abelian group is made available. In this conjunction, the Abelian group
15 arithmetic will be represented by plus sign (+). Further, in conjunction with the arithmetics for X and Y which differ from each other, "X + Y" will be referred to as the addition arithmetic. Furthermore, "X + X" will be referred to as the doubling arithmetic and represented by
20 "2X".

In order to facilitate computations involved in the elliptic curve cryptography, a point (X, Y) on an elliptic curve in the affine coordinate system may also be expressed in terms of the projective coordinates. At
25 this juncture, let's suppose the projective coordinate system in which $[X, Y, Z] = [\lambda^2 X, \lambda^3 Y, \lambda Z]$ applies valid for a given $\lambda \neq 0$. Then, there can be established such correspondences between the affine coordinates and the

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projective coordinates as mentioned below. Namely, the affine coordinates (x, y) can be represented by the projective coordinates $[x, y, 1]$ while the projective coordinates $[X, Y, Z]$ can be represented by the affine coordinates $(X/(Z)^2, Y/(Z)^3)$. Further, in the projective coordinate system, it applies valid that $-[X, Y, Z] = [X, -Y, Z]$.

In the elliptic curve cryptography, an elliptic curve in a finite field is made use of for making usable a set of points which constitutes a finite field of the elliptic curve. In this conjunction, the order of the elliptic curve is represented by a number of points of the elliptic curve. In the following, the result of addition of "P" \underline{s} times, i.e., $P + P + \dots + P$ where the number of "P" is \underline{s} , will be referred to as the s -multiplied point of "P". When the arithmetic for determining the s -multiplied point of P is represented by " sP ", the order of the point "P" on the elliptic curve is given by $\underline{n} = 112$ which satisfies the conditions that $nP = 0$, $1 \leq m < n$ and $mP \neq 0$.

The key for the elliptic curve cryptography is composed of an elliptic curve, a base point, a public key and a private key. In more concrete, the key of the elliptic curve cryptograph is composed of coefficients \underline{a} and \underline{b} of the elliptic curve, the point P (base point) whose order is a prime number, a finite field element \underline{d} (private key) and a point Q (public key) given by a product of the base point multiplied by the private key

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(i.e., $Q = dp$). Incidentally, it is to be added that the elliptic curve, the base point and the public key are the laid-open information. Further, the public key and the private key assume respective values which differ from
5 one to another user, while the elliptic curve and the base point assume respective values which are common to the users.

In the elliptic curve cryptography, a scalar multiplication (sR) arithmetic for a given point R is
10 adopted for the data encryption, generation of a digital signature and the verification of the digital signature. The scalar multiplication can be realized through combination of the addition arithmetic and the doubling arithmetic mentioned previously. However, computation
15 for each of such addition arithmetic and doubling arithmetic necessarily requires execution of division arithmetic once. In general, division of the finite field takes lots of time. For this reason, efforts have heretofore been paid for establishing such a computation
20 method which can avoid the division arithmetic.

As an approach for evading the division of the finite field, addition arithmetic and doubling arithmetic in the projective space as well as expressions or formulae for realization thereof have already been
25 proposed. For more particulars, reference should be made to D.V. Chudnovsky and G.V. Chudnovsky: "SEQUENCES OF NUMBERS GENERATED BY ADDITION IN FORMAL GROUPS AND NEW PRIMALITY AND FACTORIZATION TESTS", Advances in Applied

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Mathematics, 7. 385-434, 1986. In this conjunction, it is noted that the computation time taken for the prime field multiplication is ordinarily by far longer than that taken for the prime field addition/subtraction.

5 Thus, the overall computation time or overhead can be evaluated on the basis of the number of arithmetic processes involved in the prime field multiplication. In that case, the addition arithmetic requires execution of the prime field multiplication (inclusive of squaring

10 arithmetic) sixteen times. In the doubling arithmetic, the prime field multiplication has to be performed ten times. For more particulars, reference is to be made to the literature cited above. Further, it is reported that for the coefficient a of the elliptic curve, residual

15 multiplication arithmetic has to be performed eight times in the case where $a = -3$.

Further, according to the teachings disclosed in P. Montgomery: "SPEEDING THE POLLARD AND ELLIPTIC CURVE METHODS OF FACTORIZATION", Mathematics of

20 Computation Vol. 48, No. 177, pp. 243-264 (1987), it is reported that when the standard form of an elliptic curve in a finite prime field, i.e., $By^2 = x^3 + Ax^2 + Bx$, is employed for addition of points $P_0(x_0, y_0)$ and $P_1(x_1, y_1)$ as given by $P_3(x_3, y_3)$ and subtraction thereof as given

25 by $P_4(x_4, y_4)$, i.e., when $P_1 + P_0 = P_3$ and $P_1 - P_0 = P_4$, then x_3 can be determined speedily from x_0, x_1, x_4 . In more concrete, it is reported that x_3 can be determined by executing six times the prime field multiplication.

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Further, in the case where the double point of P1 is given by P5(x5, y5), x5 can be determined only from x1 by performing multiplication five times. By taking advantage of this feature, x-coordinate of scalar multiple
 5 (scalar value d) of the point R can be determined from Rx in the manner described below.

Presuming that the initial value is [R, 2R] and that mR represents the x-coordinate of the point R multiplied by m , the scalar value d is exploded or
 10 developed to a bit string in the binary notation. Then, starting from the most significant bit of d , it is validated that [mR, (m+1)R] \rightarrow [2mR, 2(m+1)R] for the bit "0" of d , and [mR, (m+1)R] \rightarrow [(2m+1)R, 2(m+1)R] for the bit "1" of d , where (m+1)R - mR = R and (m+1)R + mR =
 15 (2m+1)R.

In this manner, the scalar multiplication sP can be realized by performing the prime field multiplication (inclusive of squaring) ten times (6 + 5) for each bit. Hereinafter, the procedure or algorithm described
 20 above will be referred to as the Montgomery method.

On the other hand, the standard form of an elliptic curve on the finite field of characteristic 2 (extension field of "2") is given by $y^2 + xy = x^3 + ax^2 + b$ (b \neq 0). For such elliptic curve, the scalar multiplica-
 25 tion arithmetic can be realized through combination of the addition arithmetic and the doubling arithmetic. Rules for the addition arithmetic and the doubling arithmetic are set forth in IEEE: P1363/D2 "STANDARD

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SPECIFICATION FOR PUBLIC KEY CRYPTOGRAPHY" (1998). By resorting to the arithmetic in the finite field of characteristic 2 (extension field of "2"), squaring and addition/subtraction can be realized very speedily when compared with mutually different multiplications. Thus, the computation overhead involved in the arithmetics in the finite field of characteristic 2 can be evaluated by the number of times the mutually different multiplications are to be performed. The addition arithmetic requires execution of multiplication fifteen times while the doubling arithmetic requires execution of multiplication five times. However, it should be noted that in the elliptic curve cryptography based on the finite field of characteristic 2, no arithmetic algorithm is known in which the Montgomery method is resorted to.

For the elliptic curve which can ensure security, it is necessary to set parameters a and b which allow the order $\#E(F_q)$ of the elliptic curve to have a large prime factor r . In the case where the order $\#E(F_q)$ of the elliptic curve is given by kr , the prime factor r can assume a large prime number by selecting a small integer for k . As to the method of setting the parameters of the elliptic curve having a large prime factor r as the order, reference may be made to Henri Cohen: "A COURSE IN COMPUTATIONAL ALGEBRAIC NUMBER THEORY", GTM138, Springer (1993) p. 464, Atkin's Test.

Next, problems of cipher text attack and defense against the attack will be considered. In recent

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years, trials for attacking the cipher text as well as the measures for defending the cipher text against the attacks have been studied. More specifically, as to the attack on the cipher text, there can be mentioned in addition to the classical or theoretical cryptanalysis a differential power analysis (DPA in short) which tries to decode or decrypt the cipher text by processing statistically waveform representing current consumption, a timing attack trying to decode by analyzing statistically differences in the cipher processing time and others which rely on the analyses of leak information. Of course, the measures for defending the cipher against such attacks have also been developed. However, most of the defense measures have been realized primarily by physically incorporating the defense function in hardware circuit itself destined, for example, for IC cards.

The conventional elliptic curve cryptographies described above suffer problems mentioned below. As is apparent from the foregoing, in the elliptic curve cryptography in the finite field of characteristic 2, there is known no arithmetic in which the Montgomery method is adopted. Further, in the studies concerning the elliptic curve cryptographies, importance has been put primarily on the development of high-speed execution methods and generation of such elliptic curve which can ensure security as viewed from the standpoint of cryptanalysis. By contrast, no efforts have been paid to the development of defense technologies for defending the

ciphers against the attack of the leak information analysis type. In the data decryption processing of the elliptic curve cryptology, arithmetic operation for multiplying a point (x, y) on a given elliptic curve by the private key d , i.e., $D(x, y)$, is performed. In that case, deviation information of the private key d may possibly leak, being reflected in the consumed current waveform and the cipher processing time, which will give a clue to the differential power analysis (DPA) attack and the timing attack.

SUMMARY OF THE INVENTION

In the light of the state of the art described above, it is a first object of the present invention to provide an elliptic curve cryptography method which is capable of realizing at a high speed the elliptic curve cryptography in a finite field of characteristic 2 (or extension field of "2"), in which the elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ ($b \neq 0$).

With the present invention, it is also contemplated to provide an apparatus for carrying out the method mentioned above.

A second object of the present invention is to provide an elliptic curve cryptography method which can prevent the private key information from leaking in the form of deviation information of the processing time to thereby defend the cipher text against the timing attack and the differential power analysis (DPA) attack in the

elliptic curve cryptography.

A third object of the present invention is to provide a recording medium which stores the elliptic curve cryptography method in the form of a program or
5 programs which can be executed with a computer.

In view of the first object mentioned above, there is provided according to an aspect of the present invention a method of realizing an elliptic curve cryptography in a finite field of characteristic 2
10 (extension field of "2"), in which the elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ (where $b \neq 0$) and in which addition of points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the elliptic curve composed of points defined by individual coordinate components is presumed to be represented by
15 $P_3(x_3, y_3)$ with subtraction of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ being presumed to be represented by $P_4(x_4, y_4)$. The cryptography method includes a step of inputting the coordinate component x_1 , a step of transforming the inputted coordinate component x_1 into X- and
20 Z-coordinates $[X_1, Z_1]$ of a projective space, a step of storing the coordinates $[X_1, Z_1]$ of the projective space, a step of transforming the coordinate component x_2 into coordinates $[X_2, Z_2]$ of the projective space, a step of storing the projective coordinate $[X_2, Z_2]$, a step of
25 transforming the coordinate component x_4 into coordinates $[X_4, Z_4]$ of the projective space, a step of storing the projective coordinates $[X_4, Z_4]$, a step of determining projective coordinates $[X_3, Z_3]$ from the stored projective

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coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$, a step of transforming the projective coordinates $[X_3, Z_3]$ into the coordinate component x_3 , and a step of outputting the coordinate component x_3 , whereby scalar multiplication of the point $P_1(x_1, y_1)$ is determined. Further, in a preferred mode for carrying out the present invention, the aforementioned step of determining the projective coordinates $[X_3, Z_3]$ susceptible to the transformation into the coordinate component x_3 from the stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ may include a substep of computing $B = X_1Z_2^2 + X_2Z_1^2$, a substep of storing the computed B , a substep of deciding whether or not the stored B satisfies condition that $B = 0$, a substep of outputting a point at infinity when $B = 0$ while arithmetically determining $Z_3 = Z_4B$ unless $B = 0$, a substep of storing the determined Z_3 , and a substep of arithmetically determining $X_3 = X_4B^2 + X_1X_2Z_1^2Z_2^2Z_4^2$ from the stored Z_3 .

Further, for achieving the second object mentioned previously, there is provided according to a second aspect of the present invention, an elliptic curve cryptography method which can positively prevent leakage of the private key information from the deviation information of the processing time in a decryption processing of an elliptical curve cipher in the finite field of characteristic 2. In other words, the present invention also provides a method of realizing an elliptic curve cryptography in a finite field of characteristic 2

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(extension field of "2"), in which the elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ and in which addition of points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the elliptic curve composed of points defined by individual coordinate components is presumed to be represented by $P_3(x_3, y_3)$ with subtraction of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ being presumed to be represented by $P_4(x_4, y_4)$, the method including a step of inputting the coordinate component x_1 , a step of transforming the inputted coordinate component x_1 into X- and Z-coordinates $[X_1, Z_1]$ of a projective space, a step of storing the coordinates $[X_1, Z_1]$ of the projective space, a step of transforming the coordinate component x_2 into coordinates $[X_2, Z_2]$ of the projective space, a step of storing the projective coordinates $[X_2, Z_2]$, a step of transforming the coordinate component x_4 into coordinates $[X_4, Z_4]$ of the projective space, a step of storing the projective coordinates $[X_4, Z_4]$, a step of determining projective coordinates $[X_3, Z_3]$ from the stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$, a step of transforming the projective coordinates $[X_3, Z_3]$ into the coordinate component x_3 , and a step of outputting the coordinate component x_3 , wherein the cryptography method further includes a step of generating a random number \underline{k} , a step of storing the generated random number \underline{k} , and a step of performing arithmetic operation on the individual coordinate components of the projective space and the stored random number \underline{k} after the transformation of the x-

coordinate component to the projective coordinates, to
thereby derive projective coordinates $[k^2x, k]$. In other
words, the method of varying constantly the object for
the arithmetic in the finite field of characteristic 2
5 (extension field of "2") is provided.

In another preferred mode for carrying out the
present invention, the elliptic curve cryptography method
may include a step of generating a random number k , a
step of storing the generated random number k , and a step
10 of performing arithmetic operation on the individual
coordinate components of the projective space and the
stored random number k after the transformation of the x -
coordinate component to the projective coordinates, to
thereby derive projective coordinates $[kx, k]$.

15 Further, for carrying out the elliptic curve
cryptography methods described above, there is provided
according to another aspect of the present invention, an
arithmetic apparatus for realizing an elliptic curve
cryptography in a finite field of characteristic 2
20 (extension field of "2"), in which the elliptic curve is
given by $y^2 + xy = x^3 + ax^2 + b$, which apparatus includes a
random number generation module for generating a random
number k , a projective coordinate transformation which
module receiving as inputs thereto the coordinate x_0 in
25 the finite field of characteristic 2 and the random
number k to thereby transform the coordinate x_0 into
projective coordinates $[kx_0, k] = [X_1, Z_1]$, a doubling
arithmetic module for arithmetically determining a double

point from the projective coordinates $[X_1, Z_1]$, an addition arithmetic module for determining an addition point from the projective coordinate $[X_1, Z_1]$ to output the addition point, and a scalar multiplication module
5 which receives as inputs thereto information from the projective coordinate transformation module, the doubling arithmetic module and the addition arithmetic module to thereby determine scalar multiplication of the coordinate component x_0 .

10 Furthermore, in view of the third object mentioned previously, there is provided according to a further aspect of the present invention a recording medium which stores therein a cryptography method of realizing an elliptic curve cryptography in a finite
15 field of characteristic 2 (extension field of "2"), in which the elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ and in which addition of points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the elliptic curve composed of points defined by individual coordinate components is presumed to be repre-
20 sented by $P_3(x_3, y_3)$ with subtraction of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ being presumed to be represented by $P_4(x_4, y_4)$, the program comprising a step of inputting the coordinate component x_1 , a step of transforming the inputted coordinate component x_1 into X- and
25 Z-coordinates $[X_1, Z_1]$ of a projective space, a step of storing the coordinates $[X_1, Z_1]$ of the projective space, a step of transforming the coordinate component x_2 into coordinates $[X_2, Z_2]$ of the projective space, a step of

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storing the projective coordinates $[X_2, Z_2]$, a step of transforming the coordinate component x_4 into coordinates $[X_4, Z_4]$ of the projective space, a step of storing the projective coordinates $[X_4, Z_4]$, a step of determining
5 projective coordinates $[X_3, Z_3]$ from the stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$, a step of transforming the projective coordinates $[X_3, Z_3]$ into the coordinate component x_3 , and a step of outputting the coordinate component x_3 , whereby scalar multiplication of
10 the point $P_1(x_1, y_1)$ is determined.

The method of realizing the elliptic curve cryptography in the finite field of characteristic 2 mentioned previously can effectively be employed as the measures for preventing leakage of the private key
15 information from the deviation information of the processing time for decrypting an elliptic curve cipher text on a prime field. To this end, according to still further aspect of the present invention, there may be adopted a combination of the arithmetics (a) and (b)
20 mentioned below.

(a) In the case where the standard form of an elliptic curve in a prime field is given by $By^2 = x^3 + Ax^2 + Bx$, the scalar multiplication algorithm according to the Montgomery method is adopted for determining the
25 scalar multiplication $d(x, y)$ of the elliptic curve.

(b) In conjunction with computation for scalar multiplication $d(x, y)$, a random number k is generated upon transformation of the affine coordinates (x, y) into

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the projective coordinates for thereby effectuate the transformation $(x, y) \rightarrow [kx, ky, k]$ or $(x, y) \rightarrow [k^2x, k^3y, k]$.

By virtue of the method mentioned above, the
5 object for arithmetic in the prime field can constantly be varied by the random number.

In accordance with one aspect of the present invention there is provided a method of implementing an elliptic curve cryptography in a finite field of
10 characteristic 2 (or an extension field of "2"), in which said elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ and in which x and y are variables in an x - y coordinate system, a and b are parameters, addition of points $P1(x1, y1)$ and $P2(x2, y2)$ on said elliptic curve
15 composed of points defined by individual coordinate components is presumed to be represented by $P3(x3, y3)$ with subtraction of said points $P1(x1, y1)$ and $P2(x2, y2)$ being presumed to be represented by $P4(x4, y4)$, comprising the steps of: inputting the coordinate
20 component $x1$; transforming said inputted coordinate component $x1$ into X - and Z -coordinates $[X1, Z1]$ of a projective space where Z is a variable in the Z -coordinate; storing said coordinates $[X1, Z1]$ of said projective space; transforming said coordinate component
25 $x2$ into coordinates $[X2, Z2]$ of said projective space; storing said projective coordinate $[X2, Z2]$; transforming said coordinate component $x4$ into coordinates $[X4, Z4]$ of said projective space; storing said projective coordinates $[X4, Z4]$; determining projective coordinates
30 $[X3, Z3]$ from said stored projective coordinates $[X1,$

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$Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$; transforming said projective coordinates $[X_3, Z_3]$ into said coordinate component x_3 ; and outputting said coordinate component x_3 , whereby scalar multiplication of said point $P_1(x_1, y_1)$ is
5 determined.

In accordance with another aspect of the present invention there is provided an apparatus implementing an elliptic curve cryptography in a finite field of characteristic 2 (or an extension field of
10 "2"), in which x and y are variables in an x - y coordinate system, a and b are parameters, said elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$, comprising: random number generating means for generating a random number k ; projective coordinate transformation means
15 receiving as inputs thereto coordinate x_0 of said finite field of characteristic 2 and said random number k , to thereby transform said coordinate x_0 into projective coordinates $[kx_0, k] = [X_1, Z_1]$; doubling arithmetic means for arithmetically determining a double point from
20 said projective coordinates $[X_1, Z_1]$; addition arithmetic means for determining an addition point from said projective coordinate $[X_1, Z_1]$ where Z is a variable in the Z -coordinate to thereby output said addition point; and scalar multiplication means receiving information
25 from said projective coordinate transformation means, said doubling arithmetic means and said addition arithmetic means to thereby perform scalar multiplication of the coordinate component x_0 .

In accordance with yet another aspect of the
30 present invention there is provided a computer-readable

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memory having recorded thereon statements and instructions for use in the execution in a computer of the method of implementing an elliptic curve, said statements for the execution of the above-mentioned method comprising the statements of: inputting a coordinate component x_1 ; transforming said inputted coordinate component x_1 into X- and Z-coordinates $[X_1, Z_1]$ in a projective space; storing said coordinates $[X_1, Z_1]$ of said projective space; transforming said coordinate component x_2 into coordinates $[X_2, Z_2]$ of said projective space; storing said projective coordinate $[X_2, Z_2]$ where Z is a variable in the Z-coordinate; transforming said coordinate component x_4 into coordinates $[X_4, Z_4]$ of said projective space; storing said projective coordinates $[X_4, Z_4]$; determining projective coordinates $[X_3, Z_3]$ from said stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$; transforming said projective coordinates $[X_3, Z_3]$ into said coordinate component x_3 ; and outputting said coordinate component x_3 , whereby scalar multiplication of said point $P_1(x_1, y_1)$ is determined.

In accordance with still yet another aspect of the present invention there is provided a computer-program product, comprising: a memory having computer-readable code embodied therein for implementing an elliptic curve cryptography in a finite field of characteristic 2 (or an extension field of "2"), in which said elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ and in which x and y are variables in an x - y coordinate system, a and b are parameters, addition of

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points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on said elliptic curve composed of points defined by individual coordinate components is presumed to be represented by $P_3(x_3, y_3)$ with subtraction of said points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ being presumed to be represented by $P_4(x_4, y_4)$,
5 comprising: code for inputting a coordinate component x_1 ; code for transforming said inputted coordinate component x_1 into X- and Z-coordinates $[X_1, Z_1]$ in a projective space; code for storing said coordinates $[X_1, Z_1]$ of said projective space; code for transforming said coordinate component x_2 into coordinates $[X_2, Z_2]$ of said projective space; code for storing said projective coordinate $[X_2, Z_2]$ where Z is a variable in the Z-coordinate; code for transforming said coordinate
10 component x_4 into coordinates $[X_4, Z_4]$ of said projective space; code for storing said projective coordinates $[X_4, Z_4]$; code for determining projective coordinates $[X_3, Z_3]$ from said stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$; code for transforming said projective
15 coordinates $[X_3, Z_3]$ into said coordinate component x_3 ; and code for outputting said coordinate component x_3 , whereby scalar multiplication of said point $P_1(x_1, y_1)$ is determined.

Other objects, features and advantages of the
25 present invention will become apparent from the following detailed description of the preferred or exemplary embodiments taken in conjunction with the accompanying drawings.

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BRIEF DESCRIPTION OF THE DRAWINGS

In the course of the description which follows, reference is made to the drawings, in which:

Fig. 1 is a functional block diagram for
5 illustrating processing flows in an elliptic curve
cryptograph system according to an embodiment of the
present invention;

Fig. 2 is a flow chart for illustrating a part
of a scalar multiplication procedure adopted in the
10 elliptic curve cryptography according to a first
embodiment of the present invention;

Fig. 3 is a flow chart for illustrating the
other part of the scalar multiplication procedure
mentioned just above;

15 Fig. 4 is a flow chart for illustrating an
addition procedure adopted in the elliptic curve

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cryptography according to the first embodiment of the present invention;

Fig. 5 is a flow chart for illustrating a doubling arithmetic procedure adopted in the elliptic curve cryptography according to the first embodiment of the present invention;

Fig. 6 is a flow chart for illustrating a part of a scalar multiplication procedure in the elliptic curve cryptography according to a second embodiment of the present invention;

Fig. 7 is a flow chart for illustrating the other part of the procedure mentioned just above;

Fig. 8 is a flow chart for illustrating an addition procedure in the elliptic curve cryptography according to the second embodiment of the invention;

Fig. 9 is a functional block diagram showing schematically a structure of the elliptic curve arithmetic unit of the elliptic curve cryptograph apparatus according to a sixth embodiment of the present invention;

Fig. 10 is a block diagram showing a general configuration of an elliptic curve cryptograph system to which the present invention can be applied;

Fig. 11A is a flow chart for illustrating a part of a scalar multiplication procedure in which Montgomery method is adopted according to a third embodiment of the present invention;

Fig. 11B is a flow chart for illustrating the

other part of the scalar multiplication procedure mentioned just above;

Fig. 12A is flow chart for illustrating a part of a scalar multiplication procedure according to a
5 fourth embodiment of the present invention;

Fig. 12B is flow chart for illustrating the other part of the scalar multiplication procedure mentioned just above;

Fig. 13 is a flow chart for illustrating an
10 addition procedure according to the fourth embodiment of the present invention;

Fig. 14 is a flow chart for illustrating a doubling method according to the fourth embodiment of the invention;

15 Figs. 15A and 15B are a flow chart for illustrating a scalar multiplication procedure according to a fifth embodiment of the present invention;

Fig. 16 is a flow chart for illustrating an addition procedure according to the fifth embodiment of
20 the present invention.

DESCRIPTION OF THE EMBODIMENTS

Now, the present invention will be described in detail in conjunction with what is presently considered as preferred or typical embodiments thereof by reference
25 to the drawings.

General description

First mentioned below are arithmetic algorithm

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or rules for an elliptic curve of the standard form $y^2 + xy = x^3 + ax^2 + b$ ($b \neq 0$) in a finite field of characteristic 2 of the affine coordinate system.

1) $0 + 0 = 0$

5 2) $(x, y) + 0 = (x, y)$

3) $(x, y) + (x, x + y) = 0$

4) Commutativity

$$(x_0, y_0) + (x_1, y_1) = (x_1, y_1) + (x_0, y_0)$$

5) Addition arithmetic

10 $(x_2, y_2) = (x_1, y_1) + (x_0, y_0)$

$$x_2 = a + \lambda^2 + \lambda + x_0 + x_1; y_2 = \lambda(x_1 + x_2) + x_2 + y_1;$$

$$\lambda = (y_0 + y_1) / (x_0 + x_1)$$

6) Doubling arithmetic

$$(x_2, y_2) = (x_1, y_1) + (x_1, y_1) = 2(x_1, y_1)$$

15 $x_2 = a + \lambda^2 + \lambda; y_2 = \lambda(x_1 + x_2) + x_2 + y_1; \lambda = x_1 + (y_1/x_1)$ or $x_2 = (x_1)^2 + b/(x_1)^2$

In order to facilitate the computation for the elliptic curve such as mentioned above, points (X, Y) on the elliptic curve in the affine coordinate system may be transformed to the points expressed in terms of the projective coordinates. At this juncture, let's suppose such projective coordinate system in which $[X, Y, Z] = [\lambda^2x, \lambda^3y, \lambda z]$ applies valid for a given $\lambda \neq 0$. Then, correspondence can be established between the affine coordinates and the projective coordinates as mentioned below. Namely, the affine coordinates (x, y) can be

20

25

- 20 -

expressed by the projective coordinates $[x, y, 1]$ while the projective coordinates $[X, Y, Z]$ can be expressed by the affine coordinates $(X/(Z)^2, Y/(Z)^3)$. Further, in the projective coordinate system, it applies valid that $-[X, Y, Z] = [X, XZ+Y, Z]$.

Now, the concept underlying the present invention will be described by reference to the drawings.

Figure 10 is a block diagram showing a general configuration of an elliptic curve encryption system to which the present invention can be applied. Referring to the figure, reference numeral 1001 denotes an input/output interface for an input device such as a keyboard and/or the like for inputting plain texts to be encrypted and for an output device such a display, a printer and/or the like for outputting plain texts resulting from decryption. The interface 1001 may include a storage unit such as a memory or the like for storing the plain text. For encrypting the plain text as inputted through the input/output interface, there is provided an encrypting module 1002 which is so designed as to receive as the inputs thereto an elliptic curve generated by an elliptic curve generating module 1003 and keys from a public key/private key generating module 1004. At this juncture, it is to be mentioned that the public key and the encryption key are combined in a pair, wherein which of these keys is to be made available for the encrypting module 1002 or the decrypting module 1006 depends on the practical application for which the

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cryptography system is employed, i.e., whether the cryptography system is employed, for example, for the privacy communication or for the signature/authentication communication. The cipher text resulting from the encryption is sent out through the medium of an inter-connection interface 1005. The decrypting module 1006 is designed to decrypt the cipher text into a plain text.

Figure 1 is a functional block diagram for illustrating processing flows in an elliptic curve encryption system according to an embodiment of the present invention. Incidentally, it should be mentioned that the elliptic curve encryption system according to the present invention may be provided in the form of software programmed for executing the elliptic curve cryptography. In that case, the software may be installed in an appropriate information processing apparatus from a recording medium such as a CD-ROM, FD or the like. Referring to Fig. 1, the elliptic curve used for the elliptic curve cryptography is generated by the elliptic curve generating module designated by 101 in this figure. The elliptic curve generated by the elliptic curve generating module 101 is inputted to the public key/private key generating module 102 which responds thereto by generating a public key 115 and a private key 116 on the basis of the elliptic curve as inputted. The encrypting module 103 receives as inputs thereto data of the plain text 113, the public key 115 and the elliptic curve to thereby output a cipher text

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112. On the other hand, the decrypting module 104 is designed to receive as inputs thereto the cipher text 112, the private key 116 and the elliptic curve to thereby output a plain text 114. Needless to say, the
5 plain text 114 outputted from the decrypting module 104 is same as the plain text 113 mentioned previously.

The elliptic curve generating module 101 is designed to generate the elliptic curve in accordance with a processing procedure described below. Through a
10 primitive polynomial setting process or submodule 105, a primitive polynomial $f(x)$ in a prime field F_2 is set. Such primitive polynomial in the prime field F_2 is described, for example, in A. Menezes, P. Oorschot and S. Vanstone: "HANDBOOK OF APPLIED CRYPTOGRAPHY", CRC
15 Press, Section 4.5.3 Primitive Polynomials (1996).

In an elliptic curve parameter setting step or submodule 106, parameters a and b for the elliptic curve $y^2 + xy = x^3 + ax^2 + b$ defined on the basis of a finite field F_q of characteristic 2 (which may also be referred
20 to as the extension field of "2") are set. For the elliptic curve which can ensure security, it is necessary that the order $\#E(F_q)$ of the elliptic curve has a large prime factor r . In the case where $\#E(F_q) = kr$ applies valid, the prime factor r can assume a large prime number
25 by selecting a small integer for k . Parenthetically, concerning the method of generating an elliptic curve having a large prime factor r as the order, reference may be made to Henri Cohen: "A COURSE IN COMPUTATIONAL

ALGEBRAIC NUMBER THEORY", GTM138, Springer (1993) p. 464,
 Atkin's Test. At this juncture, it should however be
 mentioned that the elliptic-curve primitive polynomial
 setting method can equally be realized by resorting to
 5 other elliptic curve the order of which has a large prime
 factor.

A base point generating submodule 107 is
 designed to determine a generator of a cycling subgroup
 having the prime factor \underline{r} mentioned above as the order in
 10 the Abelian group on the elliptic curve. By way of
 example, in the case where $\#E(Fq) = kr$ applies valid, a
 given point (x_1, y_1) on the elliptic curve $E(Fq)$ in the
 finite field of characteristic 2 is determined in a first
 step. Subsequently, in a second step, $G = (x_1, y_1)$ is
 15 set as the base point on the conditions that $r(x_1,$
 $y_1) = 0$ and when $k(x_1, y_1) \neq 0$. Otherwise, the first
 step mentioned just above is resumed.

At this juncture, it is to be noted that the
 expression $r(x_1, y_1)$ means execution of the scalar
 20 multiplication (multiplication by \underline{r} or r -multiplication)
 for the point (x_1, y_1) . Incidentally, the arithmetic for
 the scalar multiplication (r -multiplication) will be
 elucidated later on in conjunction with the elliptic
 curve arithmetic submodule 109.

25 Through the procedure described above, the
 primitive polynomial $f(x)$, the parameters \underline{a} and \underline{b} of the
 elliptic curve $y^2 + xy = x^3 + ax^2 + b$, the base point G and
 the order \underline{r} of the base point have been generated which

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are the information destined to be laid open for the general public.

The public key/private key generating module 102 is designed to generate the public key and the private key in accordance with the procedure described below. On the presumption that the primitive polynomial $f(x)$, the parameters a and b of the elliptic curve $y^2 + xy = x^3 + ax^2 + b$ and the base point G are inputted to the public key/private key generating module 102 and that a public key Q and a private key d are outputted therefrom, a random number which satisfies the condition that $2 < d < r-1$ is generated in a first step, whereon the public key $Q = dG$, i.e., a scalar multiplication (d-multiplication) of the base point G is determined.

The public key is the information to be laid open to the general public while the private key represents the information to be secreted. The problem of determining the private key d on the basis of the public key Q and the base point G is what is known as the discrete logarithm problem and requires for the solution thereof such an amount of computation which is on the exponential order of bit-length of the base point on the elliptic curve. Consequently, in case the order r is a large prime number, e.g. when the prime factor r is greater than the 159-th power of "2", it is the impossible in practice to determine the private key d from the public key Q and the base point G . This is the principle underlying the elliptic curve cryptography. In this

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conjunction, the method of arithmetically determining the public key Q is known in the art, as disclosed, for example, in D.V. Chudnovsky and G.V. Chudnovsky:

"SEQUENCES OF NUMBERS GENERATED BY ADDITION IN FORMAL
5 GROUPS AND NEW PRIMALITY AND FACTORIZATION TESTS",
Advances in Applied Mathematics, 7. 385-434, 1986.

In the encrypting module 103, the plain text 113 is translated to the cipher text 112 in accordance with the procedure which will be described below. On the
10 presumption that a plain text M , the public key Q , the primitive polynomial $f(x)$, the parameter b of the elliptic curve $y^2 + xy = x^3 + ax^2 + b$ and the base point G are inputted to the encrypting module 103 and that a cipher text C is outputted therefrom, a random number k
15 is generated in a first step by the random number generating submodule 108, whereon in the second step, the base point G and the random number k generated in the first step undergo arithmetic operation for determining kG , i.e., (kx_1, ky_1) , in the elliptic curve arithmetic
20 submodule 109. In a third step, the public key Q and the random number k generated in the first step undergo arithmetic operation for determining kQ , i.e., (kx_2, ky_2) in the elliptic curve arithmetic submodule 109. In a fourth step, arithmetic operation $M \times \text{or} \times 2$ is executed in
25 the data encryption processing submodule 110, the result of which is set as M' . In a fifth step, arithmetic operation $x_1 \parallel y_1 \parallel M'$ is executed, as a result of which the cipher text C is outputted from the data encryption

submodule 110.

The elliptic curve arithmetic submodule 109 is designed to execute a scalar multiplication (kR) arithmetic for a given point R to thereby determine the x-coordinate. Owing to such arrangement, the private key information can be protected against leakage from deviation (difference) information of the processing time or period for the decryption of the elliptic curve cipher text in the finite field of characteristic 2. In the following, the scalar multiplication method will be elucidated.

Scalar multiplication method according to first embodiment

Figure 2 and 3 in combination illustrate in a flow chart the scalar multiplication method according to a first embodiment of the present invention.

It is presumed that a projective coordinate component X_0 of the x-coordinate of a given point R and a scalar value m are inputted and that a projective coordinate component X_m of the x-coordinate of a point corresponding to m -multiple of R is to be outputted. On this assumption, the scalar value m and the projective coordinate component X_0 of the x-coordinate are inputted (step 202). In the succeeding steps 203 to 205, data stirring is performed by multiplying the individual projective coordinates by the random number. More specifically, the random number k is generated in the step 203, whereon k^2X_0 is arithmetically determined by

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5 multiplying the projective coordinate component X_0 by the random number k and assigned to X_1 in the step 204 while the random number k itself is assigned to Z_1 in the step 205. In succeeding steps 206 to 208 and 301, preparation is made for the scalar multiplication. In more concrete, $[X_1, Z_1]$ is assigned to $[X_4, Z_4]$ in the step 206, being followed by the step 206 where $[X_1, Z_1]$ is inputted to the doubling process (illustrated in Fig. 5), the output of which is then assigned to $[X_2, Z_2]$ in the step 207.

10 Further, in a step 208, the scalar value m is transformed to a binary bit string $h_i h_{i-1} \dots h_0$, where the most significant bit h_i is "1" and thus "1" is assigned to i in a step 301 shown in Fig. 3. Through processing steps 302 to 309 (see Fig. 3), the addition method and the doubling method are controlled in dependence on whether one bit of the scalar value m is "0" or "1" to thereby realize the scalar multiplication. More specifically, "i-1" is assigned to i in the step 302, which is followed by the step 303 where $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted

15 to the addition process (illustrated in Fig. 4), the output of which is assigned to $[X_3, Z_3]$ in the step 303. At this juncture, when $h_i == 0$ (i.e., when the step 304 results in affirmation "Yes"), the processing proceeds to the step 305 while it proceeds to the step 307 when

20 $h_i == 1$, i.e., when the decision step 304 results in negation "No". In the step 305, $[X_1, Z_1]$ is inputted to the doubling arithmetic or process (Fig. 5), the output from which is assigned to $[X_1, Z_1]$. In the step 306, $[X_3,$

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$Z_3]$ is assigned to $[X_2, Z_2]$, whereon the processing proceeds to the step 309. On the other hand, when the decision step 304 results in "No", $[X_2, Z_2]$ is inputted to the doubling arithmetic or process illustrated in Fig. 5, the output of which is assigned to $[X_2, Z_2]$ (step 307). In the step 308, $[X_3, Z_3]$ is assigned to $[X_1, Z_1]$, whereupon the processing proceeds to the step 309. In the case where $i > 0$, i.e., the step 309 results in "Yes", when the step 302 is resumed. If otherwise, i.e., when the decision step 309 results in "No", the processing proceeds to a step 310. Subsequently, the projective coordinates are transformed to the x-coordinate of the (x, y) coordinate system. Finally, $X_1/(Z_1)^2$ is assigned to the projective coordinate component X_m (step 310) to be ultimately outputted (step 311).

Next, description will be directed to the addition method or arithmetic. It is presumed that as the projective space coordinates of a point on the elliptic curve, it applies valid that $[X, Y, Z] = [\lambda^2x, \lambda^3y, \lambda Z]$ for a given $\lambda \neq 0$. At this juncture, let's consider the points $P_0 = (x_0, y_0) = [X_0, Y_0, Z_0]$ and $P_1 = (x_1, y_1) = [X_1, Y_1, Z_1]$ as the points on the elliptic curve. Additionally, it is presumed that the sum of the points P_0 and P_1 and the difference therebetween are given by $P_3 = (x_3, y_3) = [X_3, Y_3, Z_3]$ and $P_4 = (x_4, y_4) = [X_4, Y_4, Z_4]$, respectively.

Expressing mathematically,

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$$P1 + P0 = P3,$$

$$P1 - P0 = P4,$$

$$x3 = a + (\lambda_3)^2 + \lambda_3 + x0 + x1; \lambda_3 = (y0 + y1)/(x0 + x1),$$

$$5 \quad x4 = a + (\lambda_4)^2 + \lambda_4 + x0 + x1; \lambda_4 = (x0 + y0 + y1)/(x0 + x1),$$

$$\lambda_3 + \lambda_4 = (x0) / (x0 + x1),$$

$$(\lambda_3)^2 + (\lambda_4)^2 = (x0)^2 / (x0 + x1)^2, \text{ and}$$

$$10 \quad x3 + x4 = ((x0)^2 + (x0)(x0 + x1)) / (x0 + x1)^2 \\ = (x0 \ x1) / (x0 + x1)^2.$$

From the above, the following relation can be derived.

$$x3 + x4 = (x0 \ x1) / (x0 + x1)^2 \quad \dots (1)$$

Subsequently, relations in the projective coordinate system are derived.

15 Replacing "x1" and "x0" in the expression (1) by "x1 = X₁/(Z₁)²" and "x0 = X₀/(Z₀)²", respectively, then

$$X_3/(Z_3)^2 = X_4/(Z_4)^2 + ((X_0/(Z_0)^2)(X_1/(Z_1)^2))/(X_0/(Z_0)^2 + X_1/(Z_1)^2)^2 \\ = X_4/(Z_4)^2 + ((X_0(Z_0)^2)(X_1(Z_1)^2))/(X_0(Z_1)^2 + X_1(Z_0)^2)^2 \\ 20 \quad = ((X_4\beta^2) + Z_4^2 (X_0Z_0^2)(X_1Z_1^2))/(Z_4^2\beta^2)$$

where $\beta = X_0Z_1^2 + X_1Z_0^2$.

From the above expression, there can be derived:

$$X_3 = X_4\beta^2 + Z_4^2 (X_0Z_1^2)(X_1Z_0^2) \quad \dots (2)$$

$$25 \quad Z_3 = Z_4\beta \quad \dots (3)$$

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On the presumption that $mR = [X_1, Y_1, Z_1]$, $(m + 1)R = [X_2, Y_2, Z_2]$, $R = [X_4, Y_4, Z_4]$ and $(2m + 1)R = [X_3, Y_3, Z_3]$, the addition arithmetic will be elucidated below.

Addition method according to first embodiment

5 Figure 4 is a flow chart for illustrating the addition method according to the first embodiment of the present invention. The projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted, whereby coordinates $[X_3, Z_3]$ or a point at infinity is outputted. Thus, the

10 projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted in a step 402. Through processings in steps 403 to 407, $X_1(Z_2)^2 + X_2(Z_1)^2$ is determined for making decision whether or not the result of the addition arithmetic

15 represents the point at infinity. Interim results S_1 , S_2 and B provide preparation for the realization of the expressions (2) and (3) mentioned above. More specifically, $X_1(Z_2)^2$ is assigned to S_1 in the step 403 and $X_2(Z_1)^2$ is assigned to S_2 in the step 404 whereupon $S_1 + S_2$ is assigned to B in the step 405. When $B == 0$ in the

20 step 406 (i.e., when the decision step 406 results in "Yes"), the processing proceeds to the step 407. If otherwise (i.e., when the decision in the step 406 results in "No"), the processing proceeds to the step 408. In the step 407, the point at infinity is

25 outputted, whereon the processing comes to an end (step 413). Through the processing steps 408 to 411 executed when the decision step 406 results in "No", the coordinates $[X_3, Z_3]$ are determined in accordance with the

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expressions (2) and (3) mentioned hereinbefore. In more concrete, Z_4B is assigned to Z_3 in the step 408 and $(Z_4)^2S_1S_2$ is assigned to S in the step 409 with X_4B^2 being assigned to M in the 410, whereupon $M + S$ is assigned to X_3 in the step 411, and $[X_3, Z_3]$ is outputted in the step 412. Through the procedure described above, the addition arithmetic can be realized by sextuple multiplications of the mutually different variables. In other words, X_3 can be arithmetically determined from X_1, X_2 and X_4 at a high speed.

Next, description will turn to the doubling method. Let's represent a double point of the point P_1 by P_2 and presume that $P_1 = (x_1, y_1) = [X_1, Y_1, Z_1]$, and that $P_2 = (x_2, y_2) = [X_2, Y_2, Z_2]$. The doubling expression is given by $x_2 = (x_1)^2 + b/(x_1)^2$. Accordingly, by placing $x_1 = X_1/(Z_1)^2$ and $x_2 = X_2/(Z_2)^2$ in the doubling expression as follows,

$$\begin{aligned} X_2/(Z_2)^2 &= (X_1/(Z_1)^2)^2 + b/(X_1/(Z_1)^2)^2 \\ &= X_1^2/(Z_1)^4 + (b(Z_1)^4)/(X_1)^2 \\ &= (X_1^4 + b(Z_1)^8)/(X_1^2Z_1^4) \end{aligned}$$

there can be derived the following relations.

$$X_2 = X_1^4 + bZ_1^8 \quad \dots (4)$$

$$Z_2 = X_1Z_1^2 \quad \dots (5)$$

The doubling method based on the expressions mentioned above will be described.

Doubling method according to first embodiment

Figure 5 is as flow chart for illustrating the doubling method according to the first embodiment of the

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present invention. Referring to the figure, it is presumed that $Q = [X_1, Z_1]$ and \underline{b} are inputted, whereby $2Q = [X_2, Z_2]$ or alternatively the point at infinity is to be outputted. In a step 502, X_1 and Z_1 are inputted. In the succeeding steps 503 and 504, decision is made whether or not $X_1 == 0$ or $Z_1 == 0$ is valid in order to make decision as to whether the doubling arithmetic results in the point at infinity. Namely, when $X_1 == 0$ or $Z_1 == 0$ in the step 503 (i.e., when the decision step 503 results in "Yes"), the processing proceeds to the step 504. If otherwise (i.e., when the decision step 503 results in "No"), the processing proceeds to a step 505. In the step 504, the point at infinity is outputted. In the succeeding steps 505 to 507, the coordinates $[X_2, Z_2]$ are determined in accordance with expressions (4) and (5) mentioned previously. More specifically, in the step 505, Z_1^2 is assigned to S . In the step 506, $X_1 S$ is assigned to Z_2 . In the step 507, $X_1^4 + b(S)^4$ is assigned to X_2 (step 507). In the step 508, the coordinates $[X_2, Z_2]$ are outputted. Through the procedure described above, the addition arithmetic can be realized by executing twice the multiplication of mutually different variables. Accordingly, in the scalar multiplication method, the addition arithmetic can be realized by executing (6 + 2 = 8)-times the multiplication of mutually different variables per bit of the scalar value \underline{d} . In other words, the projective coordinate X_3 can be arithmetically determined very speedily from X_1 , X_2 and X_4 .

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Now turning back to Fig. 1, the decrypting module 104 is designed to transform the cipher text 112 into the original plain text 114 through the procedure described below. Of course, the cipher text 112 and the plain text 114 are same with regard to the content. On the presumption that the cipher text $C \leftarrow x1\|y1\|M'$, private key d , primitive polynomial $f(x)$, parameter b of the elliptic curve $y^2 + xy = x^3 + ax^2 + b$ and the base point G are inputted, whereby the plain text M is outputted, the following steps are executed.

step 1: $(x2, y2) \leftarrow d(x1, y1)$ (by the data decryption processing submodule 111)

step 2: plain text $M \leftarrow M' \text{ xor } x2$

The step 1 can be executed in accordance with the procedure described hereinbefore by reference to Figs. 2 and 3.

Through the procedure described above, determination of the x -coordinate equivalent to the scalar (d) multiplication of given coordinates (x, y) can be realized by executing eight-tuple mutually different multiplication processings for each bit of d independent of the bit pattern thereof. Furthermore, by setting for the given x -coordinate of d as the initial value for the scalar multiplications $[kx^2, k]$ where k represents a random number, object for the arithmetic can constantly be varied. Additionally, owing to combination of the procedures described in the foregoing, no bit pattern of d can make appearance in the deviation (difference) of

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the $d(x, y)$ -processing time, which in turn means that any private key information can be protected against leakage in terms of the deviation information of the $d(x, y)$ -processing time. In addition, this feature indicates
 5 that in the DPA (Differential Power Analysis) for realizing the cryptanalysis by making use of deviations of current, voltage, electric power for the encryption processing as well, the private key information is protected against leakage in terms of the deviation
 10 information of the current, voltage or electric power for the $d(x, y)$ processing.

Next, description will be made of a second embodiment of the invention which can further speed up the arithmetic operations involved in the elliptic curve
 15 cryptography when compared with the first embodiment described above. Representing the coordinate transformation from the affine coordinates to the projective coordinates by $(x, y) \rightarrow [x, y, 1]$, it can apply valid that $Z_4 = 1$. By placing $Z_4 = 1$ in the expressions (2) and
 20 (3), there can be derived the following expressions:

$$X_3 = (X_4\beta^2) + (X_0(Z_1)^2)(X_0(Z_1)^2) \quad \dots (6)$$

$$Z_3 = \beta \quad \dots (7)$$

By making use of the above expressions, the scalar multiplication method and the addition method can
 25 be carried out in the manners described below.

Scalar multiplication method according to second embodiment

Figures 6 and 7 in combination illustrate in a

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flow chart, a processing procedure for the scalar multiplication method according to the second embodiment of the present invention. It is presumed that a projective coordinate component X_0 of the x-coordinate of a given point R and a scalar value \underline{m} are inputted for 5 thereby outputting a projective coordinate component X_m of the x-coordinate of a point corresponding to m-multiplication or m-tuple of R. To this end, the scalar value \underline{m} and the projective coordinate component X_0 of the 10 x-coordinate are inputted in the step 602. In the succeeding steps 603 and 604, transformation of X_0 to the projective coordinate is performed. More specifically, in the step 603, X_0 is assigned to X_1 . In the step 604, "1" is assigned to Z_1 . In the processing steps 605 to 15 607, preparation is made for the scalar multiplication. In more concrete, coordinates $[X_1, Z_1]$ are assigned to $[X_4, Z_4]$ in the step 605 to thereby allow $[X_1, Z_1]$ to be inputted to the doubling arithmetic (Fig. 5), the output of which is assigned to $[X_2, Z_2]$ in the step 606. In the 20 step 607, $h_i h_{i-1} \dots h_0$ are set as the binary bit string representing the scalar value \underline{m} , in which the most significant bit h_i is "1", and thus "1" is assigned to \underline{i} in a step 701 shown in Fig. 7. In the succeeding processing steps 702 to 709, the addition method and the 25 doubling method are controlled in dependence on whether one bit of the scalar value \underline{m} is "0" or "1", to thereby determine the scalar multiplication. More specifically, in the step 702, "i-1" is assigned to \underline{i} while in the step

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703, $[X_1, Z_1]$, $[X_2, Z_2]$ and X_0 are inputted to the addition method (Fig. 8), the output of which is assigned to $[X_3, Z_3]$. When $h_i == 0$ (i.e., when the decision step 704 results in affirmation "Yes"), the processing proceeds to the step 705 while it proceeds to the step 707 when $h_i == 1$, i.e., when the decision step 704 results in negation "No". In the step 705, $[X_1, Z_1]$ is inputted to the doubling method (Fig. 5), the output from which is assigned to $[X_1, Z_1]$. In the succeeding step 706, $[X_3, Z_3]$ is assigned to $[X_2, Z_2]$, whereupon the processing proceeds to the step 709. On the other hand, in the step 707, $[X_2, Z_2]$ is inputted to the doubling method (Fig. 5), the output of which is assigned to $[X_2, Z_2]$. In the succeeding step 708, $[X_3, Z_3]$ is assigned to $[X_1, Z_1]$, whereupon the processing proceeds to the decision step 709. In case the decision step 709 results in that $i > 0$ (i.e., when the step 709 results in "Yes"), the step 702 is resumed. On the other hand, when $i \leq 0$, i.e., when the decision step 709 results in "No", the processing proceeds to the step 710 where $X_1/(Z_1)^2$ is assigned to the projective coordinate component X_m to be ultimately outputted.

Addition method according to second embodiment

Figure 8 is a flow chart for illustrating the addition method according to the second embodiment of the invention. It is presumed that the projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted and that $[X_3, Z_3]$ or the point at infinity is to be outputted.

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Thus, the projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted in a step 802. Through the processings in subsequent steps 803 to 807, $X_1(Z_2)^2 + X_2(Z_1)^2$ is computed for making decision whether or not the result of the addition represents the point at infinity. Interim results S_1 , S_2 and B provide preparation for realization of the expressions (6) and (7) mentioned previously. More specifically, $X_1(Z_2)^2$ is assigned to S_1 in the step 803. In the step 804, $X_2(Z_1)^2$ is assigned to S_2 . In the step 805, $S_1 + S_2$ is assigned to B . When it is decided that $B = 0$ in the step 806 (i.e., when decision in the step 806 results in "Yes"), the processing proceeds to the step 807. If otherwise (i.e., when the decision in the step 806 results in "No"), the processing proceeds to the step 808. In the step 807, the point at infinity is outputted, whereon the processing proceeds to the step 813. Through the processing steps 808 to 811, the projective coordinates $[X_3, Z_3]$ are determined in accordance with the expressions (6) and (7) mentioned hereinbefore. In more concrete, B is assigned to Z_3 in the step 808. In the step 809, $S_1 S_2$ is assigned to S . In the step 810, $X_4 Z_3^2$ is assigned to M . In the step 811, $M + S$ is assigned to X_3 . Finally, in the step 812, $[X_3, Z_3]$ is outputted.

Through the procedure described above, the addition arithmetic can be realized by executing four times the multiplication of mutually different variables. Thus, it is apparent understood that with the addition

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arithmetic or method according to the second embodiment of the invention, the number of times the multiplication is required to be executed can be decreased when compared with the addition method according to the first
5 embodiment of the invention described hereinbefore. Parenthetically, it should be added that the doubling arithmetic according to the second embodiment of the invention can be realized by making use of the doubling method according to the first embodiment of the
10 invention.

The method of protects the private key information against leakage in terms of the deviation information of the processing time as described hereinbefore can also be carried out with the elliptic
15 curve in a prime field in addition to the elliptic curve in the finite field of characteristic 2 (extension field of "2").

Next, description will be made of a third embodiment of the present invention which is directed to
20 a method of preventing leakage of the private key information from the deviation information of the processing time by adopting the Montgomery method on the presumption that the elliptic curve in the prime field is represented by $By^2 = x^3 + Ax^2 + Bx$.

25 As is disclosed in P. Montgomery: "SPEEDING THE POLLARD AND ELLIPTIC CURVE METHODS OF FACTORIZATION", Mathematics of Computation Vol. 48, No. 177, pp. 243-264 (1987), presuming that the addition of points $P_0(x_0, y_0)$

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and $P1(x1, y1)$ and the subtraction therebetween are given by:

$$P3 (x3, y3); P4 (x4, y4);$$

$$P1 + P0 = P3;$$

$$5 \quad P1 - P0 = P4;$$

then, $x3$ can speedily be determined from $x0, x1$ and $x4$ by resorting to the elliptic curve of the standard form $By^2 = x^3 + Ax^2 + Bx$ in the prime field. In more concrete, x^3 can be determined by performing six times the
10 multiplications of the prime field as follows:

Presuming that

$$(x3, y3) \rightarrow [X_3, Z_3] \text{ and that } (x4, y4) \rightarrow [X_4, Z_4],$$

then

$$X_3 \leftarrow Z_4[(X_1 - Z_1)(X_0 + Z_0) + (X_1 + Z_1)(X_0 - Z_0)]^2,$$

15 and

$$Z_3 \leftarrow X_4[(X_1 - Z_1)(X_0 + Z_0) - (X_1 + Z_1)(X_0 - Z_0)]^2.$$

Further, for the doubling arithmetic, expressions mentioned below apply valid:

$$P5 = 2P1; (x1, y1) \rightarrow [X_1, Z_1];$$

$$20 \quad 4X_1Z_1 \leftarrow (X_1 + Z_1)^2 - (X_1 - Z_1)^2;$$

$$X_5 \leftarrow (X_1 + Z_1)^2(X_1 - Z_1)^2; Z_5 \leftarrow (4X_1Z_1)[(X_1 - Z_1)^2 + ((A + 2)/4)(4X_1Z_1)]$$

Furthermore, when the double point of $P1$ is given by $P5(x5, y5)$, then $x5$ can be determined only from
25 $x1$ by executing relevant multiplication five times. By taking advantage of this feature, the x -coordinate of scalar multiple (scalar value d) of the point R can be determined from Rx , as follows.

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Presuming that the initial value is given by
 $[R, 2R]$ and that mR represents the x-coordinate of \underline{m}
multiplication of the point R , the scalar value \underline{d} is
developed to the binary bit string. Then, starting from
5 the most significant bit of \underline{d} ,

$[mR, (m+1)R] \rightarrow [2mR, 2(m+1)R]$ for the bit
of $\underline{d} = "0"$,
and

10 $[mR, (m+1)R] \rightarrow [(2m+1)R, 2(m+1)R]$ for the bit
of $\underline{d} = "1"$

Hence

$(m+1)R - mR = R$, and

$(m+1)R + mR = (2m+1)R$.

15 Scalar multiplication method according to third
embodiment

Figures 11A and 11B are flow charts for
illustrating the scalar multiplication method in which
the Montgomery method is adopted according to the third
embodiment of the present invention. Referring to the
20 figures, it is presumed that a projective coordinate
component X_0 of the x-coordinate of a given point R and a
scalar value \underline{m} are inputted and that a projective
coordinate component X_m of the x-coordinate of a point
corresponding to m -multiplication of R is to be
25 outputted. To this end, the scalar value \underline{m} and the
projective coordinate component X_0 of the x-coordinate are
inputted in the step 1102 shown in Fig. 11A. In the

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succeeding steps 1103 to 1105, data is stirred through multiplication of the individual coordinates in the projective coordinate system by the random number. More specifically, the random number k is generated in the step 1103, whereon kX_0 is determined by multiplying the projective coordinate component X_0 of the x-coordinate by the random number k , and then kX_0 is assigned to X_1 in the step 1104 while the random number k being assigned to Z_1 in the step 1105. In succession, $[X_1, Z_1]$ is assigned to $[X_4, Z_4]$ (step 1106). Subsequently, $[X_1, Z_1]$ is inputted to the doubling method (i.e., Montgomery's doubling arithmetic), the output of which is assigned to $[X_2, Z_2]$ (step 1107). Further, the scalar value m is transformed to the binary bit string $h_i h_{i-1} \dots h_0$ (step 1108), where the most significant bit h_i is "1". Thus "1" is assigned to i in the step 1109 shown in Fig. 11B. In a succeeding step 1110, " $i-1$ " is assigned to i , which is then followed by a step 1111 where $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted to the addition method (Montgomery's addition arithmetic), the output of which is assigned to $[X_3, Z_3]$ (step 1111). When $h_i == 0$ in the step 1112 (i.e., when the decision step 1112 results in affirmation "Yes"), the processing proceeds to a step 1113 while it proceeds to a step 1115 when $h_i == 1$, i.e., when the decision step 1112 results in negation "No". In the step 1113 shown in Fig. 11B, $[X_1, Z_1]$ is inputted to the doubling method (Montgomery's doubling arithmetic), the output from which is assigned to $[X_1, Z_1]$. In the succeeding step 1114, $[X_3,$

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Z_3] is assigned to $[X_2, Z_2]$, whereon the processing proceeds to a step 1117. On the other hand, when the decision step 1112 results in "No", $[X_2, Z_2]$ is inputted to the doubling method (Montgomery's doubling
5 arithmetic), the output of which is assigned to $[X_2, Z_2]$ (step 1115). Further, $[X_3, Z_3]$ is assigned to $[X_1, Z_1]$ in the step 1116, whereupon the processing proceeds to a step 1117. In the case where $i > 0$, i.e., the step 1117 results in "Yes", the step 1110 is resumed. If
10 otherwise, i.e., when the decision step 1117 results in "No", the processing proceeds to a step 1118 where $X_1/(Z_1)$ is assigned to the projective coordinate component X_m to be ultimately outputted in the step 1119, whereupon the processing comes to an end (step 1120).

15 Through the procedure described above, determination of the x-coordinate corresponding to the scalar (d) multiplication of a given coordinate (x, y) can be realized by executing eleven times the mutually different multiplications for each bit of d. Further-
20 more, by setting for the given x-coordinate the initial value for scalar multiplication $[kx, k]$ where k represents a random number, the private key information can be protected against leakage in terms of the deviation information of the $d(x, y)$ processing time. In
25 addition, this feature indicates that for the DPA (Differential Power Analysis) trial for performing the cryptanalysis by making use of deviation information concerning the of current, voltage, electric power for

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the encryption processing, the private key information can be protected against leakage in terms of the deviation information of the current (voltage, electric power) involved in processing $d(x, y)$.

5 Furthermore, for the elliptic curve $y^2 = x^3 + ax + b$ in the prime field, an elliptic curve may be constituted such that the Abelian group defined by the rational points between $By^2 = x^3 + Ax^2 + Bx$ and $y^2 = x^3 + ax + b$ is same, whereon the coordinates (x, y) given by the
 10 elliptic curve $y^2 = x^3 + ax + b$ in the prime field is transformed to $By^2 = x^3 + Ax^2 + Bx$, to thereby determine the scalar multiplication through the procedure described hereinbefore, the result of which is then transformed to $y^2 = x^3 + ax + b$.

15 Next, description will be directed to a fourth embodiment of the present invention. In the case of the elliptic curve cryptography according to the first embodiment of the invention, it has been presumed that $[X, Y, Z] = [\lambda^2 X, \lambda^3 Y, \lambda Z]$ applies valid for the given
 20 projective coordinate $\lambda \neq 0$. However, the teachings of the present invention can also be implemented with the projective coordinate system in which $[X, Y, Z] = [\lambda X, \lambda Y, \lambda Z]$ applies valid.

25 Scalar multiplication method according to fourth embodiment

Figures 12A and 12B are flow charts for illustrating the scalar multiplication method according to the fourth embodiment of the present invention.

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Referring to the figures, it is presumed that a projective coordinate component X_0 of the x-coordinate of a given point R and a scalar value \underline{m} are inputted and that a projective coordinate component X_m of the x-coordinate of a point corresponding to m-multiplication of R (i.e., the point corresponding to the product of \underline{m} and R) is to be outputted. On the presumption, the scalar value \underline{m} and the projective coordinate component X_0 of the x-coordinate are inputted in the step 1202 shown in Fig. 12A. In the succeeding steps 1203 to 1205, data is stirred through multiplication of the individual projective coordinates by the random number. More specifically, the random number \underline{k} is generated in the step 1203, whereon kX_0 is determined by multiplying the projective coordinate component X_0 of the x-coordinate by the random number \underline{k} , and then kX_0 is assigned to X_1 in the step 1204 while the random number \underline{k} itself being assigned to Z_1 in the step 1205. In succession, $[X_1, Z_1]$ is assigned to $[X_4, Z_4]$ (step 1206). Subsequently, $[X_1, Z_1]$ is inputted to the doubling arithmetic, the output of which is assigned to $[X_2, Z_2]$ (step 1207). Further, the scalar value \underline{m} is transformed to the binary bit string $h_i h_{i-1} \dots h_0$ (step 1208), where the most significant bit h_i is "1". Thus, "1" is assigned to \underline{i} in the step 1209 shown in Fig. 12B. In a succeeding step 1210, "i-1" is assigned to \underline{i} , which is then followed by a step 1011 where $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted to the addition arithmetic, the output of which is assigned to $[X_3, Z_3]$. When $h_i == "0"$ in

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the step 1212 (i.e., when the decision step 1212 results in affirmation "Yes"), the processing proceeds to a step 1213 while it proceeds to a step 1215 when $h_i == "1"$, i.e., when the decision step 1212 results in negation "No". In the step 1213 shown in Fig. 12B, $[X_1, Z_1]$ is inputted to the doubling arithmetic, the output from which is assigned to $[X_1, Z_1]$. In the succeeding step 1214, $[X_3, Z_3]$ is assigned to $[X_2, Z_2]$, whereon the processing proceeds to a step 1217. On the other hand, when the decision step 1212 results in "No", $[X_2, Z_2]$ is inputted to the doubling arithmetic, the output of which is assigned to $[X_2, Z_2]$ (step 1215). Further, $[X_3, Z_3]$ is assigned to $[X_1, Z_1]$ in the step 1216, whereupon the processing proceeds to a step 1217. In the case where $i > 0$, i.e., when the step 1217 results in "Yes", the step 1210 is resumed. If otherwise, i.e., when the decision step 1217 results in "No", the processing proceeds to a step 1218 where $X_1/(Z_1)$ is assigned to the projective coordinate component X_m to be ultimately outputted in the step 1219, whereupon the processing comes to an end (step 1220).

It is presumed that in conjunction with the projective space coordinate of a point on the elliptic curve, it applies valid that $[X, Y, Z] = [\lambda x, \lambda y, \lambda z]$ for a given $\lambda \neq 0$. At this juncture, let's consider points $P_0 = (x_0, y_0) = [X_0, Y_0, Z_0]$ and $P_1 = (x_1, y_1) = [X_1, Y_1, Z_1]$ as the points on the elliptic curve. Additionally, it is presumed that the sum and the

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difference of the points P0 and P1 are given by P3 =
 (x3, y3) = [X3, Y3, Z3] and P4 = (x4, y4) = [X4, Y4, Z4],
 respectively.

Namely,

$$5 \quad P1 + P0 = P3, \text{ and}$$

$$P1 - P0 = P4$$

Subsequently, relations in the projective
 coordinate system are derived from the expression (1)
 mentioned hereinbefore in conjunction with the first
 10 embodiment of the invention, i.e., $x3 + x4 = (x0 x1) /$
 $(x0 + x1)^2$.

Replacing x1 and x0 appearing in the expression
 (1) by X_1/Z_1 and X_0/Z_0 , respectively, then

$$15 \quad \begin{aligned} X_3/Z_3 &= X_4/Z_4 + ((X_0/Z_0)(X_1/Z_1)) / (X_0/Z_0 + X_1/Z_1)^2 \\ &= X_4/Z_4 + ((X_0Z_0)(X_1Z_1)) / (X_0Z_1 + X_1Z_0)^2 \\ &= ((X_4\beta^2) + Z_4 (X_0Z_0)(X_1Z_1)) / (Z_4\beta^2) \end{aligned}$$

where $\beta = X_0Z_1 + X_1Z_0$.

From the above expression, there can be
 derived:

$$20 \quad X_3 = X_4\beta^2 + Z_4 (X_0Z_1)(X_1Z_0) \quad \dots (2)'$$

$$Z_3 = Z_4\beta^2 \quad \dots (3)'$$

On the presumption that $mR = [X_1, Y_1, Z_1]$, $(m$
 $+ 1)R = [X_2, Y_2, Z_2]$, $R = [X_4, Y_4, Z_4]$ and $(2m + 1)R = [X_3,$
 $Y_3, Z_3]$, an addition method according to the fourth
 25 embodiment of the present invention will be elucidated
 below.

Addition method according to fourth embodiment

Figure 13 is a flow chart for illustrating an addition method according to the fourth embodiment of the present invention. It is assumed that projective

5 coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted, whereby $[X_3, Z_3]$ or the point at infinity is outputted. Thus, projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted in a step 1302. Subsequently, X_1Z_2 is assigned to S_1 in a step 1303. Further, X_2Z_1 is assigned

10 to S_2 in a step 1304, whereon $S_1 + S_2$ is assigned to B in a step 1305. When $B = 0$ in a step 1306 (i.e., when decision in the step 1306 results in "Yes"), the processing proceeds to a step 1307. If otherwise (i.e., when the decision in the step 1306 results in "No"), the processing

15 proceeds to a step 1308. In the step 1307, the point at infinity is outputted, and then a step 1313 is executed. On the other hand, when the decision step 1306 results in "No", Z_4B^2 is assigned to Z_3 in a step 1308. Further, $(Z_4)^2S_1S_2$ is assigned to S in a step 1309. Subsequently,

20 X_4B^2 is assigned to M in a step 1310 while $M + S$ is assigned to X_3 in a step 1311, whereon $[X_3, Z_3]$ is outputted in a step 1312.

Through the procedure described above, the addition arithmetic can be realized by executing six

25 times the multiplication of mutually different variables.

Next, description will turn to the doubling method. Let's represent a double point of P_1 by P_2 and presume that $P_1 = (x_1, y_1) = [X_1, Y_1, Z_1]$ and $P_2 = (x_2, y_2)$

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= $[X_2, Y_2, Z_2]$. The doubling expression is given by $x_2 = (x_1)^2 + b/(x_1)^2$. Accordingly, in the doubling arithmetic formulae $x_2 = (x_1)^2 + b/(x_1)^2$, x_1 is replaced by X_1/Z_1 with x_2 being replaced by X_2/Z_2 .

5 Namely,

$$\begin{aligned} X_2/Z_2 &= (X_1/Z_1)^2 + b/(X_1/Z_1)^2 \\ &= X_1^2/(Z_1)^2 + (bZ_1^2)/(X_1)^2 \\ &= (X_1^4 + b(Z_1)^8)/(X_1^2Z_1^2) \end{aligned}$$

10 Thus, there can be derived the following relations.

$$X_2 = X_1^4 + bZ_1^4 \quad \dots (4)$$

$$Z_2 = X_1^2Z_1^2 \quad \dots (5)$$

The doubling method based on the expressions mentioned above will be described below.

15 Doubling method according to fourth embodiment

Figure 14 is a flow chart for illustrating a doubling method according to the fourth embodiment of the invention. It is presumed that $Q = [X_1, Z_1]$ and b are inputted for thereby outputting $2Q = [X_2, Z_2]$ or the point at infinity. More specifically, $[X_1, Z_1]$ and b are inputted in a step 1402. When $X_2 == 0$ or $Z_2 == 0$ (i.e., when the decision in the step 1403 results in "Yes"), the processing proceeds to a step 1404. If otherwise (i.e., when the decision step 1403 results in "No"), the processing proceeds to the step 1405. In the step 1404, the point at infinity is outputted. In the step 1405, Z_1^2 is assigned to Z_2 . In the step 1406, X_1^2S is assigned to S . In the step 1407, $X_1^4 + bS$ is assigned to X_2 , which is

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then followed by a step 1408 where $[X_2, Z_2]$ is outputted. Through the procedure described above, the addition arithmetic can be realized by executing twice the multiplication of mutually different variables.

5 Through the procedure described above, determination of the x-coordinate corresponding to the scalar (d) multiplication of given coordinates (x, y) can be realized by executing eight times the multiplication processing for each bit of d . Furthermore, by setting
10 $[kx, k]$ for the given x-coordinate as the initial value for the scalar multiplication, where k represents a random number, the private key information can be protected against leakage in terms of the deviation information of the $d(x, y)$ processing time. Further,
15 this feature indicates that in the DPA (Differential Power Analysis) for realizing the cryptanalysis, the private key information can also be prevented from leakage as the deviation (or difference) information of the current (voltage, electric power) involved in the
20 processing of $d(x, y)$.

Next, description will be directed to a fifth embodiment of the present invention. In the case of the elliptic curve cryptography according to the second embodiment of the invention, it has been presumed that
25 $[X, Y, Z] = [\lambda^2 X, \lambda^3 Y, \lambda Z]$ applies valid for the given projective coordinate $\lambda \neq 0$. However, the teachings of the present invention can also be implemented with the projective coordinate system in which $[X, Y, Z] = [\lambda X,$

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$\lambda Y, \lambda Z]$ applies valid.

When the transformation from the affine coordinates to the projective coordinates can be given by $(x, y) \rightarrow [x, y, 1]$, then it applies valid that $Z_4 = 1$.

5 Scalar multiplication method according to fifth embodiment

Figures 15A and 15B are flow charts for illustrating the scalar multiplication method according to the fifth embodiment of the present invention.

10 Referring to the figures, it is presumed that a projective coordinate component X_0 of the x-coordinate of a given point R and a scalar value \underline{m} are inputted and that a projective coordinate component X_m of the x-coordinate of a point corresponding to m-multiplication
 15 of R (i.e., the point corresponding to the product of \underline{m} and R) is to be outputted. On the presumption, the scalar value \underline{m} and the projective coordinate component X_0 of the x-coordinate are inputted in the step 1502 shown in Fig. 15A. X_0 is assigned to X_1 in the step 1504. In a
 20 succeeding step 1505, "1" is assigned to Z_1 . In succession, $[X_1, Z_1]$ is assigned to $[X_4, Z_4]$ in a step 1506. Subsequently, $[X_1, Z_1]$ is inputted to the doubling arithmetic, the output of which is assigned to $[X_2, Z_2]$ (step 1507). Further, the scalar value \underline{m} is transformed
 25 to the binary bit string $h_i h_{i-1} \dots h_0$ (step 1508), where the most significant bit h_i is "1". Thus, "1" is assigned to \underline{i} in the step 1509 shown in Fig. 15B. In a succeeding step 1510, "i-1" is assigned to \underline{i} , which is then followed

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by a step 1511 where $[X_1, Z_1]$, $[X_2, Z_2]$ and Z_4 are inputted to the addition arithmetic, the output of which is assigned to $[X_3, Z_3]$. When $h_i == "0"$ in the step 1512 (i.e., when the decision step 1512 results in affirmation "Yes"), the processing proceeds to a step 1513 while it proceeds to a step 1515 when $h_i == "1"$, i.e., when the decision step 1512 results in negation "No". In the step 1513 shown in Fig. 15B, $[X_1, Z_1]$ is inputted to the doubling arithmetic, the output from which is assigned to $[X_1, Z_1]$. In the succeeding step 1514, $[X_3, Z_3]$ is assigned to $[X_2, Z_2]$, whereon the processing proceeds to a step 1517. On the other hand, when the decision step 1512 results in "No", $[X_2, Z_2]$ is inputted to the doubling arithmetic, the output of which is assigned to $[X_2, Z_2]$ (step 1515). Further, $[X_3, Z_3]$ is assigned to $[X_1, Z_1]$ in the step 1516, whereupon the processing proceeds to the step 1517. When $i > 0$ in the step 1517, i.e., when the step 1517 results in "Yes", the step 1510 is resumed. If otherwise, i.e., when the decision step 1517 results in "No", the processing proceeds to a step 1518 where $X_1/(Z_1)$ is assigned to the projective coordinate component X_m which is ultimately outputted in the step 1519, whereupon the processing comes to an end (step 1520).

Addition method according to fifth embodiment

Figure 16 is a flow chart for illustrating an addition method according to the fifth embodiment of the present invention. It is assumed that projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and X_4 are inputted, whereby

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$[X_3, Z_3]$ or the point at infinity is outputted. Thus, projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and X_4 are inputted in a step 1602. In the succeeding step 1603, X_1Z_2 is assigned to S_1 . Further, X_2Z_1 is assigned to S_2 in a step 1604 with $S_1 + S_2$ being assigned to B in a step 1605. When $B == 0$ in a step 1606 (i.e., when decision in the step 1606 results in "Yes"), the processing proceeds to a step 1607. If otherwise (i.e., when decision in the step 1606 results in "No"), the processing proceeds to a step 1608. In the step 1607, the point at infinity is outputted, whereon an end step 1613 is executed. On the other hand, unless $B = 0$ in the step 1606, B^2 is assigned to Z_3 . In the succeeding step 1608, S_1S_2 is assigned to S . Further, (X_4Z^3) is assigned to M in a step 1610 while $M + S$ is assigned to X_3 in a step 1611. Finally, $[X_3, Z_3]$ is outputted in a step 1612. Through the procedure described above, the addition arithmetic can be realized by executing four times the multiplication of mutually different variables. Parenthetically, as the doubling arithmetic according to the instant embodiment of the invention, the doubling arithmetic described hereinbefore can be adopted. Additionally, the method incarnated in the instant embodiment can also find application not only to the arithmetic with the elliptic curve in the finite field of characteristic 2 but also to the arithmetic with the elliptic curve in the prime field.

Sixth embodiment

Next, description will be made of the elliptic

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curve arithmetic unit according to a sixth embodiment of the present invention. Figure 9 is a functional block diagram showing schematically a structure of the elliptic curve arithmetic unit according to the sixth embodiment of the present invention. In the figure, reference numeral 901 denotes generally an elliptic curve arithmetic unit which corresponds to the one shown in Fig. 1 and designated by the reference numeral 109. Referring to Fig. 9, inputted to the elliptic curve arithmetic unit 901 are x-coordinate X_0 of a given point, a scalar value m and a parameter b of the elliptic curve of the standard form given by $y^2 + xy = x^3 + ax^2 + b$ in the finite field of characteristic 2 (extension field of "2"), as indicated by an arrow 902, whereby x-coordinate X_m of a point corresponding to m-multiplication of above-mentioned given point is outputted from the elliptic curve arithmetic unit 901, as indicated by an arrow 903. At this juncture, it should however be mentioned that although the instant embodiment of the invention is described in conjunction with the elliptic curve in the finite field of characteristic 2, the invention can equally be implemented with the elliptic curve in the prime field.

The elliptic curve arithmetic unit 901 includes a random number generation module 904 for generating a random number k to be outputted, as indicated by an arrow 905. The random number k generated by the random number generation module 904 is inputted to a projective

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coordinate transformation module 906 together with the
x-coordinate X_0 , the scalar value \underline{m} and the parameter \underline{b}
although they are not shown in Fig. 9, to be thereby
transformed to the projective coordinates $[kX_0, k]$, which
5 is then assigned to $[X_1, Z_1]$. The projective coordinate
 $[X_1, Z_1]$ and the scalar value \underline{m} are inputted to a scalar
multiplication module 908, whereby a point given by $[X_1,$
 $Z_1]$ multiplied by \underline{m} is determined. Thus, the x-coordinate
 X_m of the point as determined is outputted from the scalar
10 multiplication module 908. In the scalar multiplication
module 908, $[X_1, Z_1]$ is first assigned to $[X_4, Z_4]$ which
may be previously stored in a memory incorporated, for
example, in the scalar multiplication module. Further,
the projective coordinates $[X_1, Z_1]$ are supplied to a
15 doubling arithmetic module 913 for determining a double
point $[X_2, Z_2]$. Subsequently, \underline{m} is developed to a binary
bit string. Every time the bit assumes "0", starting
from the more significant bit, $[X_1, Z_1]$ is supplied to the
doubling arithmetic module 913, whereon the double point
20 outputted from the doubling arithmetic module 913 is
assigned to $[X_1, Z_1]$. Subsequently, projective
coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted to
an addition arithmetic module 910, and the addition point
outputted from the addition arithmetic module 910 is
25 assigned to $[X_2, Z_2]$. On the other hand, when the bit is
"1", the projective coordinates $[X_2, Z_2]$ are outputted to
the doubling arithmetic module 913, whereon the double
point outputted from the doubling arithmetic module 913

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is assigned to $[X_2, Z_2]$. Subsequently, the projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ are inputted to the addition arithmetic module 910, and the addition point outputted from the addition arithmetic module 910
 5 is assigned to $[X_1, Z_1]$. Thus, there is derived the X_m -coordinate of the m-tuple point.

Inputted to the addition arithmetic module 910 is $[X_1, Z_1]$, $[X_2, Z_2]$, $[X_4, Z_4]$ for arithmetically determining $[X_3, Z_3]$ which satisfies the conditions that
 10 $[X_3, Z_3] = [X_2, Z_2] + [X_1, Z_1]$ and that $[X_4, Z_4] = [X_2, Z_2] - [X_1, Z_1]$. The coordinates $[X_3, Z_3]$ are then outputted from the addition arithmetic module 910.

More specifically, assigning arithmetics $S_1 \leftarrow X_1 Z_2^2$, $S_2 \leftarrow X_2 Z_1^2$ and $B \leftarrow S_1 + S_2$ are first executed.
 15 When $B == 0$, the point at infinity is outputted, whereupon the processing comes to an end. Unless $B = 0$, assigning arithmetics $Z_3 \leftarrow Z_4 B$, $S \leftarrow Z_4^2 S_1 S_2$, $M \leftarrow X_4 Z_3^2$ and $X_3 \leftarrow M + S$ are executed.

Inputted to the doubling arithmetic module 913
 20 are $[X_1, Z_1]$ and \underline{b} for arithmetically determining the coordinates $[X_2, Z_2]$ which satisfy the conditions that $[X_2, Z_2] = [X_1, Z_1] + [X_1, Z_1]$. The coordinates $[X_2, Z_2]$ are then outputted from the doubling arithmetic module 913. In the case where $X_1 == 0$ or $Z_1 == 0$, the point at infinity
 25 is outputted. If otherwise, assigning arithmetics $S \leftarrow Z_1^2$, $Z_2 \leftarrow X_1 S$ and $X_2 \leftarrow X_1^4 + b(S)^4$ are executed.

In the case of the embodiment described above, it has been assumed that the x-coordinate X_0 is

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transformed to the projective coordinates $[kX_0, k]$.
However, it goes without saying that the teachings of the
present invention can equally be applied to the
transformation of the x-coordinate X_0 to the projective
5 coordinates $[k^2X_0, k]$.

Finally, it should be added that the methods
according to the embodiments of the invention described
in the foregoing can be stored in a recording medium in
the form of a program or programs executable with a
10 computer without departing from the spirit and scope of
the present invention.

As will be appreciated from the foregoing
description, the elliptic curve encryption processing can
be executed at a significantly increased speed according
15 to the teachings of the invention when compared with the
conventional cryptograph technologies. Furthermore, by
virtue of such arrangement that the processing time for
 $d(x, y)$ does not depend on the bit pattern of d in
realization of the elliptic curve cryptography, the
20 private key information can be protected against leakage
from or in terms of the deviation information.

Many modifications and variations of the
present invention are possible in the light of the above
techniques. It is therefore to be understood that within
25 the scope of the appended claims, the invention may be
practiced otherwise than as specifically described.

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WHAT IS CLAIMED IS:

1. A method of implementing an elliptic curve cryptography in a finite field of characteristic 2 (or an extension field of "2"), in which said elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ and in which x and y are variables in an x - y coordinate system, a and b are parameters, addition of points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on said elliptic curve composed of points defined by individual coordinate components is presumed to be represented by $P_3(x_3, y_3)$ with subtraction of said points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ being presumed to be represented by $P_4(x_4, y_4)$, comprising the steps of:
 - inputting the coordinate component x_1 ;
 - transforming said inputted coordinate component x_1 into X - and Z -coordinates $[X_1, Z_1]$ of a projective space where Z is a variable in the Z -coordinate;
 - storing said coordinates $[X_1, Z_1]$ of said projective space;
 - transforming said coordinate component x_2 into coordinates $[X_2, Z_2]$ of said projective space;
 - storing said projective coordinate $[X_2, Z_2]$;
 - transforming said coordinate component x_4 into coordinates $[X_4, Z_4]$ of said projective space;
 - storing said projective coordinates $[X_4, Z_4]$;
 - determining projective coordinates $[X_3, Z_3]$ from said stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$;
 - transforming said projective coordinates $[X_3,$

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- Z_3] into said coordinate component x_3 ; and
outputting said coordinate component x_3 ,
whereby scalar multiplication of said point
 $P_1(x_1, y_1)$ is determined.
2. A method of implementing an elliptic curve
cryptography according to claim 1,
further comprising the steps of:
generating a random number k ;
storing said generated random number k ;
transforming the x -coordinates into projective
coordinates to thereby derive projective coordinates $[k^2x,$
 $k]$ through arithmetic operation of individual coordinate
components of said projective space and said stored
random number k .
3. A method of implementing an elliptic curve
cryptography according to claim 1,
further comprising the steps of:
generating a random number k ;
storing said generated random number k ;
transforming the x -coordinates into projective
coordinates to thereby derive projective coordinates $[kx,$
 $k]$ through arithmetic operation of individual coordinate
components of said projective space and said stored
random number k .
4. A method of implementing an elliptic curve
cryptography according to claim 1,
wherein the step of determining said projective
coordinates $[X_3, Z_3]$ susceptible to the transformation

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into said coordinate component x_3 from said stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ includes the substeps of:

computing $B = X_1Z_2^2 + X_2Z_1^2$;

storing said computed B ;

deciding whether or not said stored B satisfies condition that $B = 0$;

outputting a point at infinity when $B = 0$ while arithmetically determining $Z_3 = Z_4B$ unless $B = 0$;

storing said determined Z_3 ; and

arithmetically determining $X_3 = X_4B^2 + X_1X_2Z_1^2Z_2^2Z_4^2$ from said stored Z_3 .

5. A method of implementing an elliptic curve cryptography according to claim 1,

wherein the step of determining said projective coordinates $[X_3, Z_3]$ susceptible to transformation into said coordinate component x_3 from said stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ includes the substeps of:

computing $B = X_1Z_2 + X_2Z_1$;

storing said computed B ;

deciding whether or not said stored B satisfies condition that $B = 0$; and

outputting a point at infinity when $B = 0$ while determining arithmetically $Z_3 = Z_4B^2$ and $X_3 = X_4B^2 + X_1X_2Z_1Z_2Z_4$ unless $B = 0$.

6. An apparatus implementing an elliptic curve cryptography in a finite field of characteristic 2 (or an

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extension field of "2"), in which x and y are variables in an x - y coordinate system, a and b are parameters, said elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$, comprising:

5 random number generating means for generating a random number k ;

projective coordinate transformation means receiving as inputs thereto coordinate x_0 of said finite field of characteristic 2 and said random number k , to
10 thereby transform said coordinate x_0 into projective coordinates $[kx_0, k] = [X_1, Z_1]$;

doubling arithmetic means for arithmetically determining a double point from said projective coordinates $[X_1, Z_1]$;

15 addition arithmetic means for determining an addition point from said projective coordinate $[X_1, Z_1]$ where Z is a variable in the Z -coordinate to thereby output said addition point; and

scalar multiplication means receiving
20 information from said projective coordinate transformation means, said doubling arithmetic means and said addition arithmetic means to thereby perform scalar multiplication of the coordinate component x_0 .

7. A computer-readable memory having recorded
25 thereon statements and instructions for use in the execution in a computer of the method of claim 1, said statements for the execution of the method comprising the statements of:

inputting a coordinate component x_1 ;

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transforming said inputted coordinate component
x1 into X- and Z-coordinates $[X_1, Z_1]$ in a projective
space;

storing said coordinates $[X_1, Z_1]$ of said
5 projective space;

transforming said coordinate component x2 into
coordinates $[X_2, Z_2]$ of said projective space;

storing said projective coordinate $[X_2, Z_2]$
where Z is a variable in the Z-coordinate;

10 transforming said coordinate component x4 into
coordinates $[X_4, Z_4]$ of said projective space;

storing said projective coordinates $[X_4, Z_4]$;

determining projective coordinates $[X_3, Z_3]$ from
said stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and
15 $[X_4, Z_4]$;

transforming said projective coordinates $[X_3,$
 $Z_3]$ into said coordinate component x3; and

outputting said coordinate component x3,
whereby scalar multiplication of said point
20 $P_1(x_1, y_1)$ is determined.

8. A computer-readable memory according to claim 7,
said statements for the execution of the method
further comprising the statements of:

generating a random number \underline{k} ;

25 storing said generated random number \underline{k} ;

transforming the x-coordinates into projective
coordinates to thereby derive projective coordinates
 $[k^2x, k]$ through arithmetic operation of individual
coordinate components of said projective space and said
30 stored random number \underline{k} .

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9. A computer-readable memory according to claim 7,
said statements for the execution of the method
further comprising the statements of:
generating a random number k;
5 storing said generated random number k;
transforming the x-coordinates into projective
coordinates to thereby derive projective coordinates [kx,
k] through arithmetic operation of individual coordinate
components of said projective space and said stored
10 random number k.
10. A computer-readable memory according to claim 7,
wherein the statement of determining said
projective coordinates [X₃, Z₃] susceptible to
transformation into said coordinate component x₃ from
15 said stored projective coordinates [X₁, Z₁], [X₂, Z₂] and
[X₄, Z₄] includes further the statements of:
computing $B = X_1Z_2^2 + X_2Z_1^2$;
storing said computed B;
deciding whether or not said stored B satisfies
20 condition that $B = 0$;
outputting a point at infinity when $B = 0$ while
determining arithmetically $Z_3 = Z_4B$ unless $B = 0$;
storing said determined Z₃; and
determining arithmetically X₃
25 $= X_4Z_3^2 + X_1X_2Z_1^2Z_2^2Z_4^2$ from said stored Z₃.
11. A computer-readable memory according to claim 7,
wherein the statement of determining said
projective coordinates [X₃, Z₃] susceptible to
transformation into said coordinate component x₃ from

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said stored projective coordinates $[X_1, Z_1]$, $[X_2, Z_2]$ and $[X_4, Z_4]$ includes further the statements of:

computing $B = X_1Z_2 + X_2Z_1$;

storing said computed B;

5 deciding whether or not said stored B satisfies condition that $B = 0$; and

outputting a point at infinity when $B = 0$ while determining arithmetically $Z_3 = Z_4B^2$ and $X_3 = X_4B^2 + X_1X_2Z_1Z_2Z_4$ unless $B = 0$.

10 12. A computer-program product, comprising: a memory having computer-readable code embodied therein for implementing an elliptic curve cryptography in a finite field of characteristic 2 (or an extension field of "2"), in which said elliptic curve is given by $y^2 + xy = x^3 + ax^2 + b$ and in which x and y are variables in an
 15 x-y coordinate system, a and b are parameters, addition of points $P1(x1, y1)$ and $P2(x2, y2)$ on said elliptic curve composed of points defined by individual coordinate components is presumed to be represented by
 20 $P3(x3, y3)$ with subtraction of said points $P1(x1, y1)$ and $P2(x2, y2)$ being presumed to be represented by $P4(x4, y4)$, comprising:

code for inputting a coordinate component x1;

code for transforming said inputted coordinate

25 component x1 into X- and Z-coordinates $[X_1, Z_1]$ in a projective space;

code for storing said coordinates $[X_1, Z_1]$ of said projective space;

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code for transforming said coordinate
component x_2 into coordinates $[X_2, Z_2]$ of said projective
space;

code for storing said projective coordinate
5 $[X_2, Z_2]$ where Z is a variable in the Z -coordinate;

code for transforming said coordinate
component x_4 into coordinates $[X_4, Z_4]$ of said projective
space;

code for storing said projective coordinates
10 $[X_4, Z_4]$;

code for determining projective coordinates
 $[X_3, Z_3]$ from said stored projective coordinates $[X_1, Z_1]$,
 $[X_2, Z_2]$ and $[X_4, Z_4]$;

code for transforming said projective
15 coordinates $[X_3, Z_3]$ into said coordinate component x_3 ;
and

code for outputting said coordinate component
 x_3 , whereby scalar multiplication of said point $P_1(x_1,$
 $y_1)$ is determined.

FIG. 1

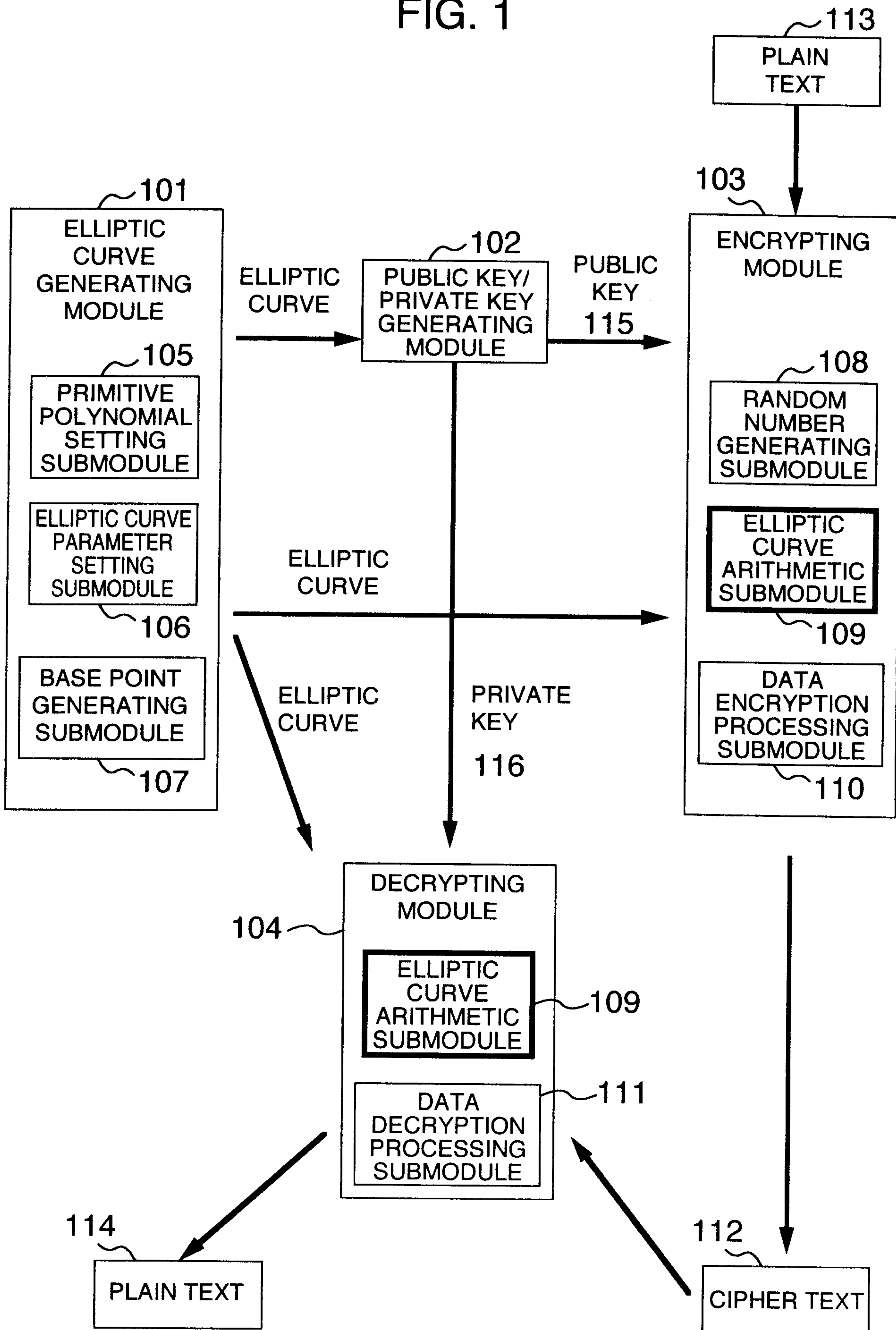


FIG. 2

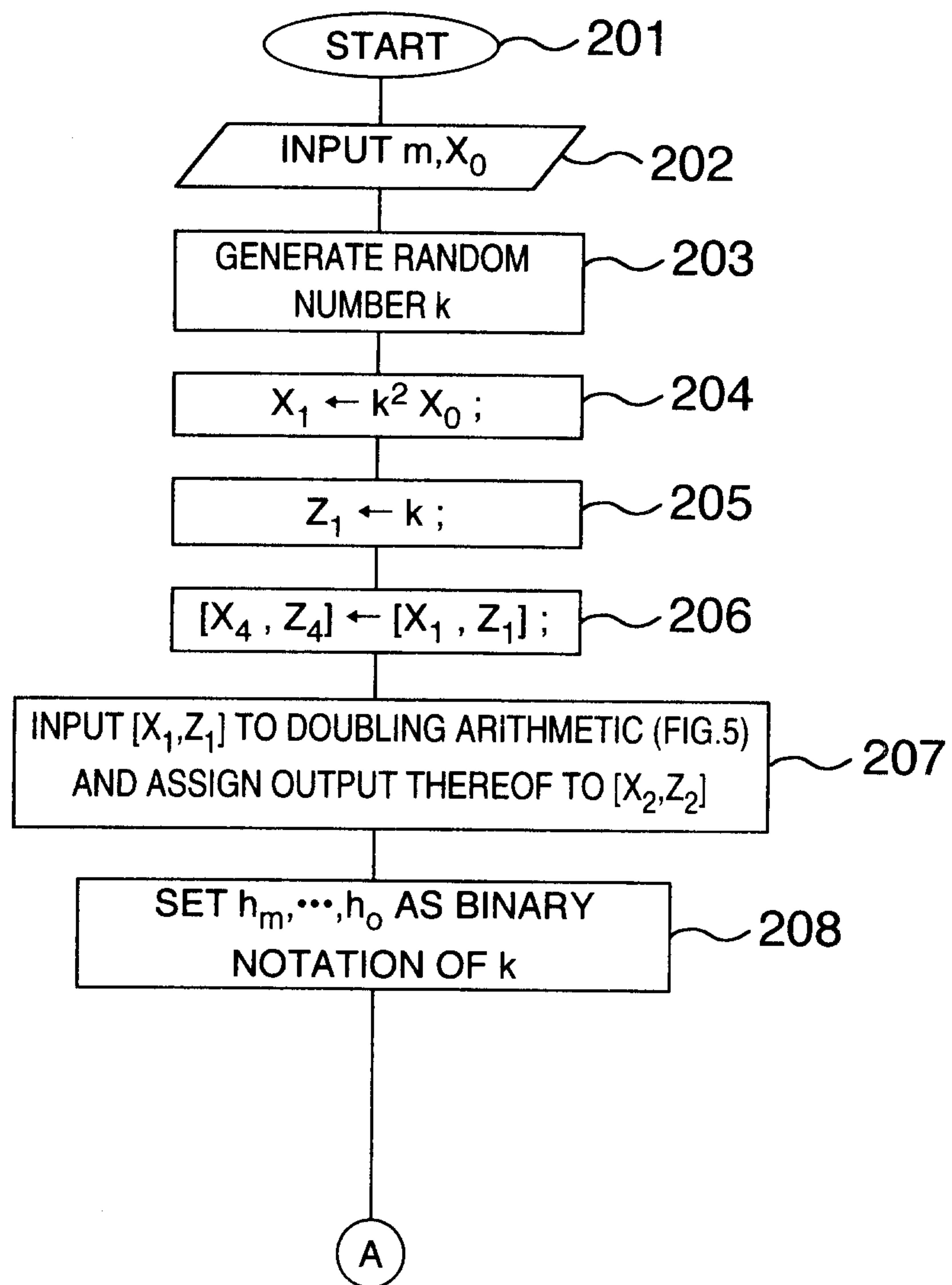


FIG. 3

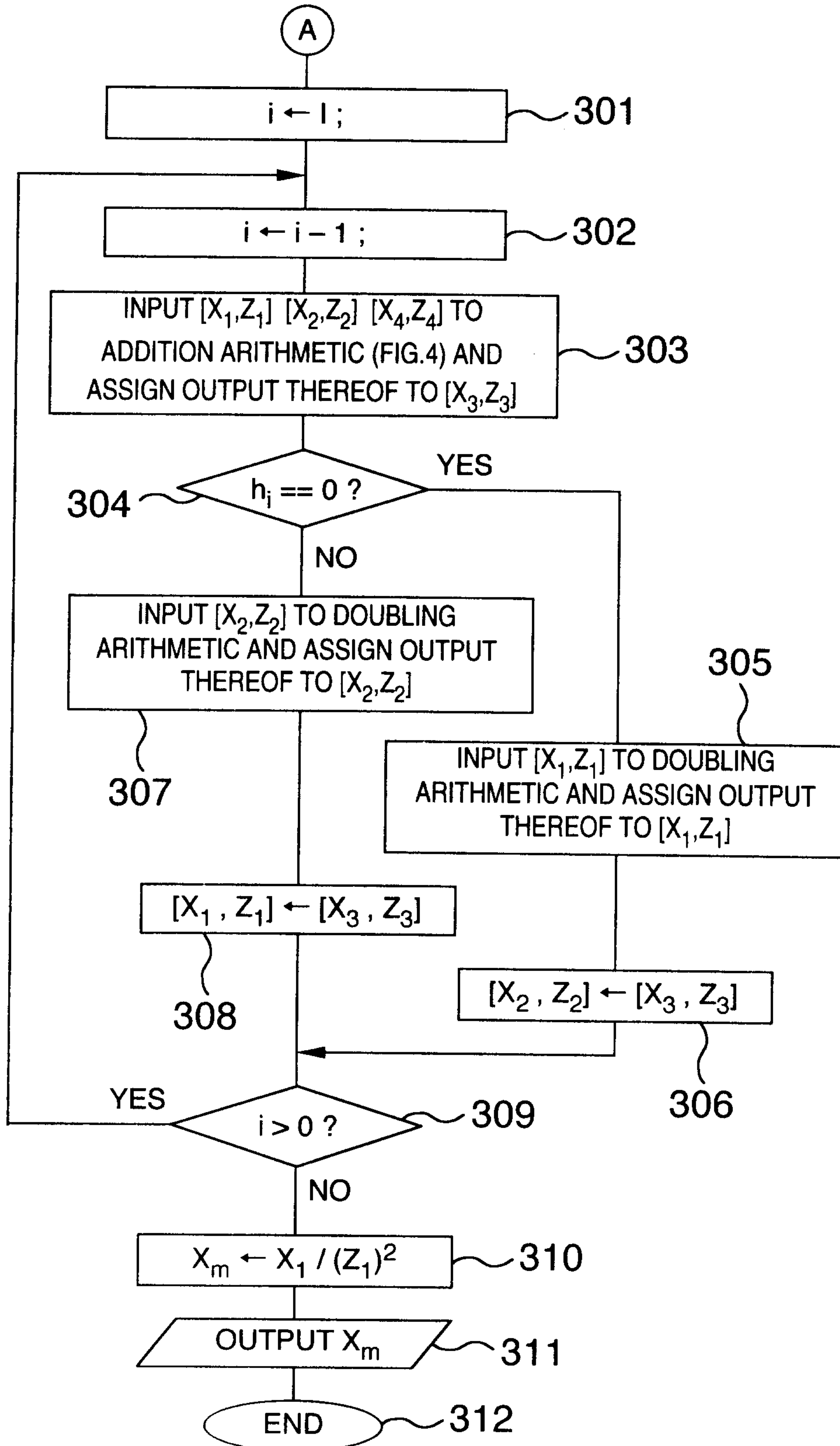


FIG. 4

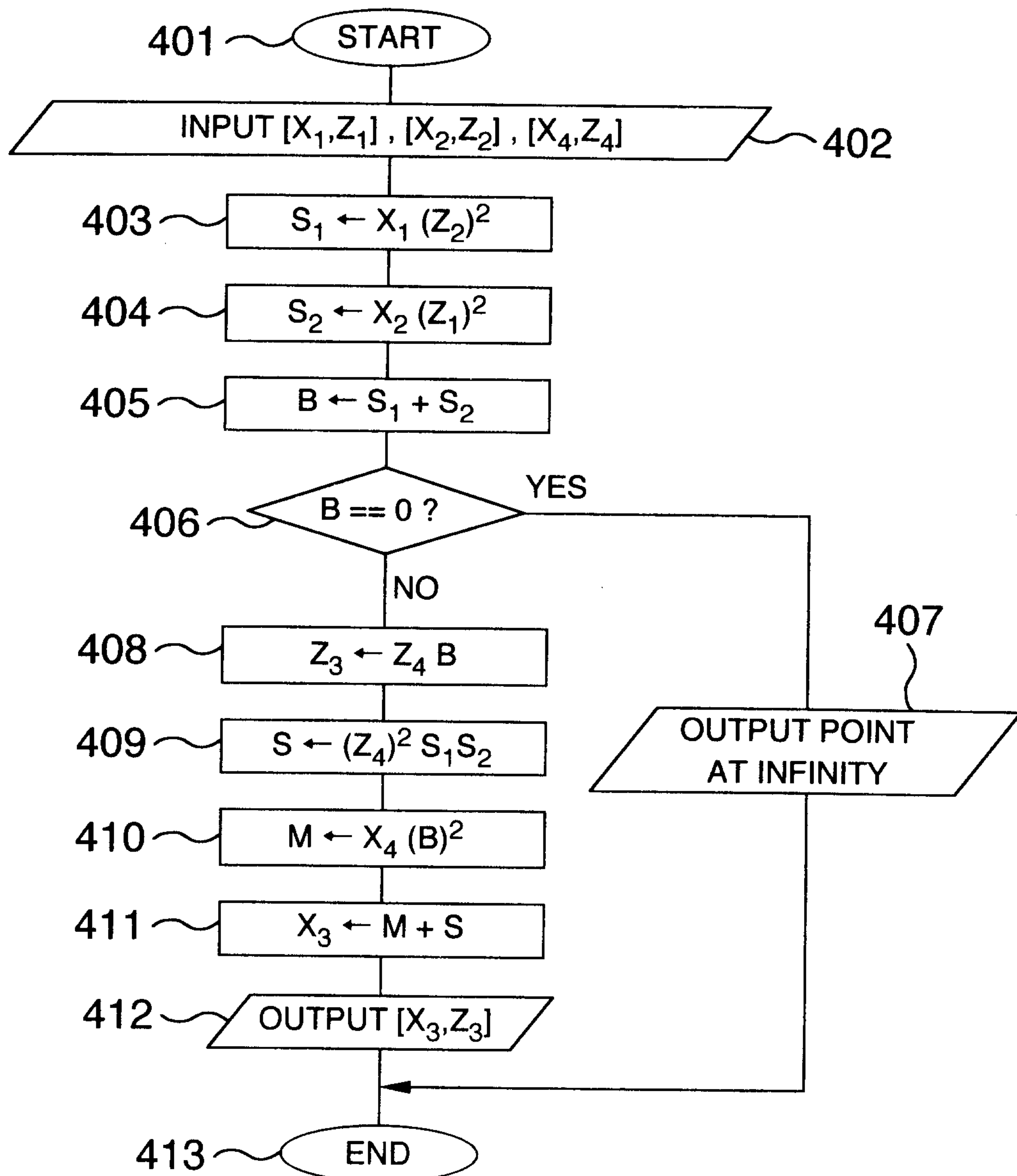


FIG. 5

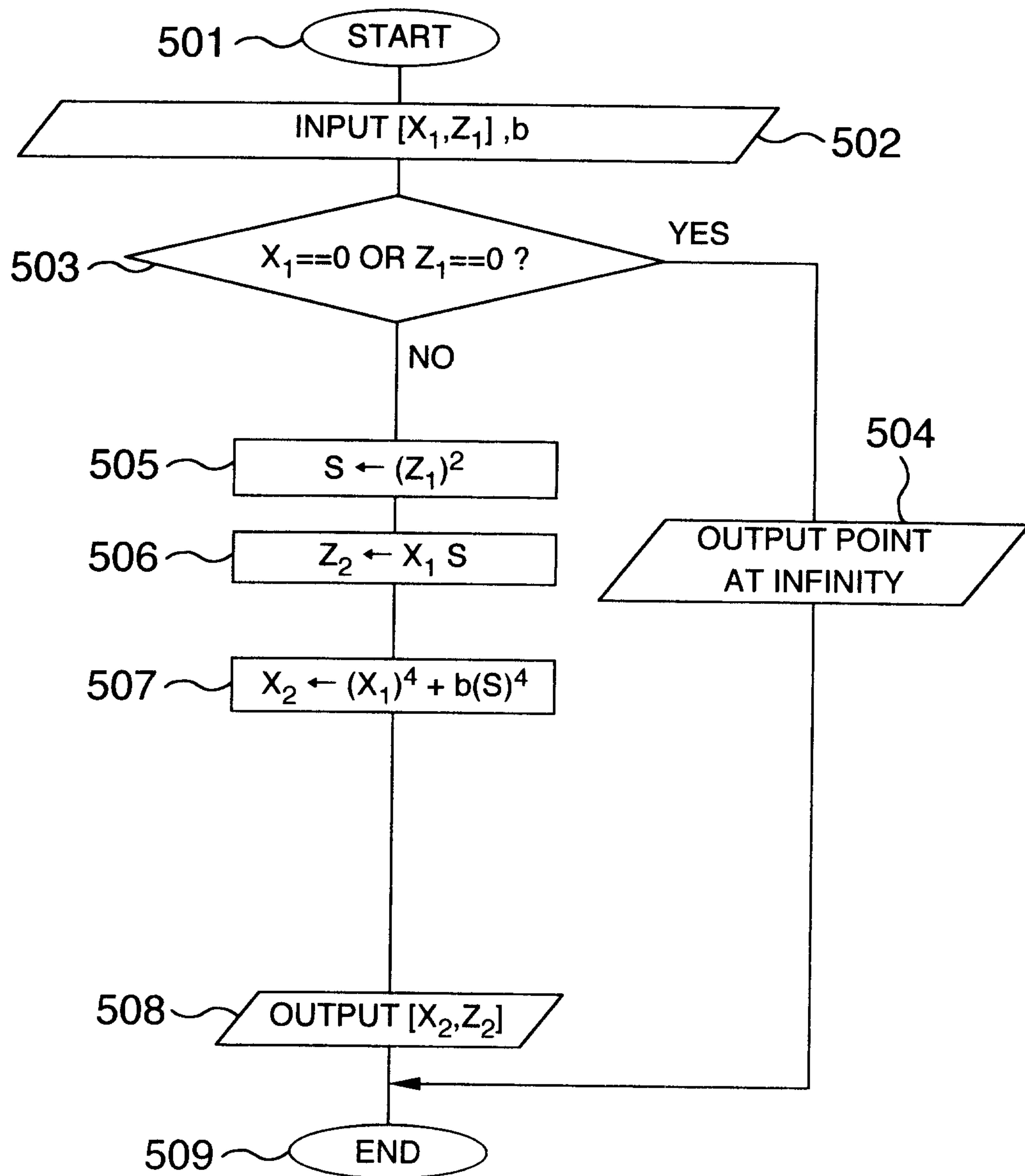


FIG. 6

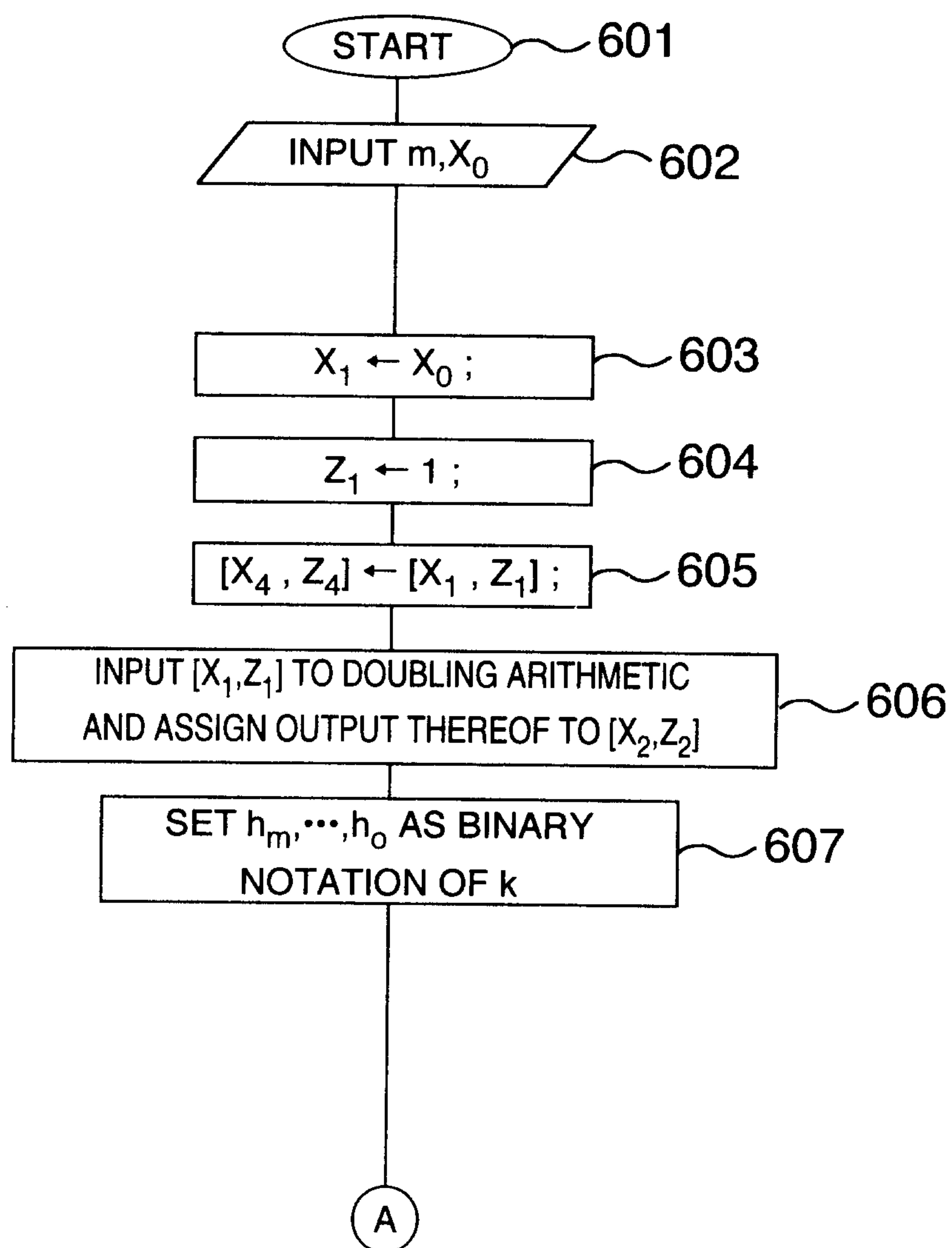


FIG. 7

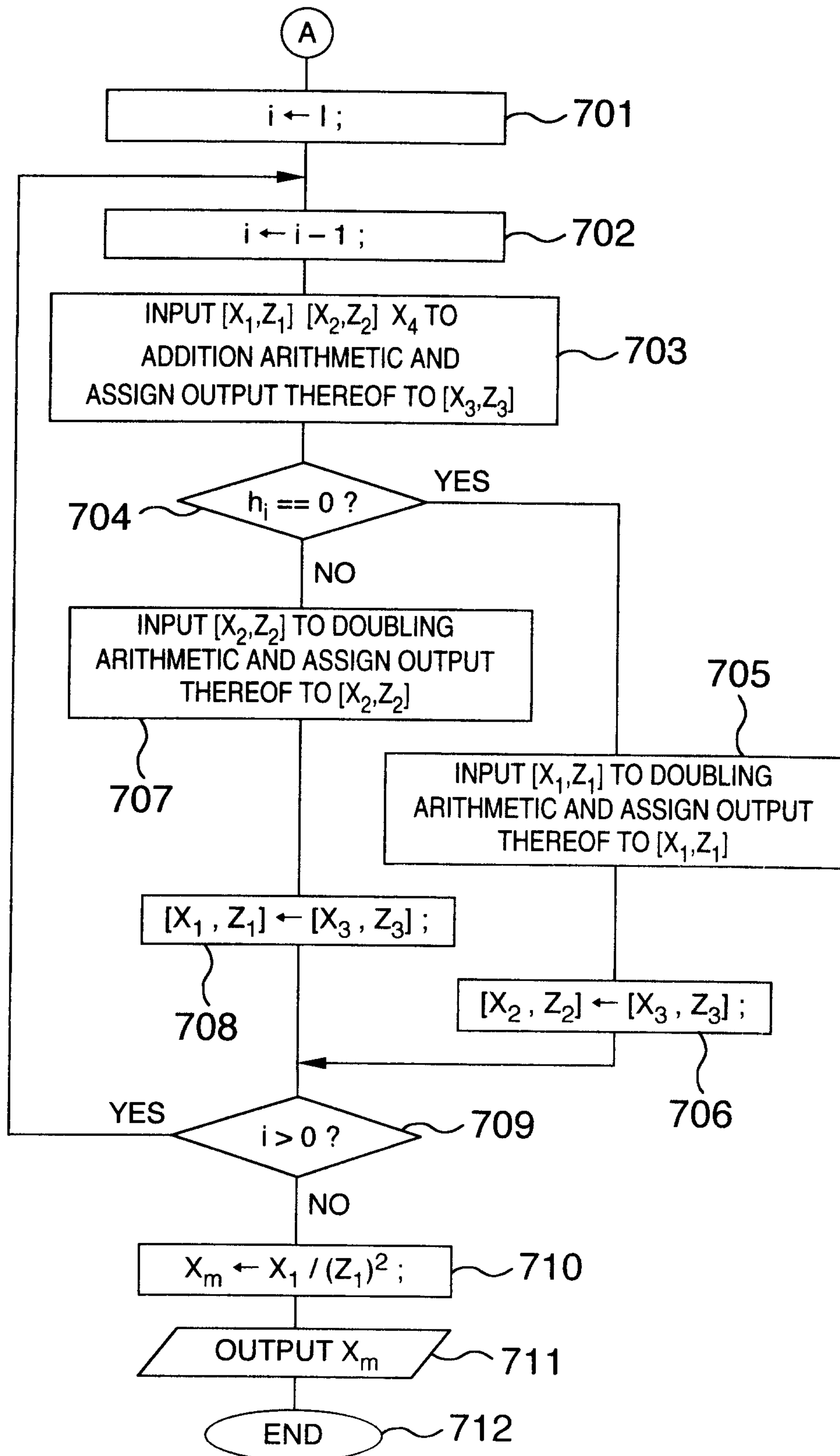


FIG. 8

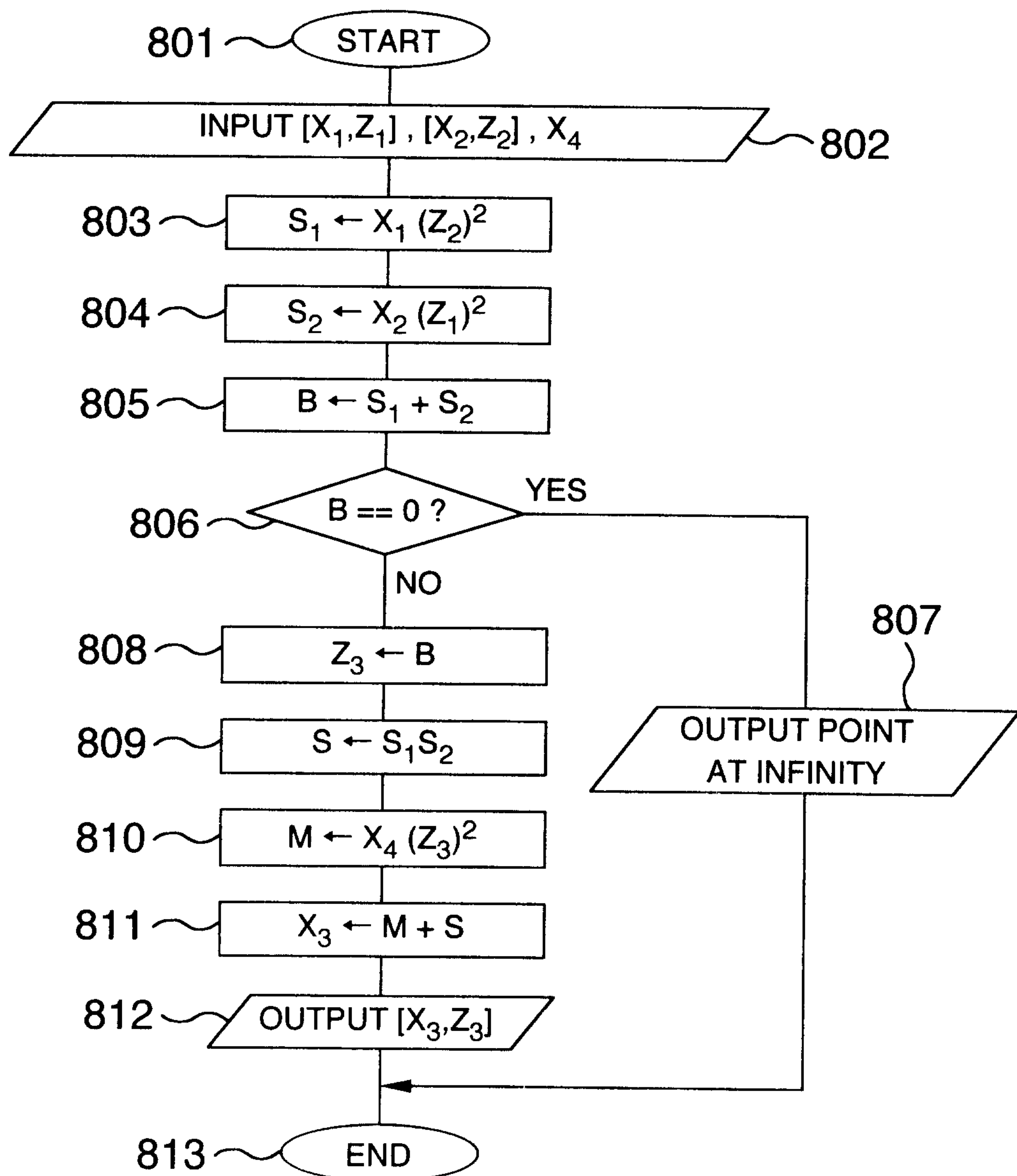
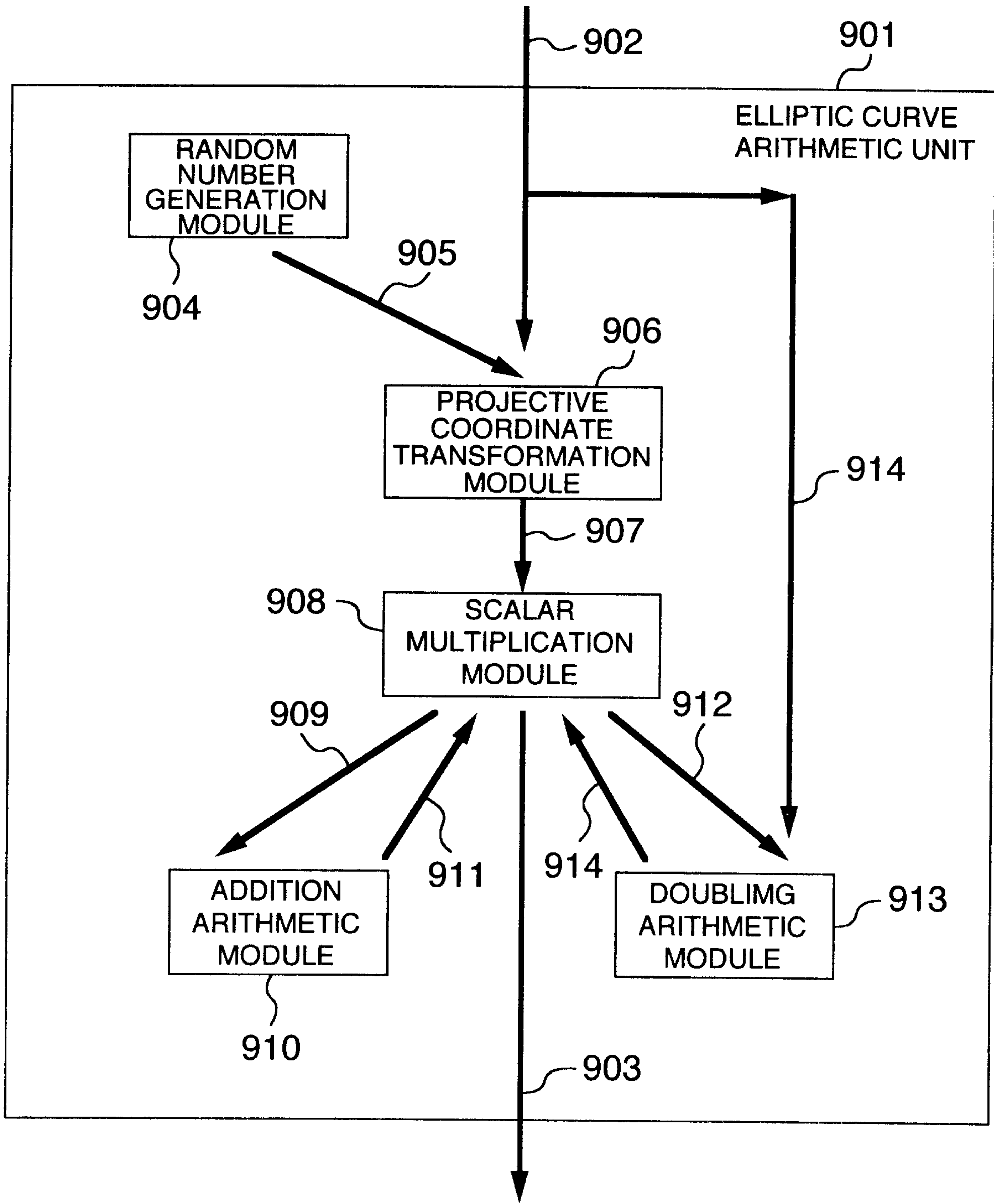


FIG. 9



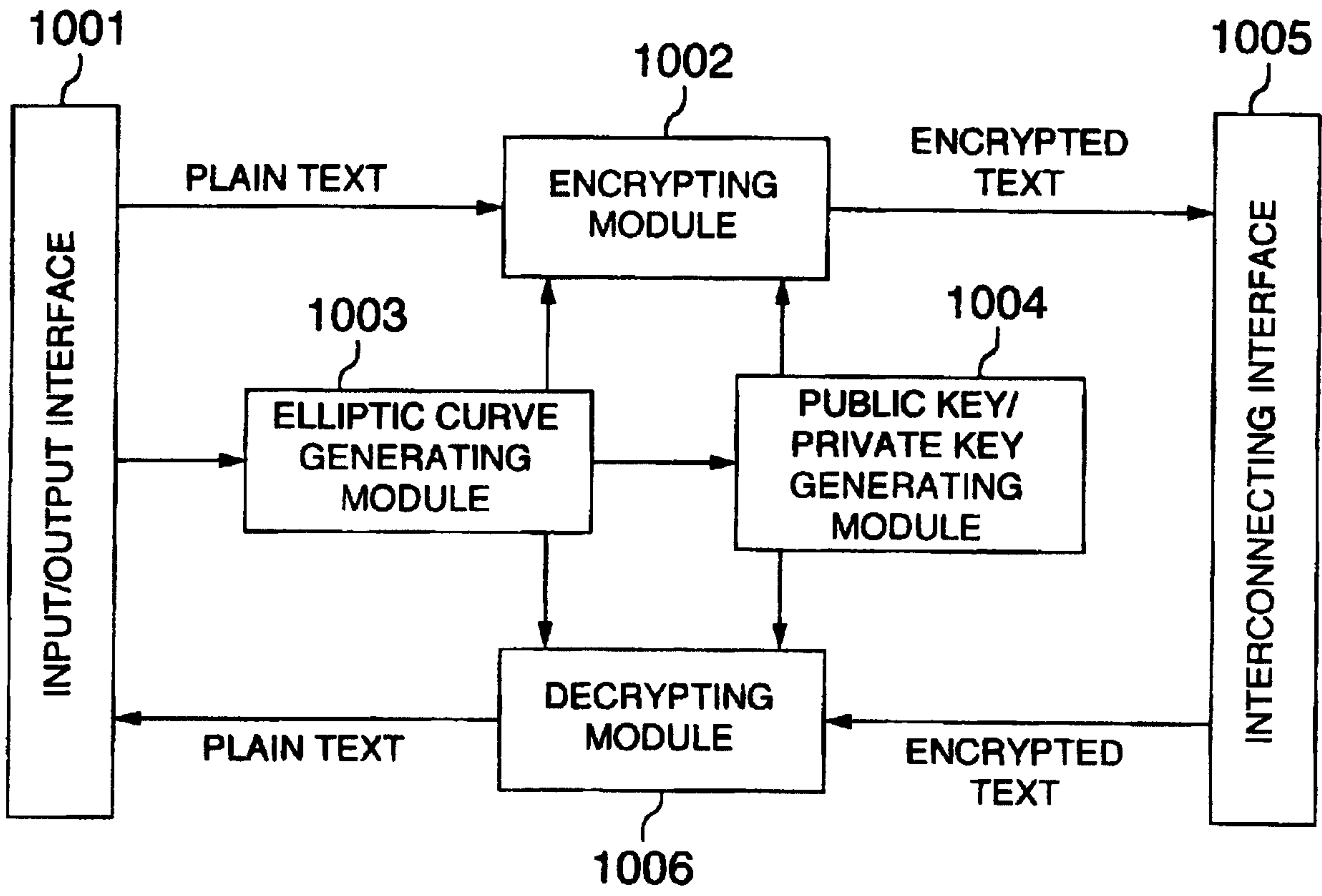


FIG. 11A

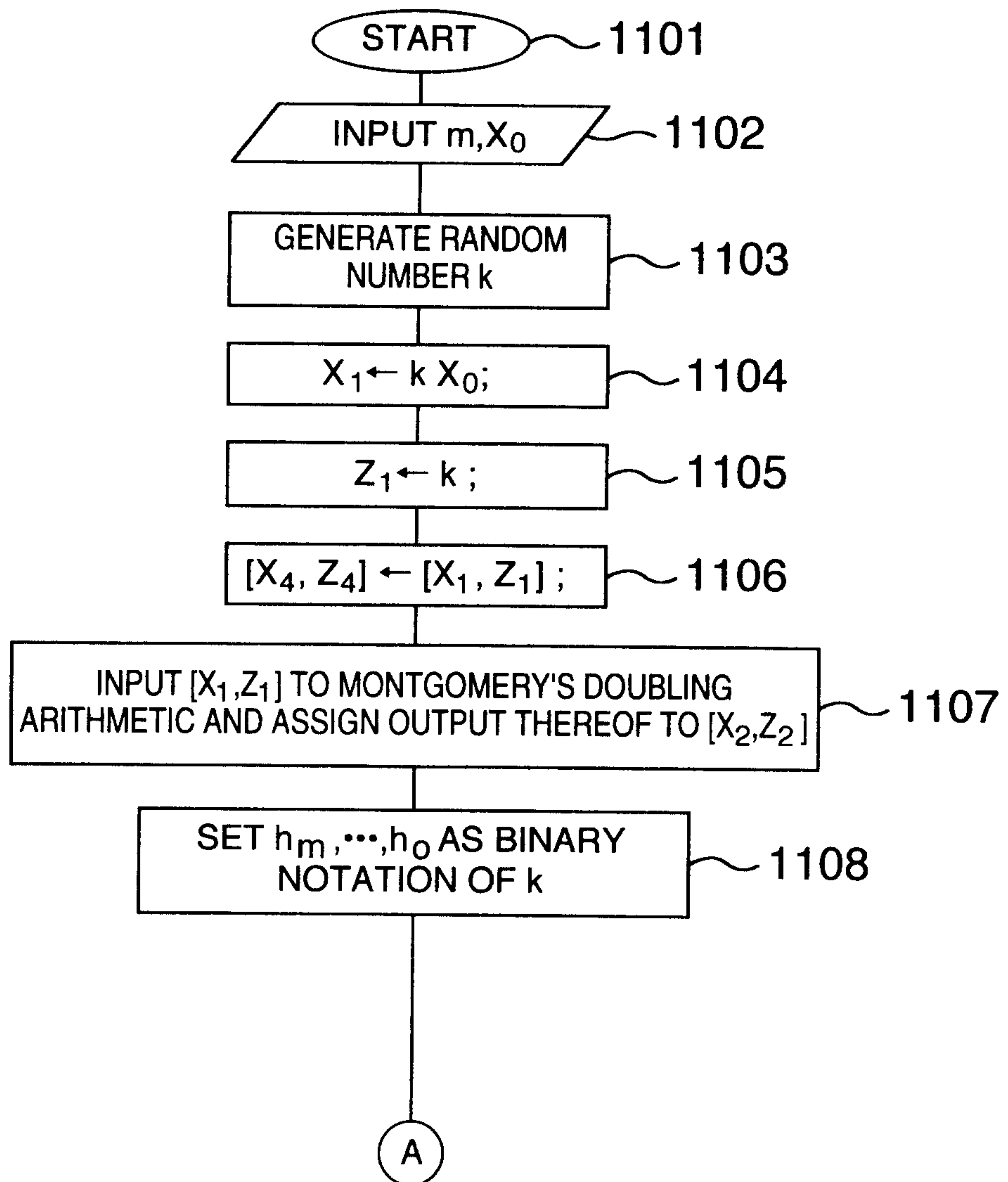


FIG. 11B

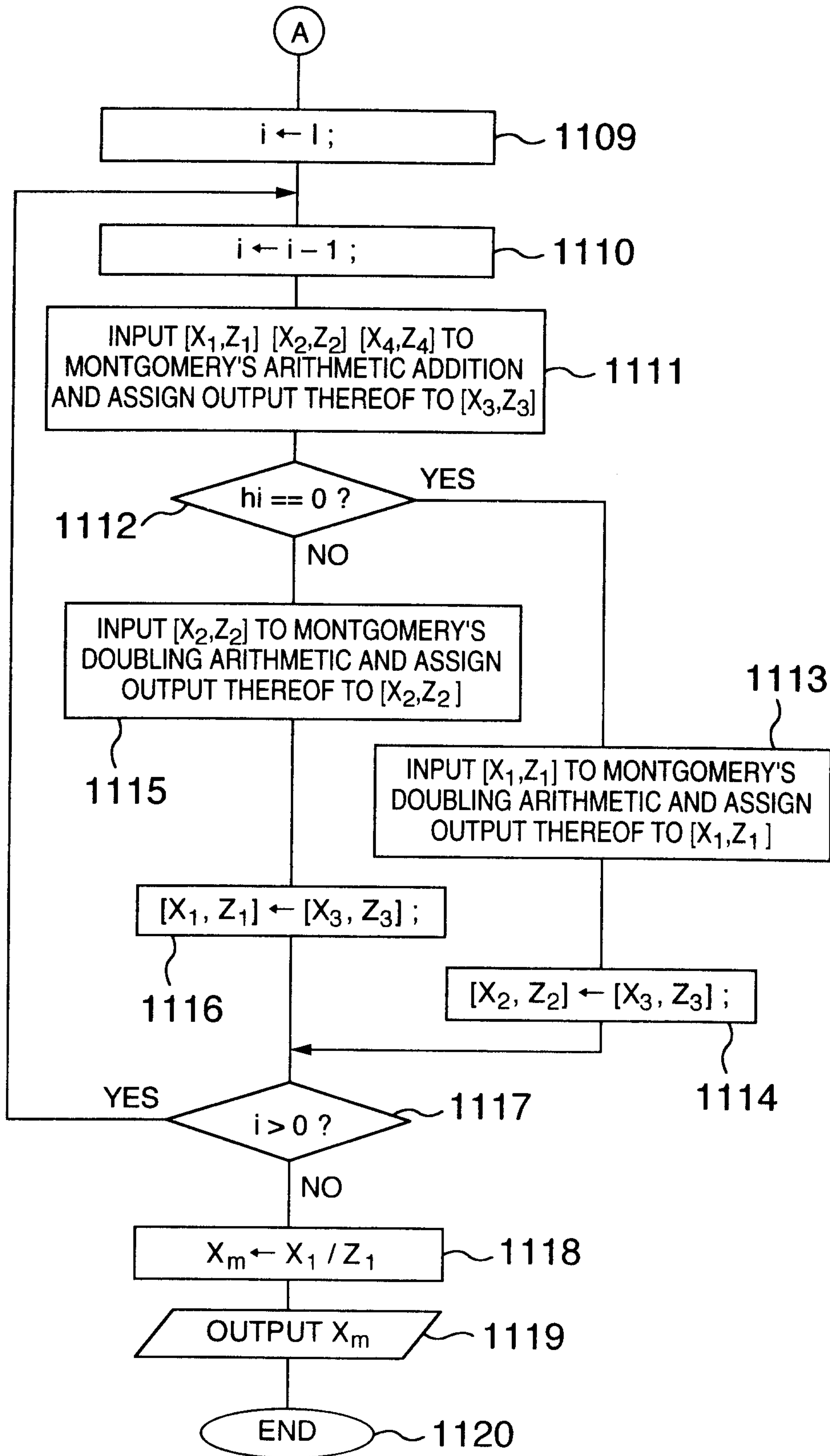


FIG. 12A

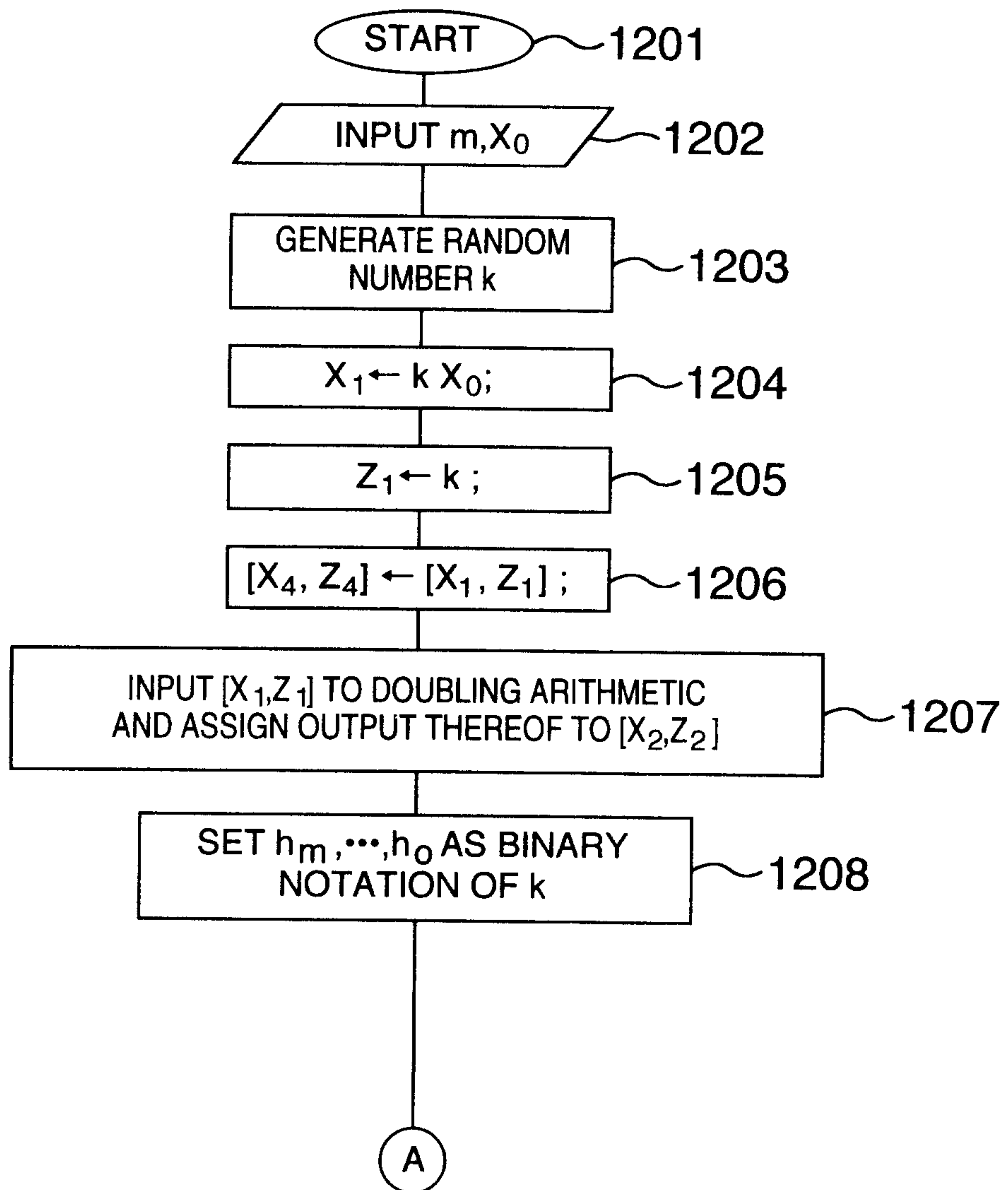


FIG. 12B

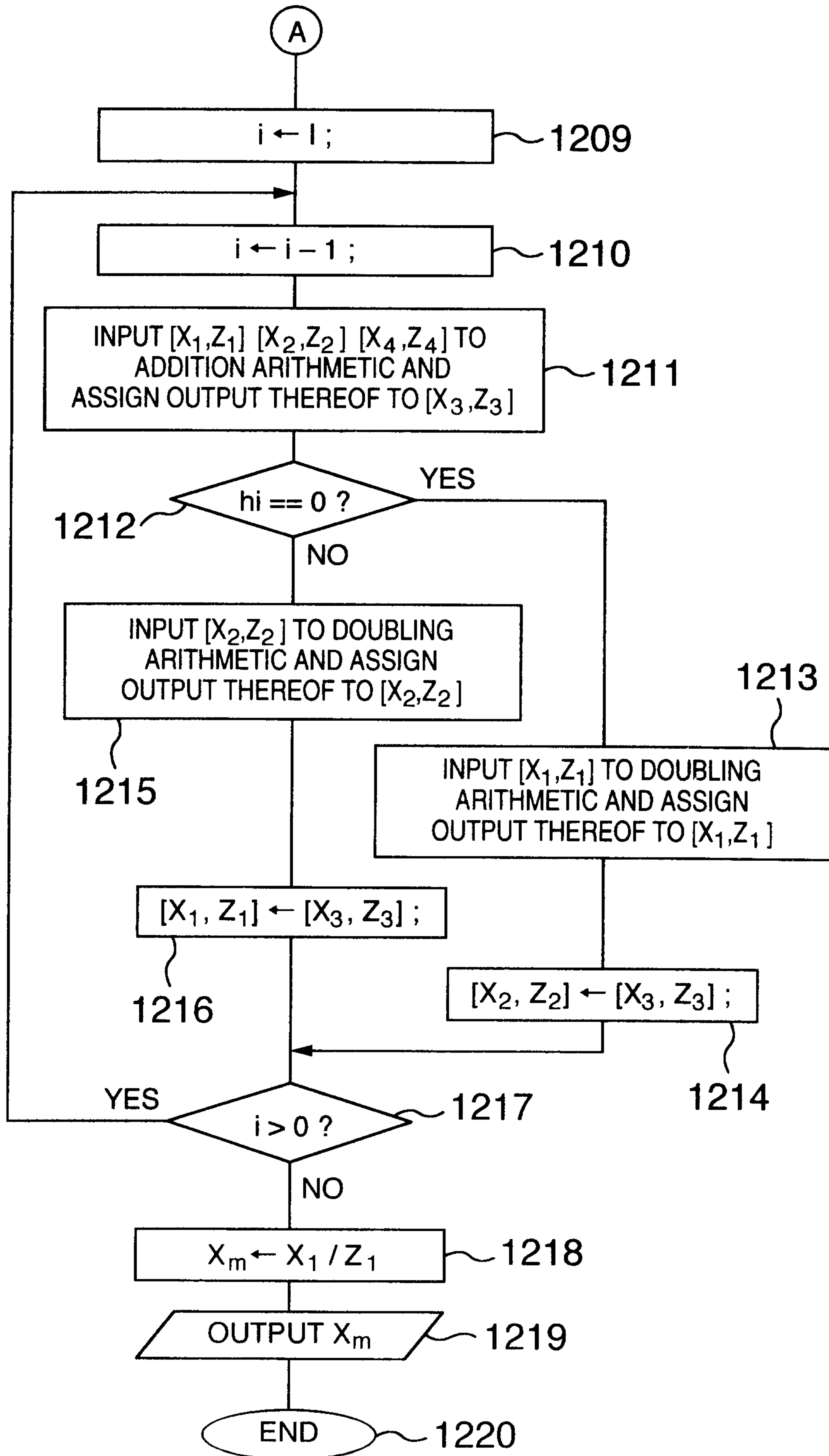


FIG. 13

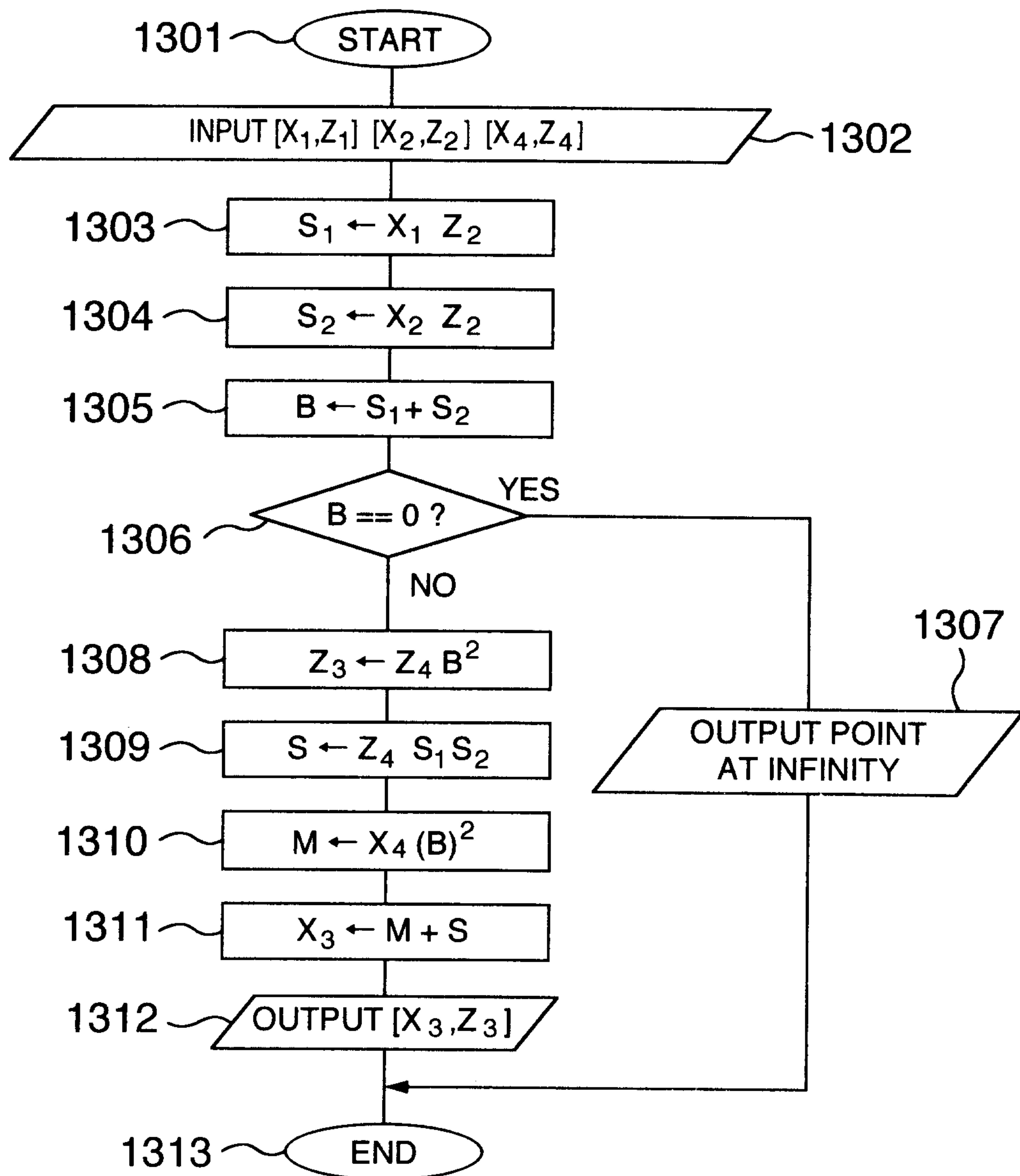


FIG. 14

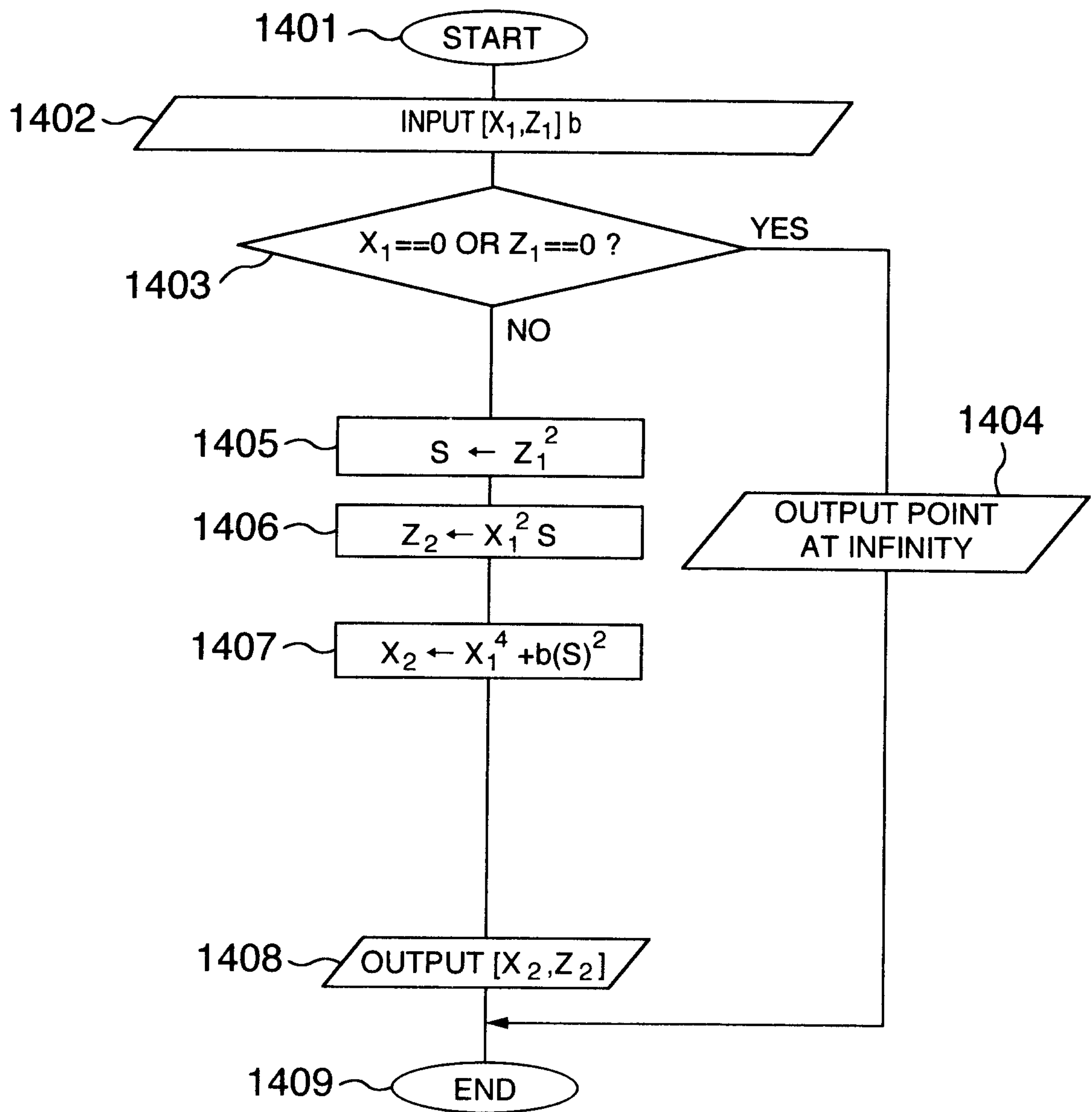


FIG. 15A

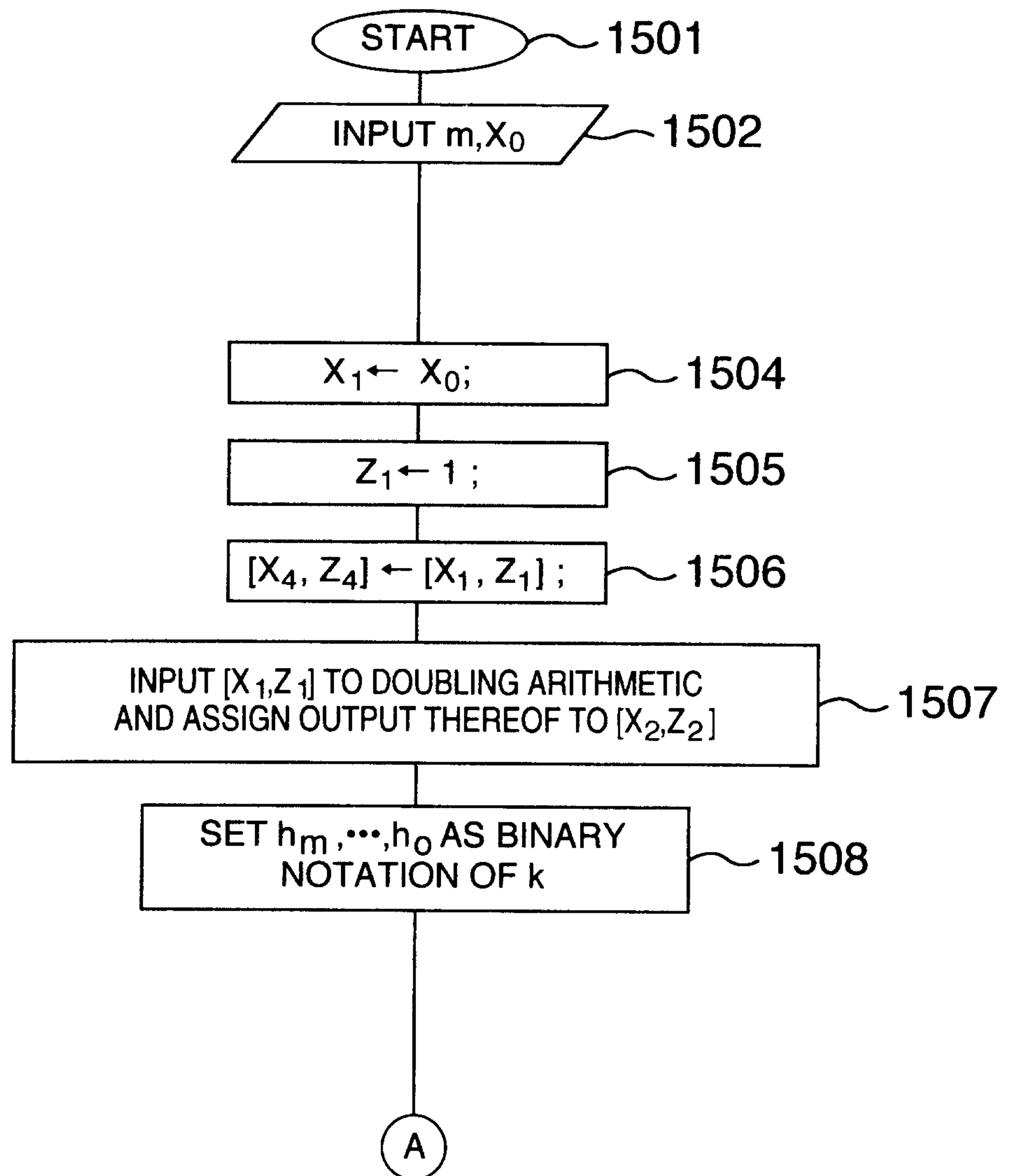


FIG. 15B

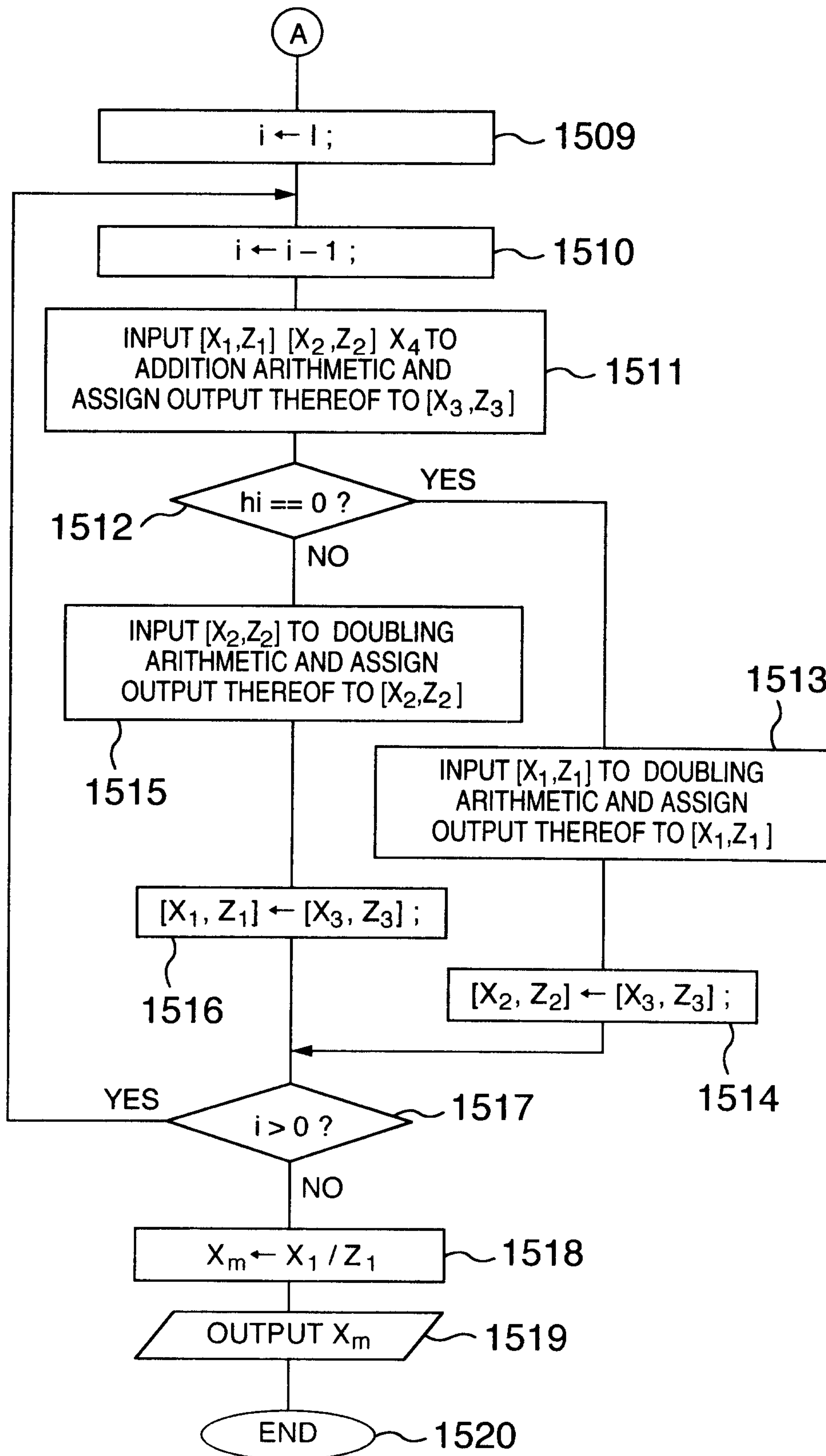


FIG. 16

