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Kerman

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(54) **FOUR SPIN COUPLERS FOR QUANTUM INFORMATION PROCESSING**

USPC 327/528
See application file for complete search history.

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H01L 39/02 (2006.01)

(57) **ABSTRACT**

Structures and techniques, using superconducting Josephson-junction based circuits, to directly engineer physical multiqubit (or “many-qubit”) interactions in a non-perturbative manner. In one embodiment, a system for multiqubit interaction includes: a multispin coupler including a plurality of loops, each loop having a pair of Josephson junctions; and a plurality of qubits each inductively coupled to the multispin coupler.

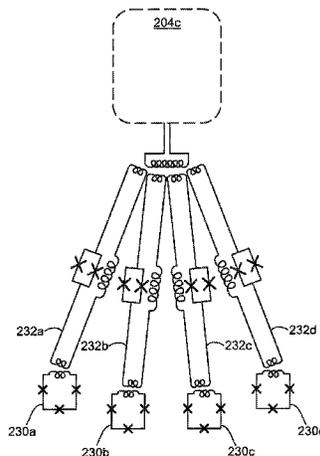
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(58) **Field of Classification Search**

CPC H03K 17/92; H03K 3/38; B82Y 10/00; G06N 99/002; H01L 39/223

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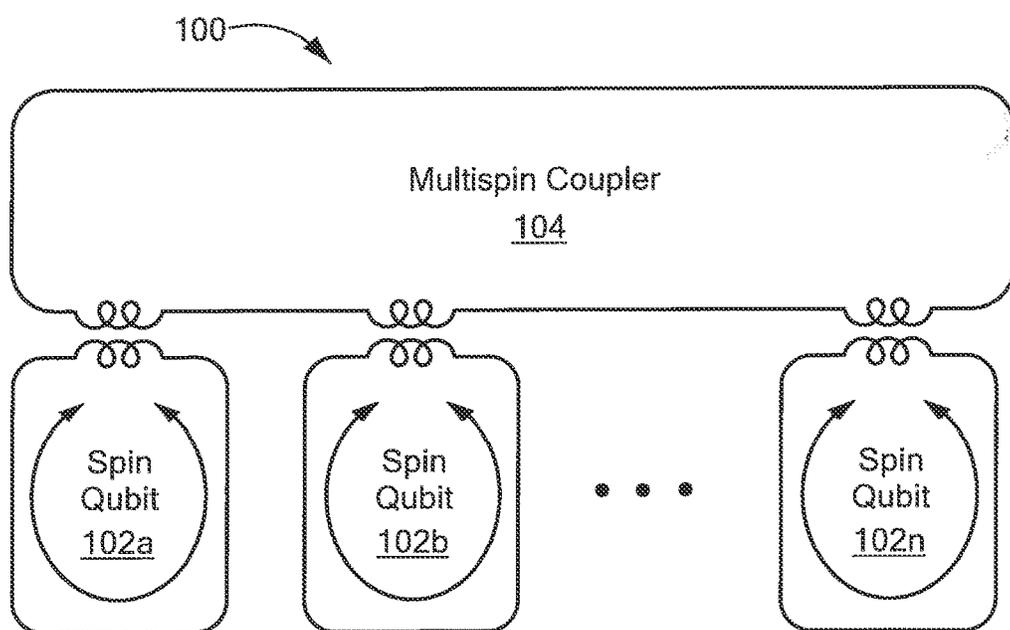


FIG. 1

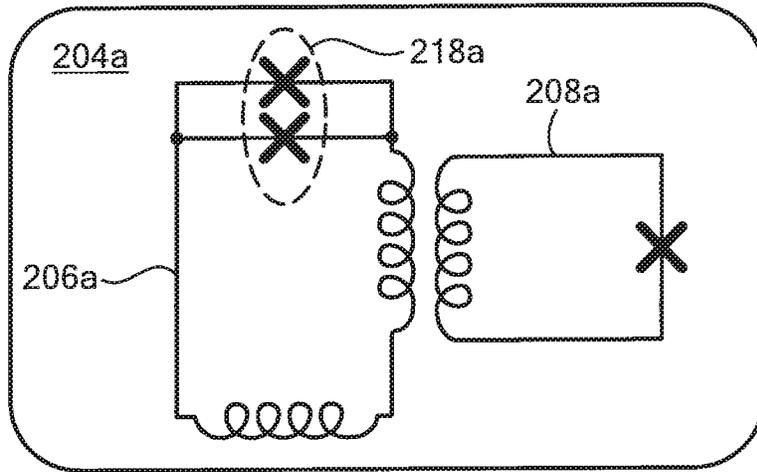


FIG. 2A

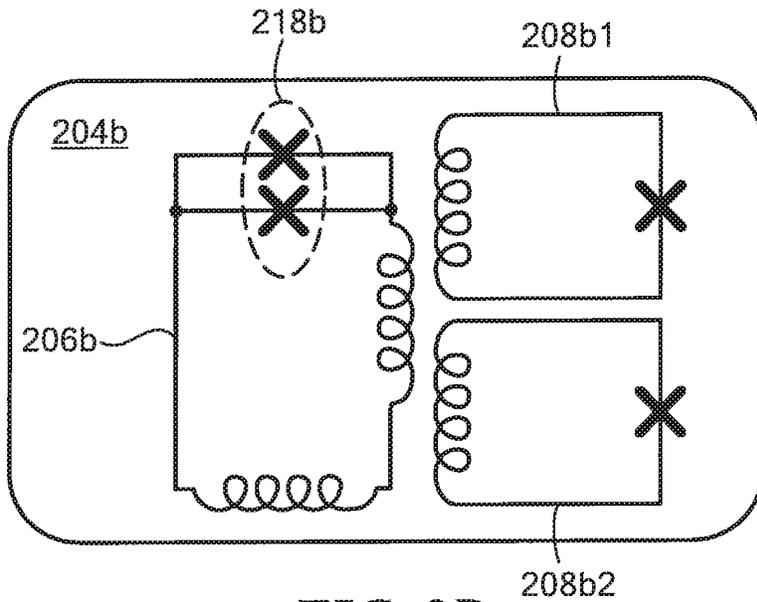


FIG. 2B

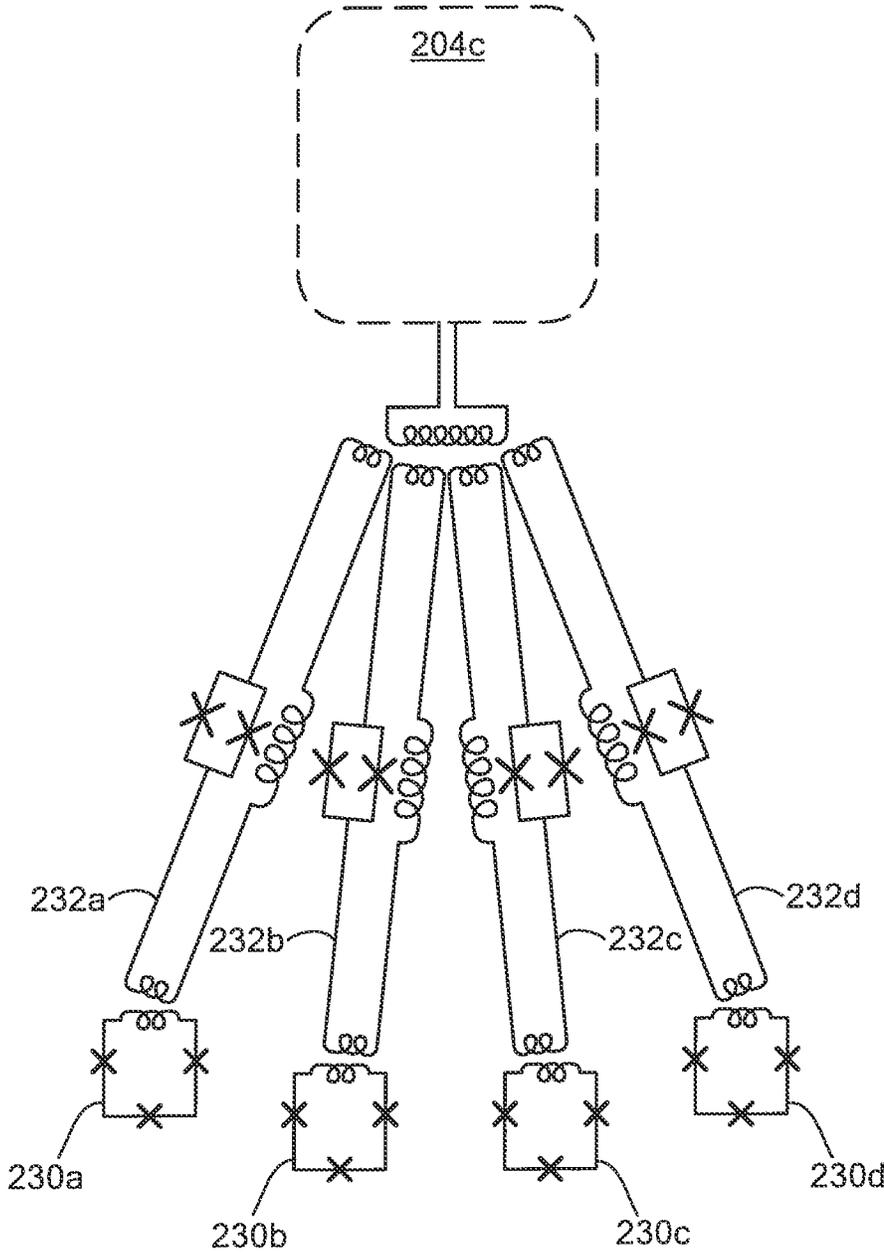


FIG. 2C

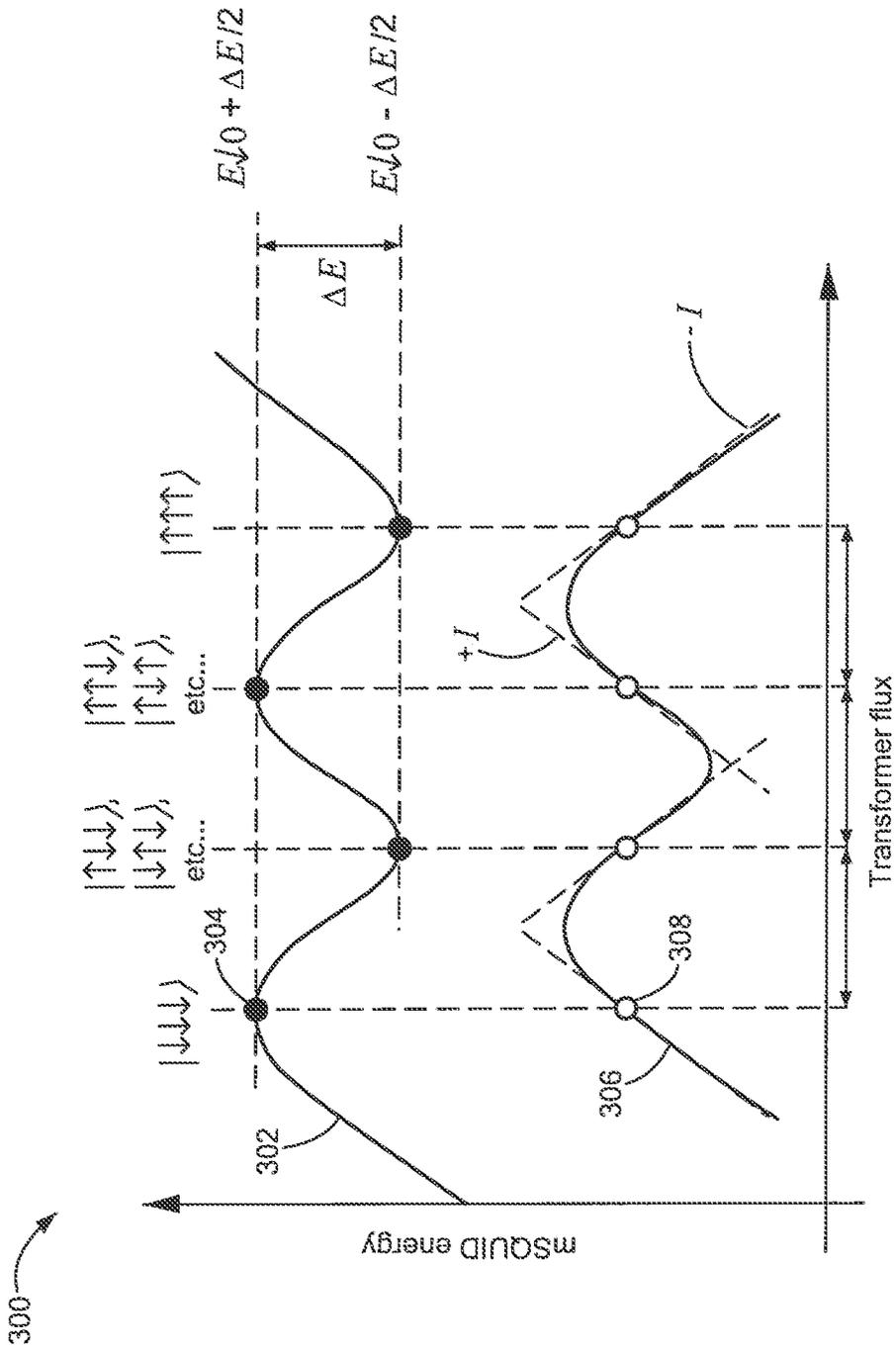
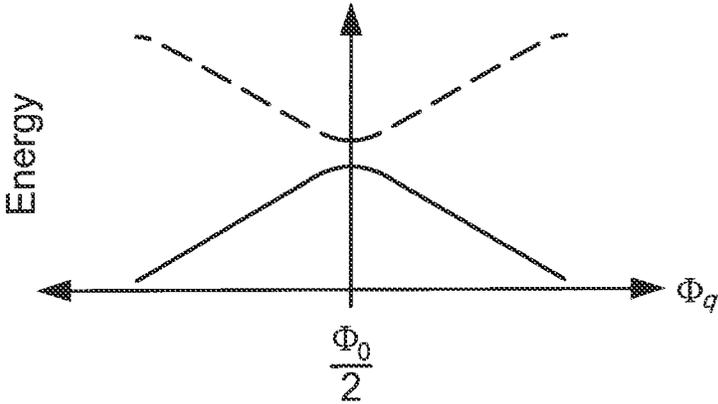
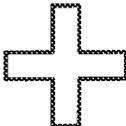


FIG. 3



Flux qubit



Inductor

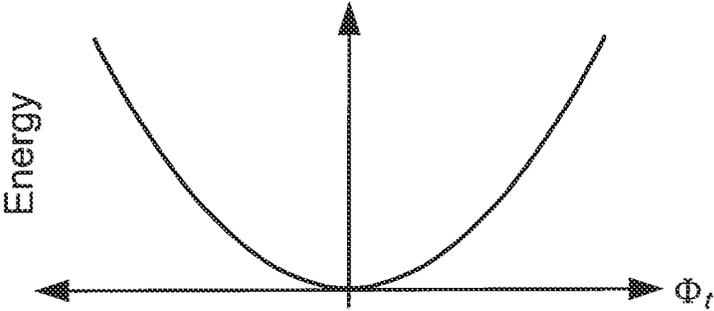


FIG. 3A

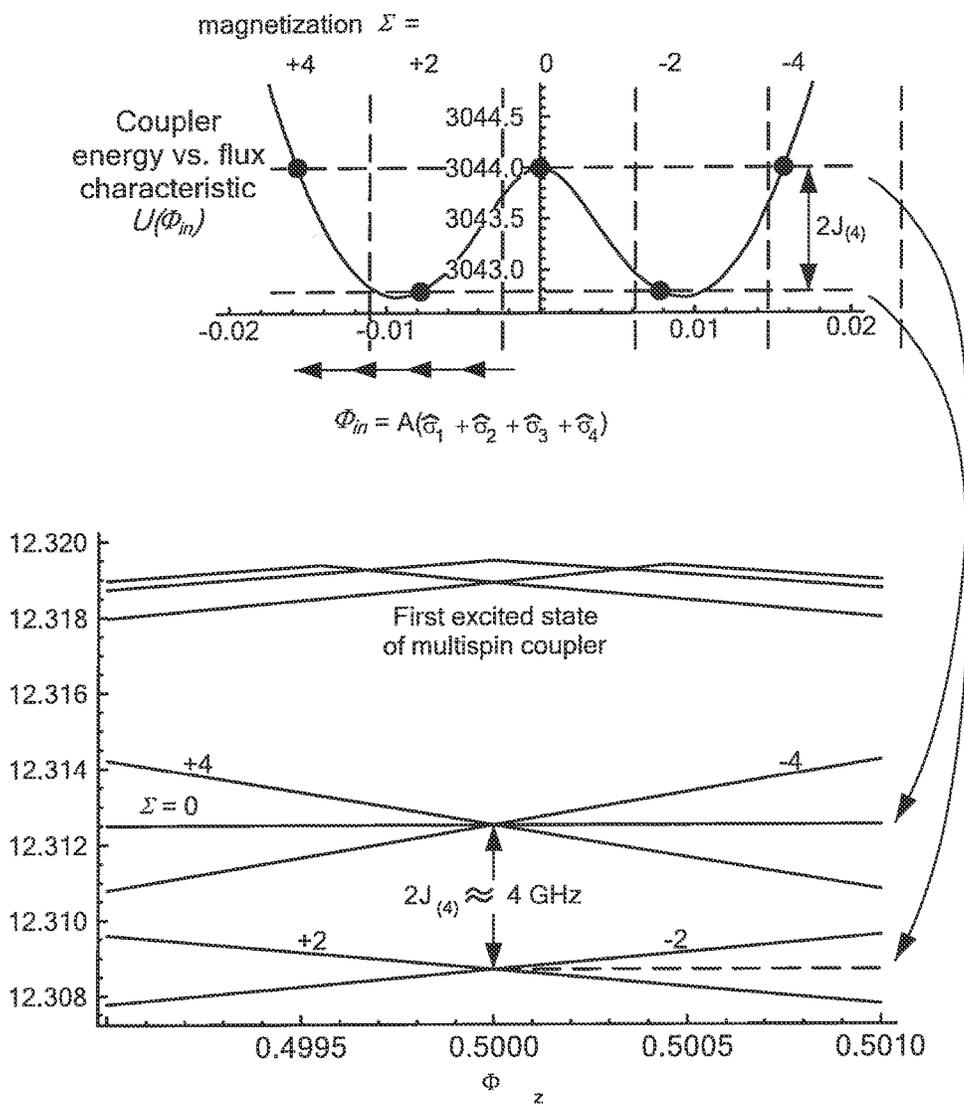


FIG. 3B

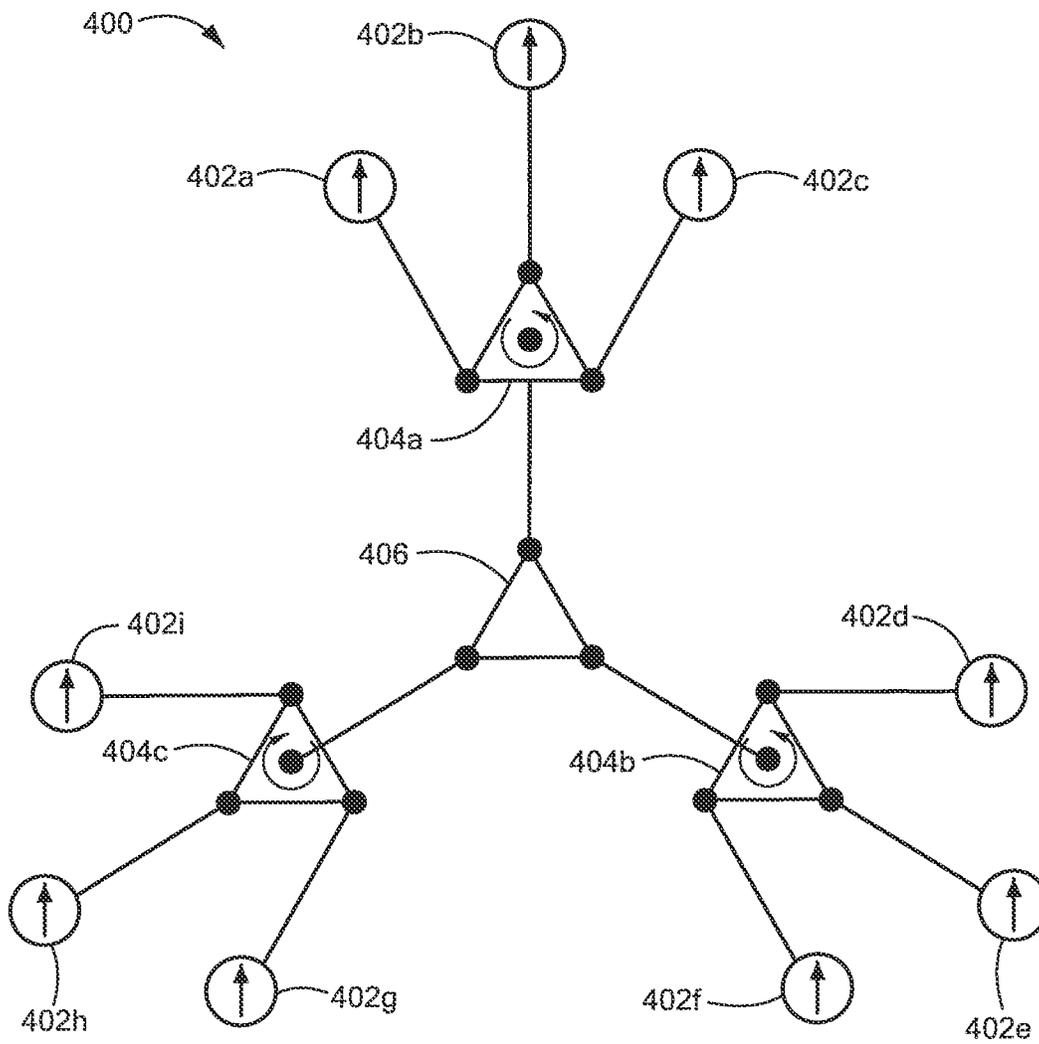


FIG. 4

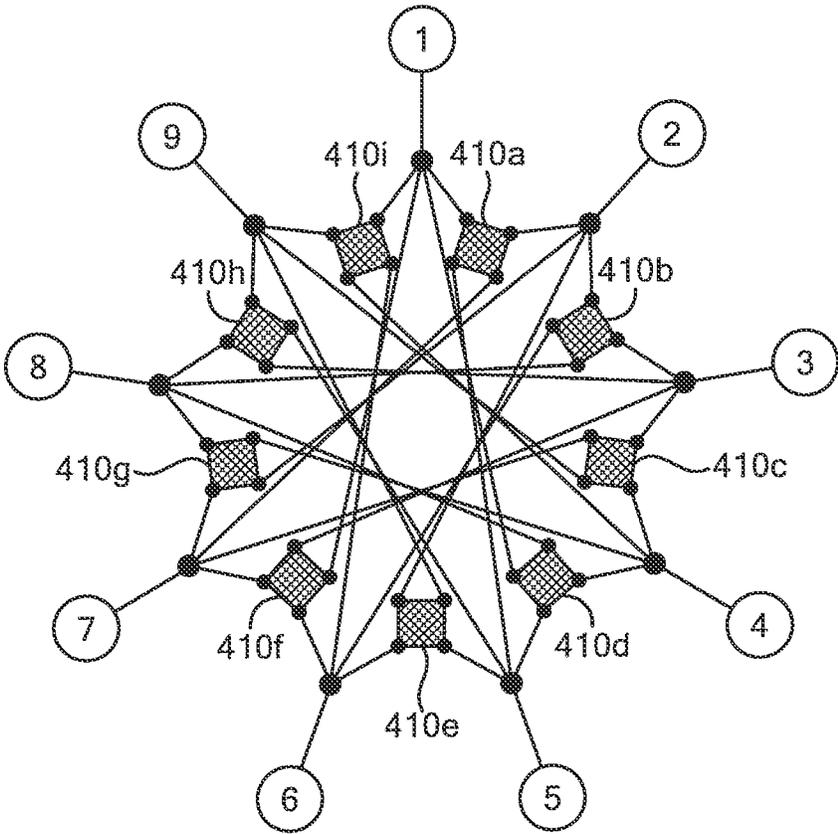


FIG. 4A

FOUR SPIN COUPLERS FOR QUANTUM INFORMATION PROCESSING

CROSS REFERENCE TO RELATED APPLICATIONS

This application is a continuation-in-part of U.S. application Ser. No. 15/354,275, filed 2016 Nov. 17, which claims the benefit under 35 U.S.C. 119(e) of U.S. Provisional Application No. 62/256,191, filed 2015 Nov. 17. The entire contents of these applications are hereby incorporated herein by reference in their entireties.

STATEMENT REGARDING FEDERALLY SPONSORED RESEARCH

This invention was made with government support under Contract No. FA8702-15-D-0001 awarded by the U.S. Air Force. The Government has certain rights in the invention.

BACKGROUND

Multiqubit quantum entanglement is a central physical resource on which the non-classical computational power of quantum information technology is based. As a result of this, known quantum information processing methods with the potential to achieve substantial performance improvement over classical techniques are built on methods for producing and exploiting large-scale quantum entanglement. The two most well-known quantum-processing paradigms are: digital quantum computing, which is expected to provide exponential performance enhancement most notably for problems in cryptography (Shor's algorithm) and quantum simulation of chemical and biological molecules (quantum phase estimation and variational eigensolver algorithms); and quantum annealing, where engineered quantum fluctuations may provide qualitative enhancement in the heuristic sampling of classical optimization problems. Also important is the recently-developed adiabatic method for simulating quantum chemistry, which encodes the quantum state properties of electronic molecular structure into the stationary states of an engineered Hamiltonian.

In all of these quantum-processing paradigms, the machinery for construction and protection of large-scale quantum entanglement relies on pairwise physical interactions between qubits, the only type available in current physical hardware. Larger-scale entanglement is then built up by combining many of these pairwise interactions, either by applying them successively in time in a pulsed manner, or by engineering many static pairwise interactions simultaneously to approximate an effective higher-order interaction perturbatively.

SUMMARY

It is appreciated herein that multiqubit interactions have engineering potential for improving the performance and scalability of quantum information processing systems. For example, in the case of digital quantum computing using surface-code-based topological encoding for fault-tolerance, the fundamental interactions that are required involve four qubits. Realizing this with two-qubit interactions requires four individual two-qubit gates and introduces an entirely new class of error processes relative to the idealized case. Block encoding in these schemes, which promises even higher performance, requires even higher-order interactions (e.g., involving more than four qubits). For quantum anneal-

ing, the quantum sampling effectiveness of complex classical configuration spaces depends exponentially on the number of spins involved in the physical interactions. Restriction to two-spin interactions only (as in existing commercial quantum information processing systems) is therefore an important reason that no evidence for quantum-enhanced performance has yet been observed in these machines.

Described herein are structures and techniques, using superconducting Josephson-junction based circuits, to directly engineer physical multiqubit (or "many-qubit") interactions in a non-perturbative manner. The overall function of the multispin coupler is to realize a potential energy vs. input flux characteristic that is nonlinear in a specific manner, such that when the state-dependent flux outputs of a plurality of qubits are combined and fed to the coupler flux input, the resulting energy or current of the coupler can be engineered to depend only on the parity of the state of the plurality of qubits. In some embodiments requiring a four-qubit interaction, the required energy vs. flux characteristic is a quartic function, with two local minima and one local maximum, symmetric around zero input flux.

According to one aspect of the disclosure, a system for multiqubit interaction includes: a multispin coupler including a plurality of loops, each loop having a pair of Josephson junctions; and a plurality of qubits each inductively coupled to the multispin coupler. In some embodiments, each loop of the multispin coupler includes an inductive element coupled between the pair of Josephson junctions and an energy storage element arranged in parallel with a first one of the pair of Josephson junctions. In certain embodiments, the multispin coupler further includes a transformer inductively coupled to each of the plurality of qubits and to each loop of the multispin coupler. In many embodiments, each of the plurality qubits includes a first loop and a second loop, the first and second loop each including a pair of Josephson junctions, the first loop further including an inductive element coupled between the pair of Josephson junctions. In some embodiments, the plurality of qubits includes three or more qubits.

In particular embodiments, the multispin coupler can be configured to operate in either an energy mode or a current mode, wherein in energy mode a total potential energy within the multispin coupler is proportional to a parity operator over each of the plurality of qubits, wherein current mode a total current circulating within the multispin coupler is proportional to the parity operator over each of the plurality of qubits. In many embodiments, the multispin coupler can be configured to operate in either an energy mode or a current mode by adjusting a magnetic flux through one or more of the loops of the multispin coupler.

According to another aspect of the disclosure, a system for multiqubit interaction includes: a first multispin coupler; a plurality of second multispin couplers each inductively coupled to the first multispin coupler, wherein the first multispin coupler and each of the plurality of second multispin couplers include a plurality of loops, each loop having a pair of Josephson junctions; and a plurality of qubits coupled to each of the second multispin couplers.

In some embodiments, each of the plurality of second multispin couplers is configured to operate in an energy mode whereby a total potential energy within each of the plurality of second multispin couplers is proportional to a parity operator over each of the plurality of qubits coupled thereto, wherein the first multispin coupler is configured to operate in a current mode whereby a total current circulating within the first multispin coupler is proportional to a parity

operator over each of the plurality of qubits coupled to each of the plurality of second multispin couplers.

In various embodiments, each of the plurality of second multispin couplers is configured to operate in a current mode whereby a total current circulating within each of the plurality of second multispin couplers is proportional to a parity operator over each of the plurality of qubits coupled thereto, and wherein the first multispin coupler is configured to operate in an energy mode whereby the total potential energy within the first multispin coupler is proportional to a parity operator over each of the plurality of qubits coupled to each of the plurality of second multispin couplers.

Other embodiments include a circuit for coupling four flux qubits. The circuit includes a spin qubit, magnetically biased at substantially one half of the magnet flux quantum, and a flux transformer, magnetically unbiased and inductively coupled to the spin qubit. The flux transformer also is inductively coupled to the four flux qubits, so that a measurement of the energy of the system has one of two values depending on a parity of the four flux qubits.

In various embodiments the spin qubit comprises a radio frequency superconducting quantum interference device (RF SQUID). The flux transformer may include a first loop having a first Josephson junction. The flux transformer also may include a second loop having the first Josephson junction and a second Josephson junction, for tuning an energy response of the circuit. Some embodiments further include four tuning couplers, each tuning coupler inductively coupled to the flux transformer and to a respective one of the four flux qubits. Each tuning coupler may be tuned to have a given magnetic interaction with the flux transformer. Some embodiments further include a second spin qubit that is magnetically unbiased and inductively coupled to the flux transformer. The second spin qubit also may include an RF SQUID. In some embodiments, the circuit is considered to include the four flux qubits inductively coupled to the flux transformer.

Still other embodiments include a method of measuring a spin parity of four flux qubits. The method requires coupling each of the four flux qubits to a flux transformer, the flux transformer being inductively coupled to a spin qubit that is magnetically biased at substantially one half of the magnetic flux quantum. The method also requires operating the circuit in a current mode, wherein a current circulating within the spin qubit is proportional to a quantum parity operator. The method also requires measuring the spin parity by sensing a magnetic flux, of the spin qubit, that has one of two values.

In some embodiments, sensing a magnetic flux includes sensing an RF SQUID. Coupling may involve inductively coupling each of the four flux qubits to a respective tuning circuit and inductively coupling each such tuning circuit to the flux transformer. In this case, the method may include tuning each tuning circuit to have a given magnetic interaction with the flux transformer, and the given magnetic interaction may produce a quartic response in the coupling circuit. Some embodiments of the method include inductively coupling the flux transformer to a second spin qubit that is magnetically unbiased, in which case the method also may include sensing a magnetic flux of the second spin qubit.

BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing features may be more fully understood from the following description of the drawings in which:

FIG. 1 is a block diagram of a system to provide multi-qubit interactions using a multispin coupler, according to an embodiment of the disclosure;

FIG. 2 is a diagram of a circuit to provide multiqubit interactions using a multiloop superconducting quantum interference device (mSQUID), according to another embodiment of the disclosure;

FIGS. 2A and 2B are diagrams of four-spin coupler circuits based on superconducting quantum interference devices (SQUIDs), according to other embodiments of the disclosure;

FIG. 2C is a diagram of a circuit to implement tunable quantum spin interactions between four qubits using a SQUID-based four-spin coupler, according to another embodiment of the disclosure;

FIG. 3 is a graph illustrating different modes of mSQUID (and general multispin coupler) operation, according to embodiments of the disclosure;

FIG. 3A illustrates the principle and mode of operation of the four-spin coupler embodiments of FIG. 2A and FIG. 2B;

FIG. 3B shows a full simulation of the quantum energy levels for the resulting circuit shown in FIG. 2C;

FIG. 4 is a diagram of an inverse paramagnetic tree structure to provide multiqubit interactions among greater than four qubits; and

FIG. 4A is a diagram of a forward paramagnetic tree structure that uses four-spin couplers.

The drawings are not necessarily to scale, or inclusive of all elements of a system, emphasis instead generally being placed upon illustrating the concepts, structures, and techniques sought to be protected herein.

DETAILED DESCRIPTION

Referring to FIG. 1, according to an embodiment of the disclosure, a system **100** includes a plurality of qubits **102a-102n** (generally denoted **102**) and a multispin coupler **104**.

In many embodiments, the qubits **102** are provided as flux qubits. As is known, flux qubits (also known as persistent current qubits) are micrometer-sized loops of superconducting metal interrupted by a number of Josephson junctions. The Josephson junction parameters may be fabricated such that a persistent current will flow continuously when an external flux is applied. The computational basis states of the qubit are defined by the circulating currents, which can flow either clockwise or counter-clockwise.

The multispin coupler **104** provides multiqubit (i.e., two or more) interactions in a non-perturbative manner. In some embodiments, the multispin coupler **104** may be provided as a multi-loop circuit.

Referring to the embodiment of FIG. 2, a circuit **200** can provide multiqubit interactions using a multispin coupler. The circuit **200** includes a plurality of qubits **202a-202n** (**202** generally) and a multispin coupler **204** provided as a multiloop DC SQUID interferometer (referred to herein as an "mSQUID"). The mSQUID **204** includes a transformer **206** to which each of the qubits **202** may be inductively coupled.

In the embodiment of FIG. 2, the qubits **202** are provided as flux qubits each having two pairs of Josephson junctions (in FIG. 2, individual Josephson junctions are denoted "X"). For example, qubit **202a** is denoted as having a first pair of Josephson junctions **210** arranged about a first loop **212**, and a second pair of Josephson junctions **214** arranged about a second loop **216**.

Each qubit **202** further includes an inductive element **215** to provide inductive coupling with the mSQUID transformer **206**. As used herein, the term “inductive element” refers to any circuit element that stores energy in a magnetic field, including linear and nonlinear inductors, and functionally equivalent circuit elements that do not store energy in a magnetic field, such as Josephson junction-based nonlinear inductors. In the embodiment of FIG. 2, the qubit inductive element is coupled between the pair of Josephson junctions **210** within the first loop **212**.

In other embodiments, the qubit inductive element **215** could be placed within the second loop **216** (e.g., between Josephson junctions **214**). In certain embodiments, a qubit may include multiple inductive elements (e.g. one in each of its two loops). Those skilled in the art will understand that each loop of a qubit has a finite self-inductance to which other loops can couple via a mutual inductance. By coupling to a loop with the larger junctions (e.g., loop **212** in FIG. 2), this is effectively coupling to a Z operator, while the loop with the smaller junctions (e.g., loop **216** in FIG. 2) is effectively coupling to an X operator.

It should be understood that the concepts and structures sought to be protected herein are not limited to the qubit structure shown in FIG. 2. For example, in some embodiments, any qubit which has a state-dependent magnetic moment can be used.

The illustrative mSQUID **204** includes the transformer **206**, a multiloop circuit **208** inductively coupled thereto, and an output **228**. The transformer **206** may include a Josephson junction pair **218**, as shown. Including Josephson junctions in the transformer can allow flux to enter and leave the transformer loop when it is cooling (so that flux is not trapped, as would happen in a closed superconducting loop). Josephson junction pair **218** may also allow the transformer **206** to be tunable.

The multiloop circuit **208** includes a plurality of loops **220a-220n** (**220** generally), each having a Josephson junction pair **222**, an inductive element (e.g., a linear or non linear inductive element) **224** to provide inductive coupling to the transformer **206**, and an energy storage element **226** (e.g., a capacitor) arranged in parallel with one of the loop Josephson junctions. To promote clarity in the figure, the Josephson junction pair, inductive element, and energy storage element are labeled only for a first loop **220a**.

It should be understood that the number of loops **220** within the circuit **208** may be selected based on requirements for a given application. In particular, the number of loops **220** determines the number of free parameters that can be used to define the shape of mSQUID’s nonlinear energy vs. flux characteristic. Thus, increasing the number of loops **200** may provide additional control required for a given application. However, it will be appreciated that increasing the number of loops **200** may require more flux from the transformer **206** for the mSQUID to function. It should also be understood that the number of loops **220** may be decreased to a single loop, as discussed below.

It will be appreciated that the design of each mSQUID loop **220** is based on that of a conventional DC SQUID, hence the name mSQUID (or multiloop SQUID). As is known, the effective Josephson potential energy of a DC SQUID can be modulated by changing the relative gauge-invariant phase between the two Josephson junctions, adjusted via the magnetic flux through the respective loop. In the case of a multiloop SQUID **204**, an arbitrary potential energy function vs. uniform flux can be realized by adjusting the individual flux bias through the multispin coupler loops **220**. In particular, the constant phase offset between each

Josephson junction can be controlled to realize arbitrary potential energy function vs. uniform flux. The result is that for flux Φ coupled to a common transformer and applied equally to all loops **220** of the mSQUID **204**, an arbitrary potential energy function $U(\Phi)$ can be engineered by appropriately tuning the offset fluxes applied to the individual loops **222**.

In operation, each qubit **202** nominally produces a state-dependent flux $\pm\Phi_q$ through the transformer **206**. For N qubits **202** coupled to transformer **206**, the total qubit-state-dependent flux (i.e., the total state-dependent flux from all qubits, which is the input to the mSQUID) then takes one of the N+1 different values: $\Phi_k \in \Phi_q \{-N, -N+2 \dots 0 \dots N-2, N\}$.

It is appreciated herein the mSQUID **204** can be dynamically adjusted such that it behaves like a current source: $(\Phi)=I\Phi$, an inductor: $U(\Phi)=\Phi^2/2L$, or any more complicated nonlinear magnetic element whose behavior can be expressed as a potential energy $U(\Phi)$ (the classical parameters I, L, etc. are related to the Taylor expansion coefficients of the energy vs. flux characteristic about a chosen bias point). Furthermore, since the internal Josephson frequencies of the multispin coupler **204** can be kept relatively large (>100 GHz), this electrical behavior can be preserved over an extremely wide frequency range. As a result, the mSQUID **204** may be used in parametric nonlinear quantum devices—such as amplifiers and frequency converters—that conventionally rely on the “bare” Josephson nonlinearity, which is relatively weak by comparison.

Referring to FIG. 3, in various embodiments, an mSQUID (e.g., mSQUID **204** in FIG. 2) can be operated in two different modes: energy mode and current mode. In energy mode, the potential energy within the mSQUID’s multiloop circuit is proportional to the N-qubit parity operator $\hat{P}_N = \prod_{i=1}^N \hat{\sigma}_i^z$ such that: $\hat{U}(\Phi_k) = \Delta E \cdot \hat{P}_N$, where $\hat{\sigma}_i^z$ is the Z operator for the ith qubit. In current mode, the total current circulating within the mSQUID’s multiloop circuit is proportional to the N-qubit parity operator $\hat{P}_N = \prod_{i=1}^N \hat{\sigma}_i^z$ such that $\hat{I}(\Phi_k) = I_m \cdot \hat{P}_N$. In some embodiments, the mSQUID **204** can be configured to operate in a given mode by adjusting the magnetic flux through the mSQUID loops **220**, as discussed above.

The graph **300** in FIG. 3 illustrates these two modes of mSQUID operation. The top curve **302** illustrates energy mode, wherein the total potential energy as a function of possible qubit flux values (denoted by circles along the curve **302**, e.g., circle **304**) is effectively proportional to a multispin (in this case, three-spin) operator: $\hat{U} = \Delta E \hat{\sigma}_1^z \hat{\sigma}_2^z \hat{\sigma}_3^z$. In energy mode, the potential energy vs. input flux can be tailored such that at the flux points accessible via the qubits’ input to the transformer, the mSQUID energy takes on one of two values, according to the total parity of the qubit state. This results in an effective interaction between the qubits.

The bottom curve **306** illustrates the current mode of operation, in which the potential energy is the same at all of the possible qubit flux values (denoted by circles along the curve **306**, e.g., circle **308**), but the mSQUID circulating current (proportional to the slope of the energy vs. flux) is proportional to a multiqubit operator: $\hat{I} = I \hat{\sigma}_1^z \hat{\sigma}_2^z \hat{\sigma}_3^z$. In current mode, the potential energy is kept constant and independent of the qubits’ input state, while the effective mSQUID circulating current (which is given by the derivative of the energy with respect to the flux) assumes only two values and is proportional to the total qubit state parity.

It should be understood that although FIG. 3 illustrates mSQUID operation for the case of three (3) qubits, the

concepts, structures, and techniques sought to be protected herein be used to effect interactions between arbitrary numbers of qubits (e.g., between N qubits, where $N > 1$).

Various embodiments of multispin couplers for use in specific applications are now described in accordance with the concepts herein. It should be appreciated that these embodiments also may be operated in both the energy mode and the current mode described above. Referring to FIG. 2A, a multispin coupler **204a** is shown for coupling four flux qubits. This capability could be used, for example, to directly realize the four-qubit operators upon which the surface-code quantum error correction scheme is based; or, it could be used to couple encoded qubits whose logical operators are weight-2 (i.e. consist of a two-qubit operator). Such distance-2 logical qubits would occur, for example, in the context of quantum error suppression. Six-qubit operators, realized using a sub-circuit of that shown in FIG. 4, could be used to directly realize interactions between two fault-tolerant logical qubits of distance 3, each having three-qubit logical operators.

The multispin coupler **204a** may be used as the multispin coupler **104** of FIG. 1 or **204** of FIG. 2 and may spin-couple the qubits **102** or **202**. The multispin coupler **204a** is like, but simpler than, the mSQUID **204**: the multiloop circuit **208** has been replaced by a single RF SQUID (i.e., a spin qubit) **208a**. In the embodiment of FIG. 2A, the spin qubit **208a** is magnetically biased substantially at its degeneracy point, i.e. one half of the magnetic flux quantum ($\Phi_0 = h/(2e)$). As shown in FIG. 3A, such a spin qubit **208a** has strictly non-classical response as approximately negatively proportional to the absolute magnitude of the input flux relative to the symmetry point at a bias of $(\Phi_0/2)$, while the transformer **206a** has approximately a positive quadratic response. Adding these responses and tuning using the loop having the Josephson junction pair **218a**, as indicated in FIG. 3B, produces a quartic energy versus flux response that can be adjusted so that it assumes one of two values depending on the total qubit state parity for the four flux qubits, as discussed above.

FIG. 2B shows an alternate embodiment of a multispin coupler **204b** having a transformer **206b** with a Josephson junction pair **218b**, a biased spin qubit **208b1**, and an unbiased RF SQUID **208b2**. In the alternate embodiment of FIG. 2B, the unbiased RF SQUID **208b2** alters the energy response of the multispin coupler **204b** and provides an additional degree of freedom for dynamic adjustment of the coupler.

Referring now to FIG. 2C, a multispin coupler **204c** is coupled to four qubits **230a**, **230b**, **230c**, **230d** (collectively, qubits **230**) using respective tuning couplers **232a**, **232b**, **232c**, **232d** (collectively tuning couplers **232**). The multispin coupler **204c** may be any multispin coupler described herein, including multispin coupler **104**, **204**, **204a**, or **204b**. The qubits **230** may be any qubits coupled as described herein, such as qubits **102** or **202**. When coupling qubits **230** as just described, it is important that the multispin coupler **204c** receives from each qubit **230** a qubit-state-dependent input flux of the correct magnitude. Therefore, some embodiments include tuning couplers **232** for matching these input fluxes from the qubits, which may also include some amplification of the qubit fluxes. The tuning couplers **232** may be independently tuned by applying a tuning magnetic field, using techniques known in the art.

FIG. 2C is only illustrative of the use of tuning couplers **232** with a multispin coupler **204c**. While only four qubits **230** and four respective tuning couplers **232** are shown in FIG. 2C, tuning couplers may be used in this way for any greater or smaller number of qubits. Any suitable circuit for

performing this flux matching may be used as a tuning coupler other than the circuits having the components indicated in FIG. 2C.

FIG. 4 illustrates how the different modes of mSQUID operation can be combined using a so-called “inverse paramagnetic tree” structure to provide higher-order multiqubit interactions. An inverse paramagnetic tree structure **400** may include a root mSQUID **406**, a plurality of intermediate mSQUIDs (e.g., intermediate mSQUIDs **404a**) coupled thereto, and a plurality of qubits (e.g., qubit **402a**) coupled to each of the intermediate mSQUIDs. In the embodiment of FIG. 4, the structure **400** includes three (3) intermediate mSQUIDs **404a-404c**, each having a group of three (3) qubits (or “spins”) **402** coupled thereto. In particular, qubits **402a-402c** are coupled to a first intermediate mSQUID **404a**, qubits **402d-402f** are coupled to a second intermediate mSQUID **404b**, and qubits **402g-402i** are coupled to a third intermediate mSQUID **404c**.

The root mSQUID **406** may be operated in a different mode than the intermediate mSQUIDs **404**. For example, each group of M (e.g., $M=3$) qubits **402** may be coupled to a respective intermediate mSQUID **404** operating in current mode, such that the total circulating current in each intermediate mSQUID **404** is proportion to a M -qubit operator product, with $M=3$ in FIG. 4. In turn, each intermediate mSQUID **404** may be coupled to a root mSQUID **406** operating in energy mode, such that the total energy in the root mSQUID **406** is proportional to an N -qubit operator product, with $N=9$ in FIG. 4. Thus, it will be appreciated that the parities of multiple groups of M qubits **402** can be combined, resulting in an effective interaction between all N qubits.

It should be understood that the tree structure **400** shown in FIG. 4 is merely one example and that an inverse paramagnetic tree structure **400** could include additional levels (i.e., more than one level of intermediate mSQUIDs **404**) and could have more than three (3) connections at each level of the tree. Moreover, the number of connections at each level may vary in some embodiments. For example, the root mSQUID could be coupled to a certain number of intermediate mSQUIDs, each of which could be coupled to a different number of qubits. Further, the structure **400** can be used to realize either X or Z operators by connecting to either the Z or X loops of the qubits **402**.

The four-spin coupler in FIGS. 2A-2C may be used in forward paramagnetic tree structures such as those disclosed in U.S. application Ser. No. 15/354,286, filed 2016 Nov. 17 and entitled “Paramagnetic Tree Coupling of Spin Qubits,” which claims the benefit under 35 U.S.C. 119(e) of U.S. Provisional Application No. 62/256,200, filed 2015 Nov. 17 having the same title. The entire contents of these applications are incorporated herein by reference. One illustrative embodiment of such use is indicated in FIG. 4A, in which nine qubits are mutually coupled by a paramagnetic tree structure. FIG. 4A shows nine four-spin couplers **410a-410i** (collectively for-spin couplers **410**) having four, four-spin couplings per spin using three paramagnetic tree levels, with the four-spin couplers are at the third level. Thus, each spin qubit 1 through 9 participates in four separate four-spin couplings involving groups of three other qubits. For example, qubit #1 is connected via the tree to four four-spin couplers **410a**, **410d**, **410f**, and **410i**, and each of these are also connected to three other qubits by going back down the tree. For example, four-spin coupler **410a** is coupled to qubits #1, #2, #5, and #7.

It is appreciated that the concepts, structures, and techniques described herein may be used to provide high-order

qubit interactions and, as such, could have transformational importance in both quantum annealing and in digital quantum information processing.

All references cited herein are hereby incorporated herein by reference in their entirety.

Having described certain embodiments, which serve to illustrate various concepts, structures, and techniques sought to be protected herein, it will be apparent to those of ordinary skill in the art that other embodiments incorporating these concepts, structures, and techniques may be used. Elements of different embodiments described hereinabove may be combined to form other embodiments not specifically set forth above and, further, elements described in the context of a single embodiment may be provided separately or in any suitable sub-combination. Accordingly, it is submitted that the scope of protection sought herein should not be limited to the described embodiments but rather should be limited only by the spirit and scope of the following claims.

What is claimed is:

1. A circuit for coupling four flux qubits, the circuit comprising:
 - a spin qubit, magnetically biased at substantially one half of the magnetic flux quantum; and
 - a flux transformer, magnetically unbiased and inductively coupled to the spin qubit;
 wherein when the flux transformer is inductively coupled to the four flux qubits, a measurement of the energy of the system has one of two values depending on a total parity of the four flux qubits.
2. The circuit of claim 1, wherein the spin qubit comprises a radio frequency superconducting quantum interference device (RF SQUID).
3. The circuit of claim 1, wherein the flux transformer comprises:
 - a first loop having a first Josephson junction; and
 - a second loop having the first Josephson junction and a second Josephson junction, for tuning an energy response of the circuit.
4. The circuit of claim 1, further comprising four tuning couplers, each tuning coupler inductively coupled to the flux transformer and to a respective one of the four flux qubits.

5. The circuit of claim 4, wherein each tuning coupler is tuned to have a given magnetic interaction with the flux transformer.

6. The circuit of claim 1, further comprising a second spin qubit that is magnetically unbiased and inductively coupled to the flux transformer.

7. The circuit of claim 6, wherein the second spin qubit comprises an RF SQUID.

8. The circuit of claim 1, further comprising the four flux qubits inductively coupled to the flux transformer.

9. A method of measuring a spin parity of four flux qubits, the method comprising:

coupling each of the four flux qubits to a flux transformer, the flux transformer being inductively coupled to a spin qubit that is magnetically biased at substantially one half of the magnetic flux quantum;

operating the circuit in a current mode, wherein a current circulating within the spin qubit is proportional to a quantum parity operator; and

measuring the spin parity by sensing a magnetic flux, of the spin qubit, that has one of two values.

10. The method of claim 9, wherein sensing a magnetic flux includes sensing a radio frequency superconducting quantum interference device (RF SQUID).

11. The method of claim 9, wherein coupling includes inductively coupling each of the four flux qubits to a respective tuning circuit and inductively coupling each such tuning circuit to the flux transformer.

12. The method of claim 11, further comprising tuning each tuning circuit to have a given magnetic interaction with the flux transformer.

13. The method of claim 12, wherein the given magnetic interaction produces a quartic response.

14. The method of claim 9, further comprising inductively coupling the flux transformer to a second spin qubit that is magnetically unbiased.

15. The method of claim 14, further comprising sensing a magnetic flux of the second spin qubit.

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