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(54) **QUANTUM NONDEMOLITION MICROWAVE PHOTON COUNTER BASED ON THE CROSS-KERR NONLINEARITY OF A JOSEPHSON JUNCTION EMBEDDED IN A SUPERCONDUCTING CIRCUIT**

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ABSTRACT

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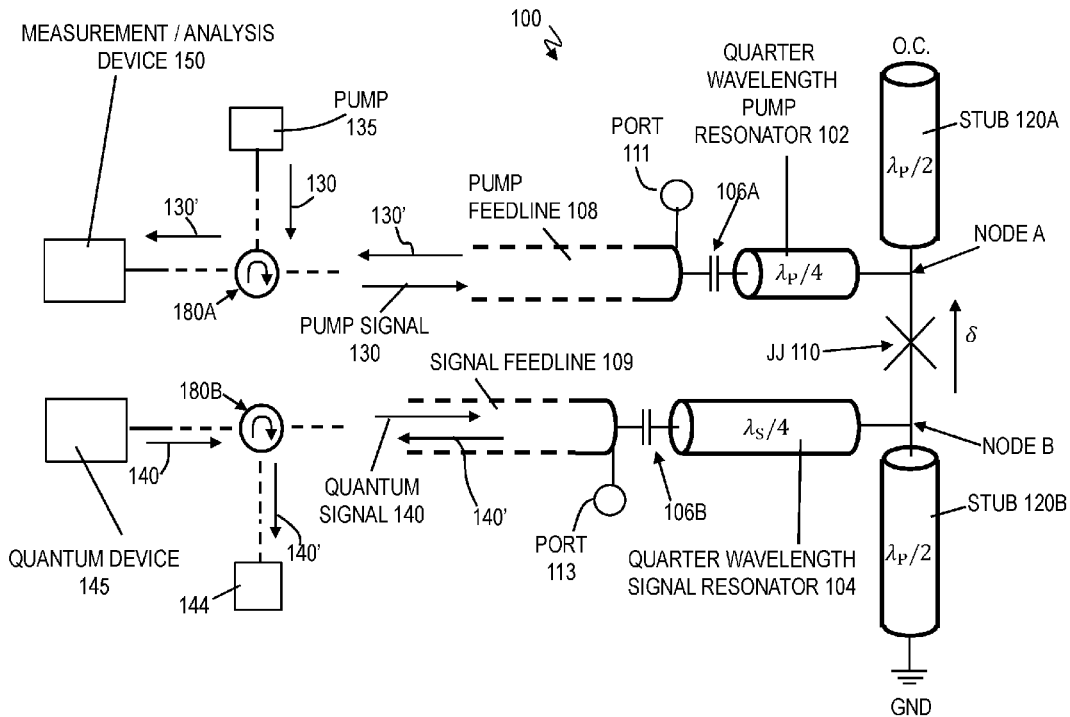
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A technique relates to a microwave device. A pump resonator, at a first pump resonator end, is connected to both a dispersive nonlinear element and a first stub. The pump resonator, at a second pump resonator end, is capacitively coupled to a pump port, where the first stub is terminated in an open circuit. A quantum signal resonator, at a first quantum signal resonator end, is connected to both the dispersive nonlinear element and a second stub. The quantum signal resonator, at a second signal resonator end, is capacitively coupled to a signal port, where the second stub is connected to ground.



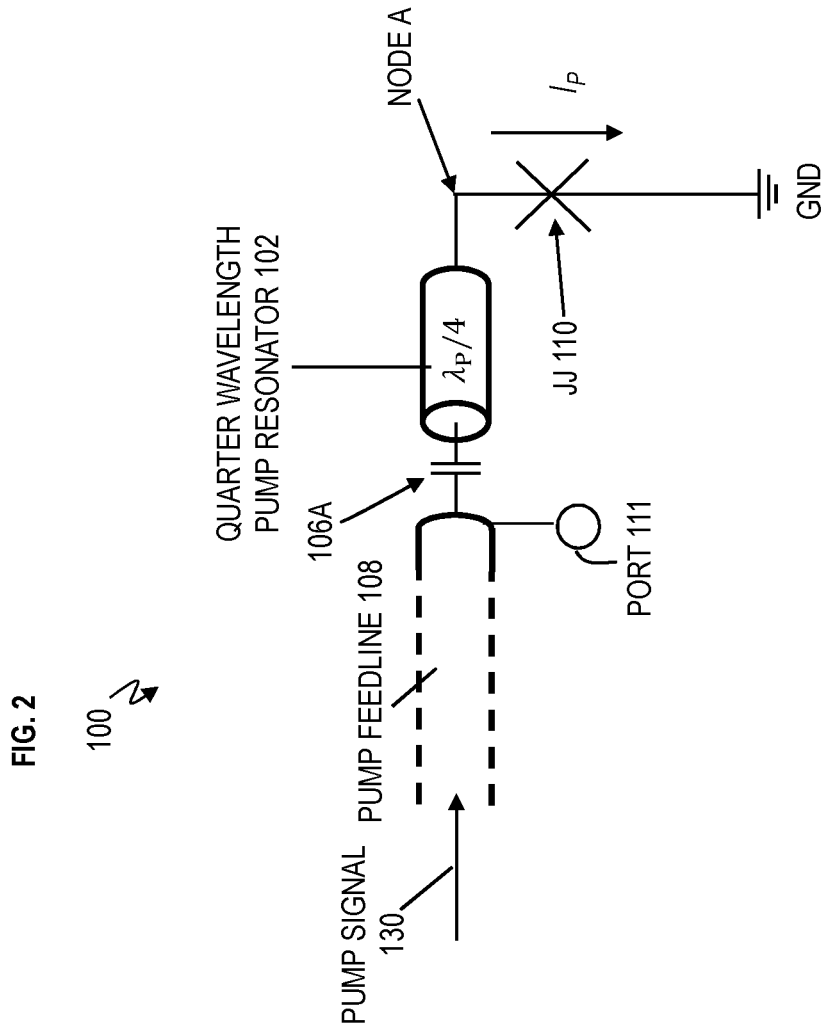


FIG. 3

100

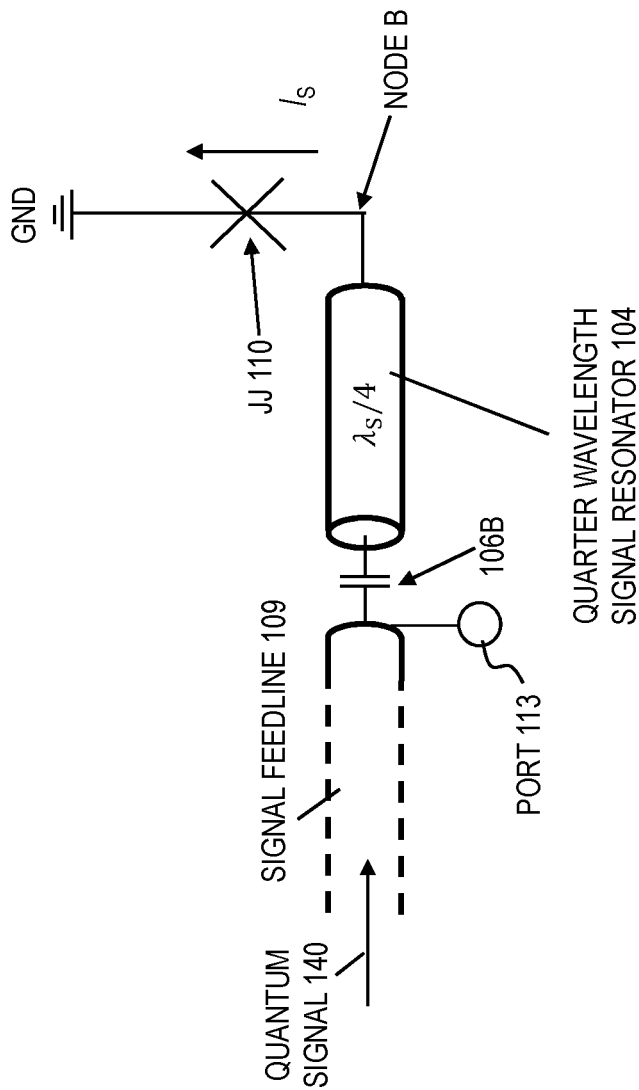


FIG. 5
100

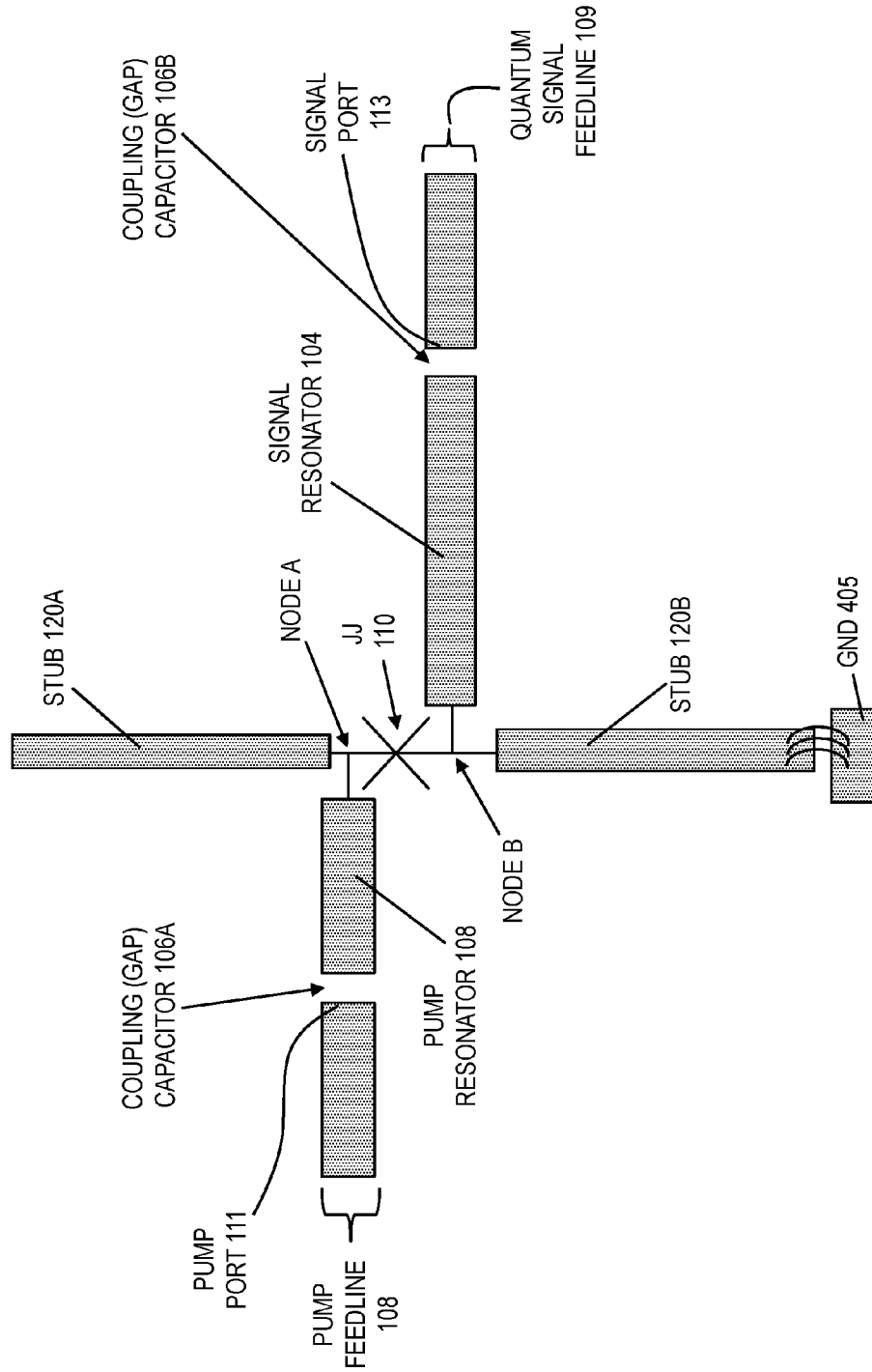


FIG. 6

600 ↗

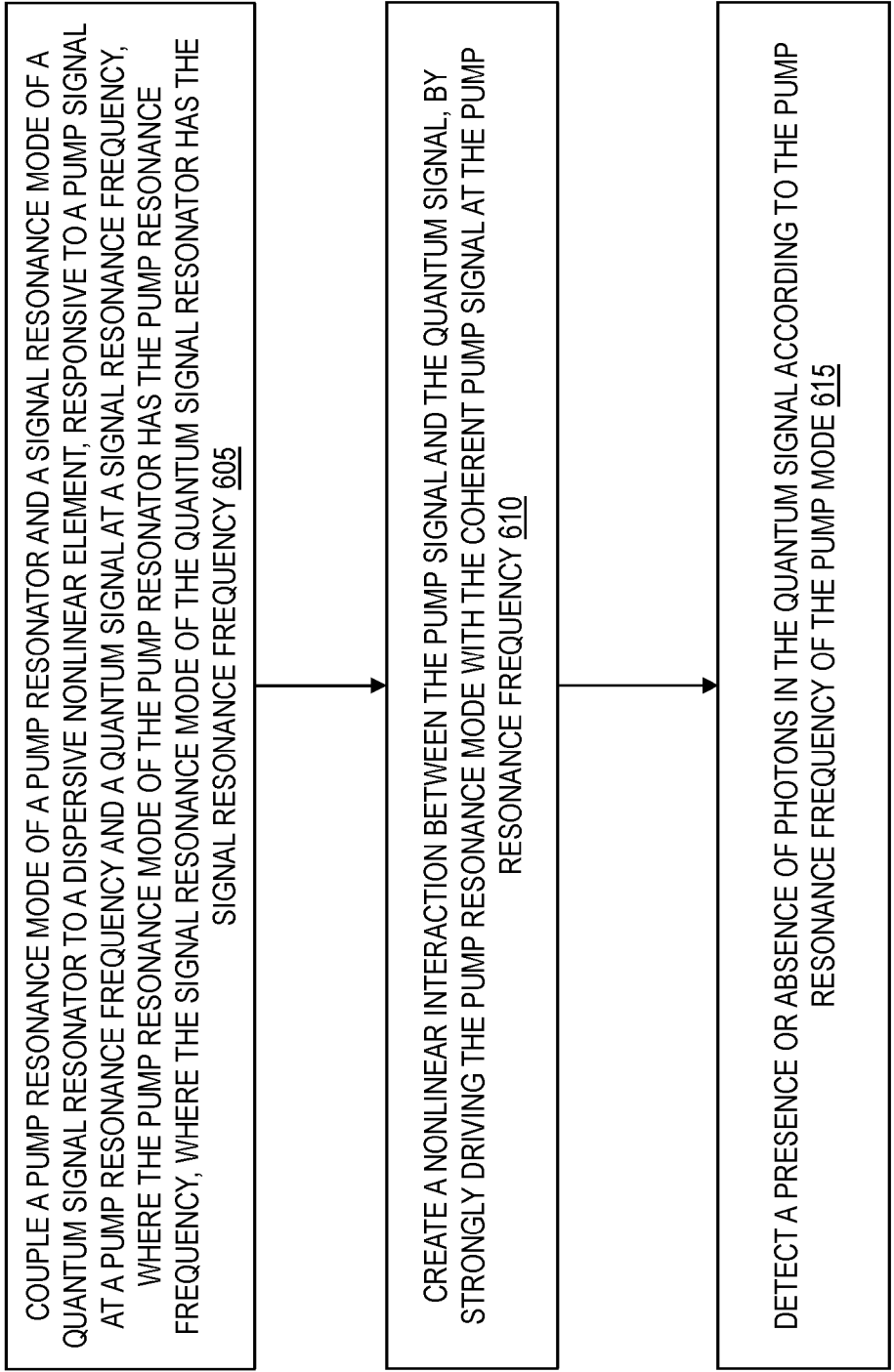
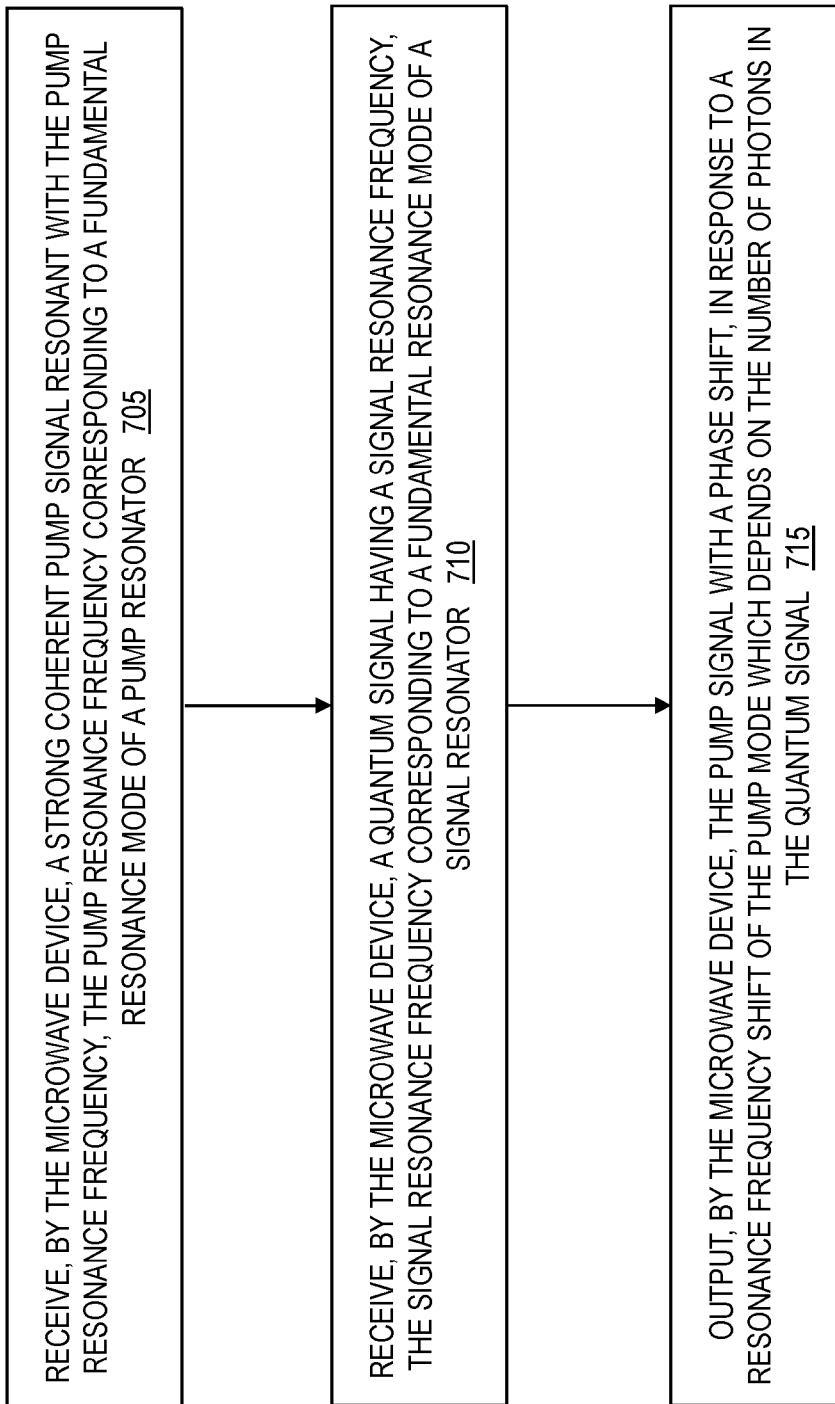


FIG. 7

700 ↗



**QUANTUM NONDEMOLITION MICROWAVE
PHOTON COUNTER BASED ON THE
CROSS-KERR NONLINEARITY OF A
JOSEPHSON JUNCTION EMBEDDED IN A
SUPERCONDUCTING CIRCUIT**

DOMESTIC PRIORITY

[0001] This application is a continuation of U.S. patent application Ser. No. 14/870,663, filed Sep. 30, 2015, the disclosure of which is incorporated by reference herein in its entirety.

BACKGROUND

[0002] The present invention relates to measurement techniques of quantum systems operating in the microwave frequency domain, such as superconducting quantum circuits, and more specifically, to detection and/or counting of single microwave photons in a nondemolition manner.

[0003] A photon is an elementary particle, the quantum of light and all other forms of electromagnetic radiation. A photon carries energy proportional to the radiation frequency and has zero rest mass.

[0004] One reason why the detection of single microwave photons is an outstanding challenge is because the energy of a single microwave photon is very small. The energy of a photon in the microwave domain, for example in the range 1-10 gigahertz, is at least 10^4 times smaller than the energy of a visible light photon.

[0005] Circuit quantum electrodynamics (cQED) is one of the leading architectures for realizing a quantum computer based on superconducting microwave circuits. It employs artificial atoms made of nonlinear superconducting devices called qubits which are dispersively coupled to microwave resonators, i.e., the frequencies of the qubits and resonators are detuned. As one example, each superconducting qubit may comprise one or more Josephson junctions shunted by capacitors in parallel with the junctions. The qubits are capacitively coupled to two-dimensional (2D) planar waveguide resonators or three-dimensional (3D) microwave cavities. The electromagnetic energy associated with the qubit is stored in the Josephson junctions and in the capacitive and inductive elements forming the qubit. To date, a major focus has been on improving lifetimes of the qubits in order to allow calculations (i.e., manipulation and readout) to take place before the information is lost due to decoherence of the qubits.

[0006] Dispersively coupling a superconducting qubit to a microwave resonator in a cQED architecture loads the resonator and makes its resonance frequency dependent on the quantum state of the qubit (i.e., the resonance frequency of the resonator is different depending on whether the qubit is in the ground or excited states). This property enables the performance of quantum nondemolition measurement of the qubit state, by sending a microwave signal on the order of a few photons to the cQED near the resonator frequency, and measuring the amplitude and/or phase of the output microwave field that carries information about the qubit state. Thus, one potential application of a working and reliable single photon detector in the microwave domain is to enable measuring this weak output signal (i.e., detecting the qubit state) inside the dilution fridge, without requiring the use of

high-gain, low-noise, and high-isolation output chains that are typically used nowadays in order to perform such measurements.

SUMMARY

[0007] According to one embodiment, a microwave device is provided. The microwave device includes a dispersive nonlinear element, and a pump resonator, at a first pump resonator end, connected to both the dispersive nonlinear element and a first stub. The pump resonator, at a second pump resonator end, is capacitively coupled to a pump port, where the first stub is terminated in an open circuit. Also, the microwave device includes a quantum signal resonator, at a first quantum signal resonator end, connected to both the dispersive nonlinear element and a second stub. The quantum signal resonator, at a second signal resonator end, is capacitively coupled to a signal port, wherein the second stub is connected to ground.

[0008] According to one embodiment, a method for nondemolition counting of photons is provided. The method includes coupling a pump resonance mode of a pump resonator and a signal resonance mode of a quantum signal resonator to a dispersive nonlinear element, responsive to a pump signal at a pump resonance frequency and a quantum signal at a signal resonance frequency. The pump resonance mode of the pump resonator has the pump resonance frequency, where the signal resonance mode of the quantum signal resonator has the signal resonance frequency. Also, the method includes creating a nonlinear interaction between the pump signal and the quantum signal, by driving the pump resonance mode with the pump signal at the pump resonance frequency, and detecting a presence or absence of photons in the quantum signal according to the pump resonance frequency which affects an output pump signal being measured.

[0009] According to one embodiment, a method of operating a microwave device is provided. The method includes receiving, by the microwave device, a pump signal at a pump resonance frequency, where the pump resonance frequency corresponds to a pump resonance mode of a pump resonator. The method includes receiving, by the microwave device, a quantum signal at a signal resonance frequency, where the signal resonance frequency corresponds to a signal resonance mode of a signal resonator, and outputting, by the microwave device, the pump signal with a phase shift, in response to a number of photons in the quantum signal.

[0010] Additional features and advantages are realized through the techniques of the present invention. Other embodiments and aspects of the invention are described in detail herein and are considered a part of the claimed invention. For a better understanding of the invention with the advantages and the features, refer to the description and to the drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

[0011] FIG. 1 is a schematic of a microwave device according to an embodiment.

[0012] FIG. 2 is a schematic of an equivalent circuit of the microwave device as seen by the pump port according to an embodiment.

[0013] FIG. 3 is a schematic of an equivalent circuit of the microwave device as seen by the signal port according to an embodiment.

[0014] FIG. 4 is a schematic of an example implementation of the microwave device using a coplanar waveguide geometry according to an embodiment.

[0015] FIG. 5 is a schematic of an example implementation of the microwave device using a microstrip geometry according to an embodiment.

[0016] FIG. 6 is a flow chart of a method for nondemolition counting and/or detection of photons using the microwave device according to an embodiment.

[0017] FIG. 7 is a flow chart of a method for the microwave device according to an embodiment.

DETAILED DESCRIPTION

[0018] In the optical frequency domain, reliable single photon detectors such as photomultipliers, microwave kinetic inductance detectors, and superconducting nanowire single-photon detector are widely used in various experiments and applications. However, one disadvantage of these devices is that they destroy (i.e., absorb) the photons that they detect.

[0019] In contrast, in the microwave domain, i.e., the gigahertz (GHz) range, reliable and practical single photon detectors are still under research and development. A working microwave photon detector based on Josephson junctions (dubbed the Josephson photomultiplier) has been experimentally researched. However, similar to single photon detectors in the optical domain this device absorbs the photons it detects. Additionally, this microwave device under development does not count the number of photons present in an incoming signal, it can only distinguish between the cases of zero photons or at least one photon in the signal.

[0020] Embodiments provide a practical scheme for a microwave device and measurement method for counting single microwave photons. Embodiments are configured to 1) detect and count the number of single photons within a certain bandwidth in the microwave domain, (i.e., in the gigahertz (GHz) range, e.g., the 1-20 GHz range), and 2) perform the detection and counting of photons in a non-demolition manner, i.e., without destroying (or absorbing) the photons being detected or counted.

[0021] Now turning to the figures, FIG. 1 is a schematic of a microwave device 100 according to an embodiment. The microwave device 100 includes a quarter-wavelength resonator 102 for the pump drive and a quarter-wavelength resonator 104 for the quantum signals. One end of the pump resonator 102 is connected to a coupling capacitor 106A, and the coupling capacitor 106A connects to a pump feedline 108. The pump feedline 108 is connected to pump port 111 and/or the pump port 111 is on the pump feedline 108. The pump feedline 108 receives a microwave pump signal 130 (i.e., a strong microwave tone), from a microwave generator 135 (denoted as pump). The other end of the pump resonator 102 connects to both a dispersive nonlinear element, e.g., Josephson junction (JJ) 110 and connects to a half-wavelength stub at the pump frequency 120A. The connection of the pump resonator 102, Josephson junction (JJ) 110, and stub 120A may be designated as node A. Opposite the node A, the stub 120A is terminated in an open circuit (O.C.). Opposite the coupling capacitor 106A, the pump feedline 108 is connected to a microwave measurement/analysis device 150. The microwave measurement/analysis device 150 is configured to measure the pump signal 130 in reflection after pump signal 130 has interacted

with microwave device 100. The microwave measurement/analysis device 150 may include and/or be connected to a computer for determining the phase shift (in the frequency) in the pump signal as discussed further herein. The microwave measurement/analysis device 150 may include and/or be connected to one or more processors, memories (e.g., computer readable storage medium) display screens, input devices (e.g., mouse, keyboard, touch screen, etc.). The pump 135 and microwave measurement/analysis device 150 are operatively connected to the microwave device 100 via a circulator 180A but are not part of the microwave device 100.

[0022] In the microwave device 100, one end of the quarter wavelength signal resonator 104 is connected to a coupling capacitor 106B, and the coupling capacitor 106B connects to a signal feedline 109. The signal feedline 109 is connected to a signal port 113 and/or the signal port 113 is on the signal feedline 109. The signal feedline 109 is configured to receive a microwave quantum signal 140, i.e., microwave signal being measured/tested, from a quantum device 145. The quantum device 145 may be a qubit, a cavity coupled to a qubit, etc. The other end of the signal resonator 104 connects to the Josephson junction (JJ) 110 and connects to a half-wavelength stub at the pump frequency 120B. The connection of the pump resonator 102, Josephson junction (JJ) 110, and stub 120B may be designated as node B. Opposite the node B, the stub 120B is terminated in a short-circuit. The signal feedline 109 may be connected to the quantum device 145 and a measurement/analysis device 144 via a circulator 180B. The measurement/analysis device 144 may be utilized for further processing. In one implementation, the device 144 may represent a 50 ohm termination. The port of the circulator 180B connected to the measurement/analysis device 144 ensures that no reflected signal is transmitted back to the quantum device 145.

[0023] The pump resonator 102 has a fundamental mode, which may be referred to as the pump mode or pump resonance mode. The pump mode of the pump resonator 102 has a resonance frequency, which may be referred to as the pump resonance frequency f_p . The pump mode of the pump resonator 102 has a wavelength λ_p , where $\lambda_p = c/f_p$ and c is the speed of light in the transmission line or waveguide used in the implementation of the device 102. The pump signal 130 applied to the pump resonator 102 is a strong coherent resonant tone (i.e., its frequency matches the resonance frequency of the pump resonator 102). The pump resonator 102 is designed to have a length corresponding to $\lambda_p/4$, which is quarter the wavelength of the pump signal. The stubs 120A and 120B are each designed to have a length corresponding to $\lambda_p/2$, which is half the wavelength of the pump signal.

[0024] The signal resonator 104 has a fundamental mode, which may be referred to as the signal mode or signal resonance mode. The signal mode of the signal resonator 104 has a resonance frequency, which may be referred to as the signal resonance frequency f_s . The quantum microwave signal 140 input to the signal resonator is a weak resonant tone on the order of a few single photons, whose frequency f_s matches the resonance frequency of the signal mode. The signal mode of the signal resonator 104 has a wavelength λ_s , where $\lambda_s = c/f_s$ and c is the speed of light in the transmission line or waveguide used in the implementation of the device.

The signal resonator **104** is designed to have a length corresponding to $\lambda_s/4$, which is a quarter the wavelength of the quantum signal.

[0025] The microwave device **100** has a frequency condition between the (pump) resonance frequency of the pump resonator **102** and the (signal) resonance frequency of the signal resonator **104**. The frequency condition is that the pump resonance frequency f_p of the pump resonator **102** is equal to twice the signal resonance frequency f_s of the signal resonator **104**. In other words, the frequency condition is $f_p=2 \cdot f_s$. Accordingly, the applied signal **130** has a frequency f_p that is twice the frequency f_s of the quantum signal **140**.

[0026] The microwave device **100** is configured such that the reflected pump signal **130** (identified as, e.g., as reflected pump signal **130'**) carries information about the number of photons present in the input quantum signal **140**, and thereby can be utilized to count the photons in the quantum signal **140**. Additionally, the reflected quantum signal **140** (identified as, e.g., reflected quantum signal **140'**) carries information about the number of photons present in the input pump signal **130**, and thereby can be utilized to count the photons in the pump signal **130**. This information about the number of photons in the quantum signal **140** is encoded in the phase shift of the reflected pump signal **130'** off port **108** as a result of the resonance frequency shift of the pump resonator **102** depending on the number of photons in the signal resonator **104**. The phase shift in the reflected pump signal **130'** is measured and analyzed by the microwave measurement/analysis device **150**.

[0027] The microwave device **100** (and/or operation via pump signal **130** and quantum signal **140**) is configured such that it can be described by the effective Hamiltonian (without the drives and feedlines) $H_{eff}=\hbar\tilde{\omega}_pN_p+\hbar\tilde{\omega}_sN_s+\hbar KN_p^2+\hbar K'N_pN_s$, where $\hbar\tilde{\omega}_pN_p$ represents the pump resonance mode term (modelled as a harmonic oscillator with ω_p the dressed resonance frequency of the pump resonance mode), $\hbar\tilde{\omega}_sN_s$ represents the signal resonance mode term (modelled as a harmonic oscillator with ω_s the dressed resonance frequency of the signal resonance mode), $\hbar KN_p^2$ represents the self-Kerr nonlinearity of the device, and $\hbar K'N_pN_s$ represents the cross-Kerr nonlinearity of the device. Further, K is the self-Kerr constant (i.e., the Kerr frequency shift per photon), and K' is the cross-Kerr constant (i.e., the cross-Kerr frequency shift per photon). Additionally, N_p is the photon number operator of the pump mode (whose eigenvalue is the number of photons in the pump resonance mode), where $N_p=\alpha_p^\dagger\alpha_p$, and N_s is the photon number operator of the signal mode (whose eigenvalue is the number of photons in the signal resonance mode), where $N_s=\alpha_s^\dagger\alpha_s$, and

$$\hbar = \frac{h}{2\pi},$$

and where h is Planck's constant. Also, α_p and α_s are quantum operators (i.e., annihilation operators associated with the pump and signal resonance modes). It is noted that sometimes in this disclosure the symbols N_p , N_s may be utilized to represent the eigenvalues of the number operators and not the number operators themselves. It is also noted that any person who is skilled in the art can easily make this distinction from the context.

[0028] FIG. 2 is a schematic of the equivalent circuit of the microwave device **100** according to an embodiment as seen by the pump port **111**. In addition to illustrating what the pump port **111** sees, FIG. 2 concurrently illustrates the circuit as seen by the incoming pump signal **130** at the pump resonance frequency f_p . Accordingly, discussion regarding the pump port **111** applies to the incoming pump signal **130**.

[0029] In the pump equivalent circuit, FIG. 2 shows the pump feedline **108** (including pump port **111**) coupled to the transmission line part of the pump resonator **102** via the coupling capacitor **106A**, and the other end of the transmission line part of the pump resonator connected to ground via the Josephson junction **110**. To explain this equivalent circuit, it is noted that 1) stub **120A**, which serves as an impedance transformer, is terminated in an open circuit and its length corresponds to half the wavelength of the pump signal **130**, thus node A sees an open circuit at the pump frequency, and 2) stub **120B**, which serves as an impedance transformer, is terminated in a short circuit and its length corresponds to half the wavelength of the pump signal **130**, thus node B sees a short circuit at the pump frequency.

[0030] One beneficial result of this pump equivalent circuit is that it shows that the pump resonance mode does not see the signal resonator **104**. In other words, the pump resonator **102** is isolated from the signal resonator **104**.

[0031] Another beneficial result is that the rf-current I_p associated with the pump resonance mode has an anti-node at the location of the Josephson junction **110**.

[0032] FIG. 3 is a schematic of the equivalent circuit of the microwave device **100** as seen by the quantum signal port **113** according to an embodiment. In addition to illustrating what the signal port **113** sees, FIG. 3 concurrently shows the equivalent circuit as seen by the incoming quantum signal **140** at the signal resonance frequency f_s . Accordingly, discussion regarding the signal port **113** applies to the incoming quantum signal **140**.

[0033] In the equivalent circuit of the microwave device **100** which is seen by the signal port, FIG. 3 shows the signal feedline **109** (including signal port **113**) coupled to the transmission line part of the signal resonator **104** via the coupling capacitor **106B**, and the other end of the transmission line part of the signal resonator **104** connected to ground via the Josephson junction **110**. Since the frequency condition for the pump frequency is $f_p=2 \cdot f_s$ (the fundamental resonance mode of the pump resonator **102** corresponds to the pump frequency f_p while the fundamental resonance mode of the signal resonator **104** corresponds to the signal frequency f_s), the signal port **113** (quantum signal **140** at the signal resonance frequency f_s) sees the opposite of the pump port **111**.

[0034] In this case (i.e., the case of the signal port), stub **120B**, which serves as an impedance transformer, is terminated in a short circuit and its length corresponds to quarter the wavelength of the signal, thus node B sees an open circuit at the signal frequency. Similarly, stub **120A**, which serves as an impedance transformer, is terminated in an open circuit and its length corresponds to quarter the wavelength of the signal, thus node A sees a short circuit at the signal frequency.

[0035] One beneficial result of this signal equivalent circuit is that it shows that the signal resonance mode does not see the pump resonator **102**. In other words, the signal resonator **104** is isolated from the pump resonator **102**.

[0036] Another beneficial result is that the rf-current I_S associated with the signal resonance mode has an anti-node at the location of the Josephson junction **110**.

[0037] It is noteworthy to clarify here based on FIGS. 2 and 3, that 1) the pump resonator **102** (ignoring the coupling capacitor and feedline) consists of the quarter-wavelength transmission line at the pump frequency shorted to ground via the Josephson junction **110**, and 2) the signal resonator **104** (ignoring the coupling capacitor and feedline) consists of the quarter-wavelength transmission line at the signal frequency shorted to ground via the Josephson junction **110**.

[0038] The microwave device **100** is configured to couple two microwave resonance modes (i.e., the pump resonance mode and the signal resonance mode) to a common dispersive nonlinear element, i.e., Josephson junction **110**.

[0039] The microwave device **100** is configured to use one mode, i.e., the pump mode at the pump resonance frequency f_p , as a photon number detector of the photons present in the second mode, i.e., the quantum signal mode at the signal resonance frequency f_s . In the microwave device **100**, the signal resonance frequency f_s of the signal mode corresponds to the microwave frequency of the microwave photons that are to be detected and/or counted.

[0040] By driving the pump mode (of the pump resonator **102**) using a strong coherent microwave tone (i.e., pump signal **130**) at the pump resonance frequency f_p , the microwave device **100** is configured to give rise to a cross-Kerr nonlinear effect in the Josephson junction **110** which leads to a nonlinear interaction between the pump and signals modes (and consequently between the pump signal **130** at the pump resonance frequency f_p and the quantum signal **140** at the signal resonance frequency f_s).

[0041] As a result of this cross-Kerr effect, the microwave device **100** is configured such that the pump resonance frequency f_p of the pump mode becomes dependent on the number of photons in the signal resonance mode at frequency f_s and vice versa.

[0042] The microwave device **100** is configured such that by monitoring the phase of the reflected pump signal **130'** at frequency f_p , the measurement/analysis device **150** can detect in a quantum nondemolition measurement the presence or absence of signal photons in the signal mode (i.e., detect the presence or absence of signal photons in the quantum signal **140** at frequency f_s).

[0043] The microwave device **100** is configured such that the number of photons in the signal mode is inferred/determined based on the size of the phase shift acquired by the output pump signal **130'** (as measured in reflection by the measurement/analysis device **150** at the pump feedline **108**). Hence, the microwave device **100** serves as a nondemolition microwave photon detector and counter. By introducing a frequency shift in the resonance frequency of the pump mode, the microwave device **100** neither absorbs nor destroys the signal photons in the quantum signal **140**. Rather, the quantum signal is reflected off **104'** the microwave device **100** at the signal feedline **109** after interacting with the pump signal **130** in the device **100** via the Josephson junction **110**.

[0044] It is noted that in addition to the pump and signal modes which are measured in reflection and explained in detail above, the microwave device **100** has also two common resonance modes which can be measured in transmission between the pump and signal ports. However, these common resonance modes do not play a role in the signal-

pump interaction described above and have frequencies that are far detuned from the pump and signal resonance modes (thus can be filtered out if necessary). For example, for a device with a pump resonance frequency around 16 GHz, and a signal resonance frequency around 8 GHz, the common modes of the device are expected to resonate at around 3 GHz, and 13 GHz.

[0045] Two beneficial advantages of the microwave device **100** which can be readily inferred from the device description are the following:

[0046] 1) the strong pump drive (i.e., pump signal **130**) which enables the detection of the signal photons is injected through a different port than the weak signal (e.g., quantum signal **140**) being detected; and

[0047] 2) the pump and signal modes are completely isolated from each other (due to the use of the stubs). They only interact through the JJ (or JJs) which connects their respective resonators. Hence, by design there should not be any direct power leakage between the pump and signal ports.

[0048] FIG. 4 is a schematic of the microwave device **100** implemented as a coplanar waveguide according to an embodiment. In FIG. 4, a pump feedline **108** is connected to the pump resonator **102** by the coupling capacitor **106A**. The pump feedline **108** and the pump resonator **102** are made of superconductors formed on a low-loss dielectric substrate. The coupling capacitor **106A** is implemented as a gap capacitor between the conductors of the pump feedline **108** and pump resonator **102**. The pump resonator **102** has a length corresponding to approximately $\lambda_p/4$ (for a particular pump resonance frequency this length can vary depending on the amount of lumped inductance added by the Josephson junction **110** terminating the transmission line part of the pump resonator). A ground plane **405** is formed on both sides of the pump resonator **102** and pump feedline **108**.

[0049] A quantum signal feedline **109** is connected to the signal resonator **104** by the coupling capacitor **106B**. The signal feedline **109** and the signal resonator **104** are also made of superconductors formed on the low-loss dielectric substrate. Similarly, the coupling capacitor **106B** is implemented as a gap capacitor between the conductors of the signal feedline **109** and signal resonator **104**. The signal resonator **104** has a length corresponding to approximately $\lambda_s/4$ (for a particular signal resonance frequency this length can vary depending on the amount of lumped inductance added by the Josephson junction **110** terminating the transmission line part of the signal resonator). A ground plane **405** is formed on both sides of the signal resonator **104** and signal feedline **109**.

[0050] At node A, the pump resonator **102** is connected to the Josephson junction **110** and the stub **120A**. The other end of the stub **120A** is left open (i.e., terminated in an open circuit).

[0051] At node B, the signal resonator **104** is connected to the Josephson junction **110** and the stub **120B**. The other end of the stub **120B** is connected to the ground plane **405**. The stubs **120A** and **120B** are superconducting transmission lines implemented in this embodiment in the form of a coplanar waveguide on the low-loss dielectric substrate, and the center conductor of the stubs **120A** and **120B** each have a length corresponding to $\lambda_p/2$.

[0052] The Josephson junction **110** is a dispersive nonlinear inductor, which is made of two superconducting electrodes separated by a barrier (e.g., insulating tunnel barrier). For example, one superconducting electrode of the Joseph-

son junction **110** connects to node A, while the other superconducting electrode connects to node B.

[0053] FIG. 5 is a schematic of the microwave device **100** implemented in the form of a microstrip geometry according to an embodiment. FIG. 5 is similar to FIG. 4 in that the microstrip implementation has superconductors formed on a low-loss dielectric substrate according to the microwave device **100**. One main difference between the microstrip and the coplanar waveguide implementations relates to the location of the ground plane. In the coplanar waveguide configuration (FIG. 4) the ground plane is located on the same side of the dielectric substrate as the center conductor, whereas in the microstrip configuration (FIG. 5) the ground plane is on the opposite side of the dielectric substrate.

[0054] In FIG. 5, a pump feedline **108** is connected to the pump resonator **102** by the coupling capacitor **106A**. The pump feedline **108** and the pump resonator **102** are superconductors formed on a low-loss dielectric substrate. The coupling capacitor **106A** is implemented as a gap capacitor between the conductors of the pump feedline **108** and pump resonator **102**. The pump resonator **102** has a length corresponding to $\lambda_p/4$ (for a particular pump resonance frequency this length can vary depending on the amount of lumped inductance added by the Josephson junction **110** terminating the transmission line part of the pump resonator). However, unlike FIG. 4, no ground plane is formed on both sides of the pump resonator **102** and pump feedline **108**, and instead the ground plane is formed on the other side of the dielectric substrate.

[0055] A quantum signal feedline **109** is connected to the signal resonator **104** by the coupling capacitor **106B**. The signal feedline **109** and the signal resonator **104** are also superconductors formed on the low-loss dielectric substrate. Similarly, the coupling capacitor **106B** is implemented as a gap capacitor between the conductors of the signal feedline **109** and signal resonator **104**. The signal resonator **104** has a length corresponding to $\lambda_s/4$ (for a particular signal resonance frequency this length can vary depending on the amount of lumped inductance added by the Josephson junction **110** terminating the transmission line part of the signal resonator). Unlike FIG. 4, no ground plane is formed on both sides of the signal resonator **104** and signal feedline **109**, and instead the ground plane is formed on the other side of the dielectric substrate.

[0056] At node A, the pump resonator **102** is connected to the Josephson junction **110** and the stub **120A**. The other end of the stub **120A** is left open (i.e., terminated in an open circuit).

[0057] At node B, the signal resonator **104** is connected to the Josephson junction **110** and the stub **120B**. The other end of the stub **120B** is connected to the ground plane **405**. The stubs **120A** and **120B** are superconducting transmission lines implemented in the form of a microstrip on the low-loss dielectric substrate, and the center conductor of the stubs **120A** and **120B** each have a length corresponding to $\lambda_p/2$.

[0058] In accordance with the teachings presented herein, one of ordinary skill in the art recognizes other possible implementations or variations to embodiments. In one implementation, the pump and signal resonators **102** and **104** may be equivalently implemented using lumped inductors (e.g., narrow superconducting wires or array of large Josephson junctions) and lumped capacitors (e.g., planar capacitors or interdigitated capacitors). One particular con-

dition in the various implementations is to maintain maximum RF currents of the pump and signal modes at the Josephson junction **110** location.

[0059] In another implementation, the half-wave stubs **120A** and **120B** of the microwave device **100** can also be implemented using their equivalent lumped-element circuit in the vicinity of the pump resonance frequency.

[0060] In one implementation, the single Josephson junction **110** may be replaced by an array of large Josephson junctions.

[0061] In yet another implementation, the single Josephson junction **110** may be replaced by a direct current superconducting quantum interference device (DC-SQUID) (or array of DC-SQUIDS) which enables in-situ tuning of the linear inductance of the mixing element (i.e., the inductance of the Josephson junctions in the DC-SQUID) by varying the magnetic flux threading the DC-SQUID loop (or the loops of the array of DC-SQUIDS).

[0062] According to an implementation, the microwave device **100** can be made frequency tunable by incorporating DC-SQUIDS in the device resonators, stubs, and the non-linear mixing element (i.e., the Josephson junction **110** or array of Josephson junctions).

[0063] More detail of the theory for the photon counting and detection in the microwave device **100** is discussed. For ease of understanding, sub-headings are provided below. It is understood that the sub-heading are for explanation purposes and not limitation.

[0064] I. The Energy of the Josephson Junction

[0065] A supercurrent flowing in a Josephson junction satisfies the current-phase relation given by $I_j = I_0 \sin \delta$, where I_0 is the critical current of the Josephson junction, δ is the gauge-invariant phase difference. The energy of the Josephson junction can be written as $E_j = E_J [1 - \cos \delta]$, where $E_J = I_0 \omega_0$ is the Josephson energy and $\omega_0 = \hbar/2e$ is the reduced flux-quantum (e is the electron charge). By using the trigonometric identity $\cos x = \sqrt{1 - x^2}$ we can rewrite the energy of the Josephson junction as

$$E_j = E_J \left[1 - \sqrt{1 - \left(\frac{I_j}{I_0} \right)^2} \right].$$

[0066] Expanding the expression for the energy of the Josephson junction up to fourth order in current we get

$$E_j \approx \frac{E_J}{2} \left(\frac{I_j}{I_0} \right)^2 - \frac{E_J}{24} \left(\frac{I_j}{I_0} \right)^4.$$

By substituting the junction inductance

$$L_j = \frac{E_J}{I_0^2}$$

we obtain

$$E_j \approx \frac{L_J}{2} \hat{I}_j^2 - \frac{L_J}{24} \frac{\hat{I}_j^4}{I_0^2}, \quad (\text{Eq. 1})$$

[0067] where the first term ($\propto I_j^2$) modifies the bare resonance frequencies of the pump and signal resonators while the second term ($\propto I_j^4$) represents the nonlinear mixing term.

[0068] II. Quantization

[0069] Based on the equivalent circuits of the microwave device as seen by the pump and signal ports shown in FIGS. 2, 3, the Radio frequency (RF) current flowing in the Josephson junction is $I_J = I_P - I_S$, where I_P and I_S are the rf-currents of the pump and signal microwave resonance modes flowing in the Josephson junction.

[0070] Expressing the currents I_P , I_S in terms of the quantum operators α_P , α_S which represent the annihilation operators associated with the pump and signal resonance modes gives

$$I_P = i\hat{I}_P(\alpha_P^\dagger - \alpha_P) \quad (\text{Eq. 2})$$

$$I_S = i\hat{I}_S(\alpha_S^\dagger - \alpha_S) \quad (\text{Eq. 3})$$

[0071] where \hat{I}_P , \hat{I}_S are the zero-point fluctuations (ZPF) current amplitudes given by

$$\hat{I}_P = \omega_P \sqrt{\frac{\hbar}{2Z_P}} \quad \text{and} \quad \hat{I}_S = \omega_S \sqrt{\frac{\hbar}{2Z_S}},$$

where ω_P and ω_S are the angular resonance frequencies of the pump and signal resonators, and Z_P and Z_S are the characteristic impedances of the corresponding resonators.

[0072] Using the following expressions for the angular resonance frequencies

$$\omega_P^2 = \frac{1}{L_P C_P}, \quad \omega_S^2 = \frac{1}{L_S C_S}$$

and resonator impedances

$$Z_P^2 = \frac{L_P}{C_P}, \quad Z_S^2 = \frac{L_S}{C_S}$$

the ZPF current amplitudes can be rewritten as

$$\hat{I}_P^2 = \frac{\hbar \omega_P}{2 L_P}, \quad \text{and} \quad (\text{Eq. 4})$$

$$\hat{I}_S^2 = \frac{\hbar \omega_S}{2 L_S}, \quad (\text{Eq. 5})$$

[0073] where L_P , L_S and C_P , C_S represent the inductances and capacitances of the equivalent LC circuit of the pump and signal resonators at resonance.

[0074] III. Effective Hamiltonian of the System

[0075] Without taking into account the feedlines, drives, and loss to the environment the effective Hamiltonian of the system is given by the sum

$$H_{\text{eff}} = H_{\text{res}} + E_j, \quad (\text{Eq. 6})$$

[0076] where $H_{\text{res}} = \hbar \omega_P N_P + \hbar \omega_S N_S$, and $N_P = \alpha_P^\dagger \alpha_P$, $N_S = \alpha_S^\dagger \alpha_S$ are the photon number operators for the pump and signal modes.

[0077] Substituting Eqs. 2 and 3 into the expression for E_j (i.e., Eq. 1) while using Eqs. 4 and 5, the photon number operators N_P , N_S , the imposed frequency condition $\omega_P = 2\omega_S$, and the rotating wave approximation, we can write the effective Hamiltonian of the system (Eq. 6) in the form

$$H_{\text{eff}} = \hbar \tilde{\omega}_P N_P + \hbar \tilde{\omega}_S N_S + \hbar K N_P^2 + \hbar K' N_P N_S, \quad (\text{Eq. 7})$$

[0078] where $\tilde{\omega}_P$, $\tilde{\omega}_S$ in the first and second term are the dressed angular resonance frequencies of the pump and signal modes which include the inductive loading of the resonators due to the Josephson junction (represented by the first term in Eq. 1), and K , K' in the third and fourth term which represent the self-Kerr and cross-Kerr nonlinearity correspond to the self-Kerr and cross-Kerr constants respectively.

[0079] In the derivation of Eq. 7, we have also used the fact the pump mode is driven strongly compared to the signal mode, and that the bosonic operators of the two modes α_P , α_S commute with each other and those of the same mode satisfy the usual commutation relations of the form $[\alpha_P, \alpha_P^\dagger] = 1$, $[\alpha_S, \alpha_S^\dagger] = 1$.

[0080] The self-Kerr constant in Eq. 7 is given by

$$K = -\frac{L_J}{4I_0^2} \frac{\hat{I}_P^4}{\hbar},$$

which we can rewrite in terms of the participation ratio

$$p_P = \frac{L_J}{L_P}$$

of the linear inductance of the JJ to the total inductance of the pump resonator and the plasma frequency of the JJ

$$\omega_J = \frac{I_0}{2e} \quad K = -\frac{p_P^2 \omega_P^2}{16\omega_J}. \quad (\text{Eq. 8})$$

[0081] Similarly, the cross-Kerr constant in Eq. 7 is given by

$$K' = -\frac{L_J}{I_0^2} \frac{\hat{I}_P^2 \hat{I}_S^2}{\hbar},$$

which we can rewrite in terms of p_P , ω_J , and the participation ratio

$$p_S = \frac{L_J}{L_S}$$

of the linear inductance of the JJ to the total inductance of the signal resonator

$$K' = -\frac{p_P p_S \omega_P \omega_S}{4\omega_J}. \quad (\text{Eq. 9})$$

[0082] IV. Resonance Frequency Shift Per Photon

[0083] To better understand the basic idea of the device, we rearrange the terms in Eq. 7 such that the effective Hamiltonian of the system reads

$$H_{eff} = \hbar(\tilde{\omega}_P + K N_P + K' N_S) N_P + \hbar\tilde{\omega}_S N_S. \quad (\text{Eq. 10})$$

[0084] This form shows that by operating the device in the nonlinear regime where the Kerr effect is appreciable, the self-Kerr and cross-Kerr nonlinearity cause the dressed angular resonance frequency of the pump mode to shift depending on the number of photons present in the pump and signal resonance modes. Furthermore, since the pump mode is externally driven at a certain working point, signal photons that enter the signal resonator would shift the pump resonance frequency by $K' N_S$, which is proportional to their number, thus the cross-Kerr constant K' corresponds to frequency shift per photon.

[0085] It is noted that in order to detect (i.e., resolve the presence of) a single microwave photon using this device the frequency shift per photon due to the cross-Kerr nonlinearity (i.e., K') should be equal or larger than the linewidth (i.e., bandwidth) of the pump resonance mode at the working point.

[0086] V. Design Example Using Typical Numerical Values

[0087] In a design example of the proposed microwave device **100**, feasible numerical values of the various parameters are utilized. The dressed resonance frequency for the pump mode is

$$\frac{\tilde{\omega}_P}{2\pi} = 16 \text{ GHz.}$$

The dressed resonance frequency for the signal mode is

$$\frac{\tilde{\omega}_S}{2\pi} = 8 \text{ GHz.}$$

The impedance of the resonators $Z_P = Z_S = 50\Omega$ (please note that lower characteristic impedances are also feasible and are expected to be more favorable with respect to the device performance). Using the relation

$$L_{P,S} = \frac{Z_{P,S}}{\tilde{\omega}_{P,S}},$$

we get an estimate for $L_P = 0.5$ nanoHenry (nH), $L_S = 1$ nH. Assuming $I_0 = 1$ microampere (μA), then $L_J = 0.3$ nH and

$$\frac{\omega_J}{2\pi} = 497 \text{ GHz.}$$

Using the values for $L_{P,S}$ and L_J we get an estimate for the participation ratios for the pump and signal resonators $p_P = 0.38$ and $p_S = 0.23$. Substituting these values in Eqs. 8 and 9 yields

$$\frac{K}{2\pi} \approx -4.6 \text{ MHz and } \frac{K'}{2\pi} \approx -5.6 \text{ MHz.}$$

By designing the pump resonance mode to have a linewidth smaller than these frequency shifts per photon (which is completely achievable with state-of-the-art superconducting microwave circuits), the microwave device **100** is configured to detect single quantum signal photons as measured by the measurement/analysis device **150**.

[0088] In order for the device to work properly in one implementation, it needs to satisfy two additional requirements. For the first requirement, the internal quality factor of both resonators should be as high as possible at the single photon level $>10^5$ and at least 2 orders of magnitude larger than the external quality factor of the resonators set by the coupling capacitors between the resonators and the feedlines and their characteristic impedances. This requirement is so that the signal photons being detected and the pump photons detecting them do not get lost to internal loss mechanisms in the resonators at a higher rate than the rate at which they enter and leave both resonators. One obvious consequence of this requirement is that the total quality factor of both resonators is mainly set by the external quality factor.

[0089] For the second requirement, the bandwidth of the pump resonator at the bias point should be equal or larger than the bandwidth of the signal resonator. In other words, the response time of the pump resonator should be equal or shorter than the response time of the signal resonator. This requirement is in order to allow a sufficient time for the pump photons to detect the signal photons before they leave the device through the signal feedline. It is noted that both requirements are achieved in superconducting microwave circuits discussed herein.

[0090] In one implementation, a technique to (experimentally) calibrate the value of the cross-Kerr constant K' for a certain pump drive is by varying the input power of a coherent tone applied at the signal frequency to the signal resonator while measuring—for each input power—the complex reflection parameter of the pump resonator as a function of frequency using a very weak probe (less than one photon on average) superimposed on the pump drive. By extracting the slope of the measured pump resonance frequency versus signal power and using beforehand knowledge of the signal resonance frequency and the signal resonator bandwidth, one can calculate the constant K' .

[0091] In one implementation, a technique to (experimentally) detect single signal photons using this device is by monitoring the phase of the reflected pump drive applied at the pump resonance frequency with no input signal (i.e., $N_S = 0$). When signal photons on the order of 1-3 photons enter the signal resonator and interact with the pump mode through the JJ (i.e., the nonlinear dispersive element) the resonance frequency of the pump mode shifts downwards by

a multiple number of $K'/2\pi$ that is proportional to the number of signal photons in the signal resonator. As a consequence of this resonance frequency shift of the pump mode, the phase of the reflected pump drive is correspondingly shifted as well. By measuring this phase shift one can infer the number of signal photons (on the order of 1-3) that entered the device.

[0092] In order to count in real-time a larger number of incoming signal photons, e.g., between 3 to 10, using the device, one may employ a more elaborate measurement technique. For example, continuous monitoring may be performed of the reflected phase of multiple relatively weak tones (in order not to alter the device operation) applied to the pump resonator at frequencies that are located

$$\frac{K'}{2\pi}, \frac{2K'}{2\pi}, \dots, \frac{N_S K'}{2\pi}$$

below the pump resonance frequency with no input signal (i.e., $N_S=0$). According to this method, if a phase shift is detected in the reflected weak tone that is applied at frequency

$$\frac{N_S K'}{2\pi}$$

below the pump resonance frequency with no input signal (i.e., $N_S=0$), this indicates with high probability that the incoming signal contained N_S photons.

[0093] Measuring a larger number of microwave photons beyond a few photons might not be as useful for certain quantum applications, and thus the device may not be tuned accordingly. It is also noted that the effective Hamiltonian of the system was specifically derived in the limit of very weak signal compared to the pump drive. Thus, by further increasing the strength of the signal beyond a few photons, other unwanted nonlinear terms which were neglected in the derivation of the Hamiltonian (Eq. 10) are expected to come into play. The exact number of input signal photons which the device can detect or count without a significant degradation of performance can of course vary from one device to another and from one implementation to another depending on several design parameters, such as the critical current of JJ or JJs, the participation ratios, the bandwidths of the resonators, the characteristic impedances of the resonators, and the particular implementation of the resonators.

[0094] Now turning to FIG. 6, a flow chart of a method 600 is provided for nondemolition counting and/or detection of microwave photons using the microwave device 100 according to an embodiment.

[0095] At block 605, the microwave device 100 is configured to couple a pump resonance mode (fundamental resonance mode) of the pump resonator 102 and a signal resonance mode (e.g., fundamental resonance mode) of the quantum signal resonator 104 to a dispersive nonlinear element (e.g., Josephson junction 110), responsive to the pump signal 130 at a pump resonance frequency f_p and the quantum signal 140 at a signal resonance frequency f_s . The pump resonance mode of the pump resonator 102 has the

pump resonance frequency f_p and the signal resonance mode of the quantum signal resonator has the signal resonance frequency.

[0096] At block 610, the microwave device 100 is configured to create a nonlinear interaction/mixing (i.e., via the Josephson junction 110) between the pump signal 130 and the quantum signal 140, by strongly driving the pump resonance mode (i.e., pump mode) with the coherent pump signal 130 at the pump resonance frequency f_p .

[0097] At block 615, the microwave device 100 is configured to enable detection of the presence or absence of photons in the quantum signal 140 according to the resonance frequency of the pump mode which affects the phase of the output pump signal 130' (i.e., reflected from the microwave device 100).

[0098] The microwave device 100 is configured to excite a cross-Kerr nonlinear effect in the dispersive nonlinear element, thereby causing the nonlinear interaction between the pump signal 130 and the quantum signal 140. The microwave device 100 is configured such that the pump resonance frequency of the pump mode is dependent on the number of photons in the quantum signal 140 as a result of the cross-Kerr nonlinear effect taking place in the device (this can be shown by taking into account the input-output relations of the device).

[0099] The number of photons in the quantum signal 140 is determined by a size of a frequency shift in the pump resonance frequency. The frequency shift is a multiple of a cross-Kerr coefficient. A baseline frequency shift is established, such that the frequency shift is denoted as being greater than the baseline frequency shift previously established. The frequency shift denotes the number of the photons in the quantum signal while the baseline frequency shift is established prior to receiving the quantum signal. Each multiple of the baseline frequency shift in the pump signal denotes a single photon count of the quantum signal, such that 0-N photons corresponds to 0-M multiples of the baseline frequency shift, where N is a last number of the photons and M is a last multiple of the baseline frequency shift.

[0100] FIG. 7 is a flow chart of a method 700 for the microwave device 100 according to an embodiment. Reference can be made to FIGS. 1-5.

[0101] At block 705, the microwave device 100 is configured to receive a strong coherent pump signal 130 at the pump resonance frequency f_p , where the pump resonance frequency corresponds to a pump resonance (fundamental) mode of a pump resonator 102.

[0102] At block 710, the microwave device 100 is configured to receive a quantum signal 140 at the signal resonance frequency f_s , where the signal resonance frequency corresponds to a signal resonance (fundamental) mode of a signal resonator 104.

[0103] At block 715, the microwave device 100 is configured to output the pump signal 130' with a phase shift, in response to pump resonance frequency shift of the pump mode which depends on a number of photons in the quantum signal 140.

[0104] It will be noted that various microelectronic device fabrication methods may be utilized to fabricate the components/elements discussed herein as understood by one skilled in the art. In superconducting and semiconductor device fabrication, the various processing steps fall into four

general categories: deposition, removal, patterning, and modification of electrical properties.

[0105] Deposition is any process that grows, coats, or otherwise transfers a material onto the wafer. Available technologies include physical vapor deposition (PVD), chemical vapor deposition (CVD), electrochemical deposition (ECD), molecular beam epitaxy (MBE) and more recently, atomic layer deposition (ALD) among others.

[0106] Removal is any process that removes material from the wafer: examples include etch processes (either wet or dry), and chemical-mechanical planarization (CMP), etc.

[0107] Patterning is the shaping or altering of deposited materials, and is generally referred to as lithography. For example, in conventional lithography, the wafer is coated with a chemical called a photoresist; then, a machine called a stepper focuses, aligns, and moves a mask, exposing select portions of the wafer below to short wavelength light; the exposed regions are washed away by a developer solution. After etching or other processing, the remaining photoresist is removed. Patterning also includes electron-beam lithography.

[0108] Modification of electrical properties may include doping, such as doping transistor sources and drains, generally by diffusion and/or by ion implantation. These doping processes are followed by furnace annealing or by rapid thermal annealing (RTA). Annealing serves to activate the implanted dopants.

[0109] The flowchart and block diagrams in the Figures illustrate the architecture, functionality, and operation of possible implementations of systems, methods, and computer program products according to various embodiments of the present invention. In this regard, each block in the flowchart or block diagrams may represent a module, segment, or portion of instructions, which comprises one or more executable instructions for implementing the specified logical function(s). In some alternative implementations, the functions noted in the block may occur out of the order noted in the figures. For example, two blocks shown in succession may, in fact, be executed substantially concurrently, or the blocks may sometimes be executed in the reverse order, depending upon the functionality involved. It will also be noted that each block of the block diagrams and/or flowchart illustration, and combinations of blocks in the block diagrams and/or flowchart illustration, can be implemented by special purpose hardware-based systems that perform the specified functions or acts or carry out combinations of special purpose hardware and computer instructions.

[0110] The descriptions of the various embodiments of the present invention have been presented for purposes of illustration, but are not intended to be exhaustive or limited to the embodiments disclosed. Many modifications and variations will be apparent to those of ordinary skill in the art without departing from the scope and spirit of the described embodiments. The terminology used herein was chosen to best explain the principles of the embodiments, the practical application or technical improvement over technologies found in the marketplace, or to enable others of ordinary skill in the art to understand the embodiments disclosed herein.

What is claimed is:

1. A system comprising:

a quantum device configured to output a quantum signal;
a microwave device coupled to the quantum device, the microwave device comprising:

a first resonator coupled to both a dispersive nonlinear element and a first stub; and

a second resonator coupled to both the dispersive nonlinear element and a second stub.

2. The system of claim 1, further comprising a measurement device coupled to the first resonator or the second resonator.

3. The system of claim 2, wherein the measurement device is configured to measure a reflected pump signal having interacted with the microwave device.

4. The system of claim 3, wherein the measurement device is configured to determine an absence or presence of photons in the quantum signal having been input according to a phase shift in the reflected pump signal.

5. The system of claim 4, wherein a number of photons in the quantum signal is determined by a magnitude of the phase shift in the reflected pump signal.

6. The system of claim 1, wherein the quantum device is coupled to the first resonator or the second resonator.

7. The system of claim 1, wherein the quantum device comprises a qubit and the quantum signal is associated with the qubit.

8. The system of claim 1, wherein the first stub is coupled to the dispersive nonlinear element.

9. The system of claim 1, wherein the second stub is coupled to the dispersive nonlinear element.

10. The system of claim 1, wherein one of the first and second stubs is connected to an open circuit and another one of the first and second stubs is connected to ground.

11. A microwave device comprising:

a first resonator coupled to both a dispersive nonlinear element and a first stub; and

a second resonator coupled to both the dispersive nonlinear element and a second stub.

12. The microwave device of claim 11, wherein the first resonator is operable to couple to a measurement device.

13. The microwave device of claim 11, wherein the first resonator is operative to couple to a pump signal source.

14. The microwave device of claim 11, wherein the second resonator is operable to couple to a quantum device.

15. The microwave device of claim 11, wherein the second resonator is operable to couple to a 50 ohm termination point.

16. The microwave device of claim 11, wherein the dispersive nonlinear element is at least one Josephson junction.

17. The microwave device of claim 11, wherein the dispersive nonlinear element is an array of Josephson junctions.

18. The microwave device of claim 11, wherein the first resonator comprises a pump resonance mode, the pump resonance mode having a pump resonance frequency and a pump wavelength, a length of the first resonator corresponding to a quarter wavelength of the pump wavelength.

19. The microwave device of claim 11, wherein the second resonator comprises a signal resonance mode, the signal resonance mode having a signal resonance frequency and a signal wavelength, a length of the second resonator corresponding to a quarter wavelength of the signal wavelength.

20. The microwave device of claim 11, wherein the first resonator, the second resonator, the first and second stubs,

and the dispersive nonlinear element are configured to neither destroy nor absorb any photons input in a quantum signal.

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