US 20060129800A1

(19) United States

(12) Patent Application Publication

Lauter et al.

(10) Pub. No.: US 2006/0129800 A1

(43) Pub. Date: Jun. 15, 2006

(54) CRYPTOGRAPHICALLY PROCESSING DATA BASED ON A CASSELS-TATE PAIRING

(75) Inventors: **Kristin E. Lauter**, La Jolla, CA (US); **Anne Kirsten Eisentraeger**, Ann Arbor, MI (US)

Correspondence Address:
LEE & HAYES PLLC
421 W RIVERSIDE AVENUE SUITE 500
SPOKANE, WA 99201

(73) Assignee: **Microsoft Corporation**, Redmond, WA

(21) Appl. No.: 11/011,289

(22) Filed: Dec. 14, 2004

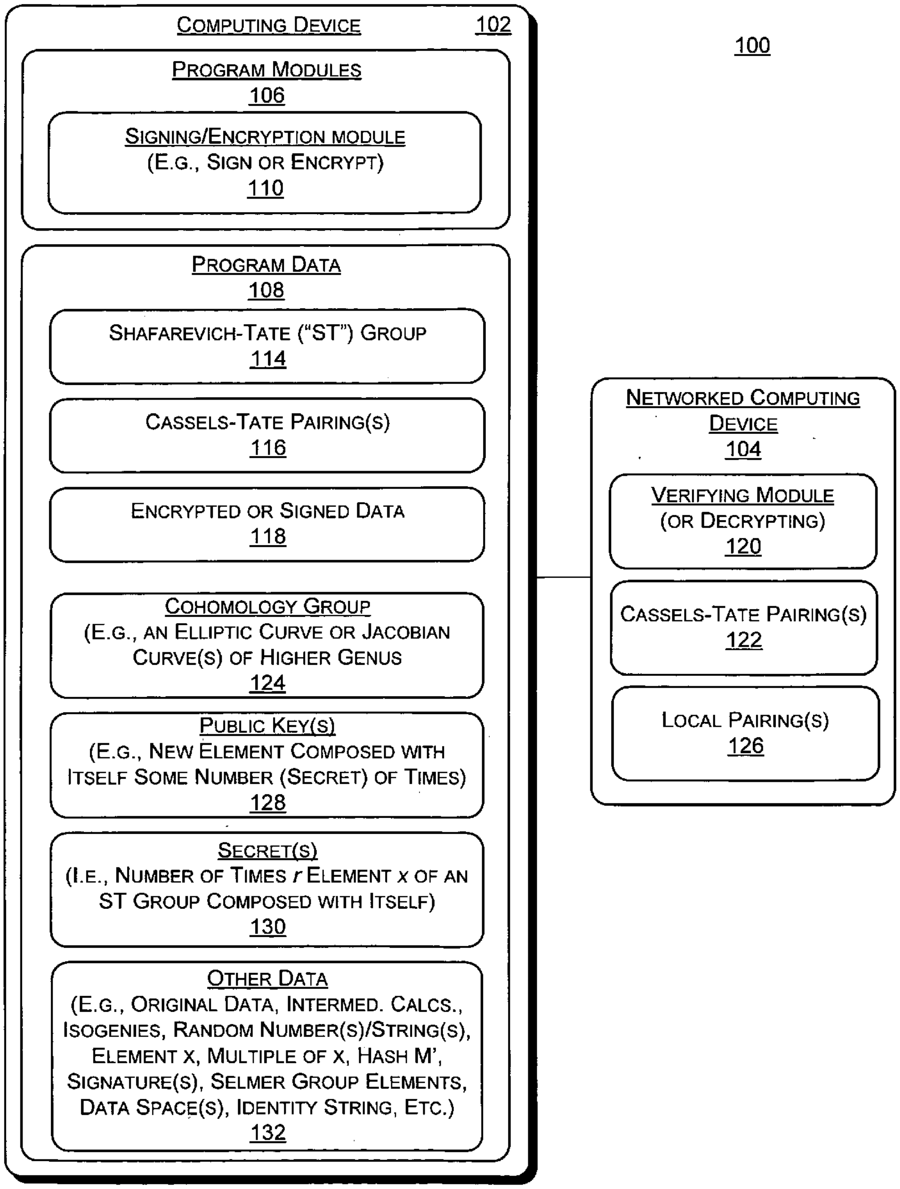
Publication Classification

(51) Int. Cl.
H04L 9/00 (2006.01)

(52) U.S. Cl. 713/151

(57) ABSTRACT

Systems and methods for cryptographically processing data as a function of a Cassels-Tate pairing are described. In one aspect, a Shafarevich-Tate group is generated from a cohomology group. A Cassels-Tate pairing is determined as a function of elements of the Shafarevich-Tate group. Data is then cryptographically processed as a function of the Cassels-Tate pairing.


COMPUTING DEVICE 102
100
PROGRAM MODULES 106
SIGNING/ENCRYPTION MODULE (E.G., SIGN OR ENCRYPT) 110
PROGRAM DATA 108
SHAFAREVICH-TATE ("ST") GROUP 114
CASSELS-TATE PAIRING(S) 116
ENCRYPTED OR SIGNED DATA 118
COHOMOLOGY GROUP (E.G., AN ELLIPTIC CURVE OR JACOBIAN CURVE(S) OF HIGHER GENUS 124
PUBLIC KEY(S) (E.G., NEW ELEMENT COMPOSED WITH ITSELF SOME NUMBER (SECRET) OF TIMES) 128
SECRET(S) (I.E., NUMBER OF TIMES r ELEMENT x OF AN ST GROUP COMPOSED WITH ITSELF) 130
OTHER DATA (E.G., ORIGINAL DATA, INTERMED. CALCS., ISOGENIES, RANDOM NUMBER(S)/STRING(S), ELEMENT x, MULTIPLE OF x, HASH M', SIGNATURE(S), SELMER GROUP ELEMENTS, DATA SPACE(S), IDENTITY STRING, ETC.) 132
NETWORKED COMPUTING DEVICE 104
VERIFYING MODULE (OR DECRYPTING) 120
CASSELS-TATE PAIRING(S) 122
LOCAL PAIRING(S) 126

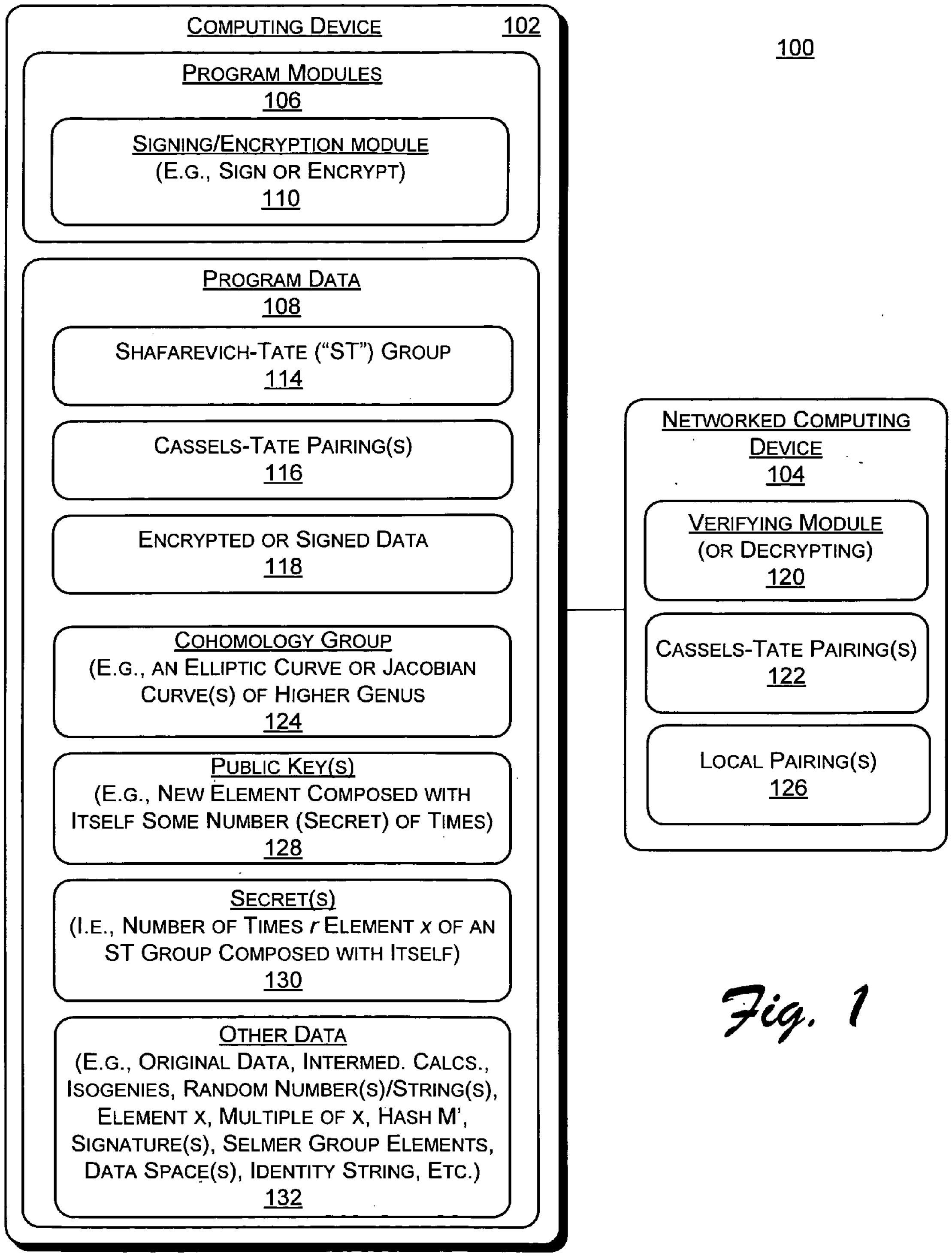

COMPUTING DEVICE 102
100
PROGRAM MODULES 106
SIGNING/ENCRYPTION MODULE (E.G., SIGN OR ENCRYPT) 110
PROGRAM DATA 108
SHAFAREVICH-TATE ("ST") GROUP 114
CASSELS-TATE PAIRING(S) 116
ENCRYPTED OR SIGNED DATA 118
COHOMOLOGY GROUP (E.G., AN ELLIPTIC CURVE OR JACOBIAN CURVE(S) OF HIGHER GENUS 124
PUBLIC KEY(S) (E.G., NEW ELEMENT COMPOSED WITH ITSELF SOME NUMBER (SECRET) OF TIMES) 128
SECRET(S) (I.E., NUMBER OF TIMES r ELEMENT x OF AN ST GROUP COMPOSED WITH ITSELF) 130
OTHER DATA (E.G., ORIGINAL DATA, INTERMED. CALCS., ISOGENIES, RANDOM NUMBER(S)/STRING(S), ELEMENT X, MULTIPLE OF X, HASH M', SIGNATURE(S), SELMER GROUP ELEMENTS, DATA SPACE(S), IDENTITY STRING, ETC.) 132
NETWORKED COMPUTING DEVICE 104
VERIFYING MODULE (OR DECRYPTING) 120
CASSELS-TATE PAIRING(S) 122
LOCAL PAIRING(S) 126

Fig. 1

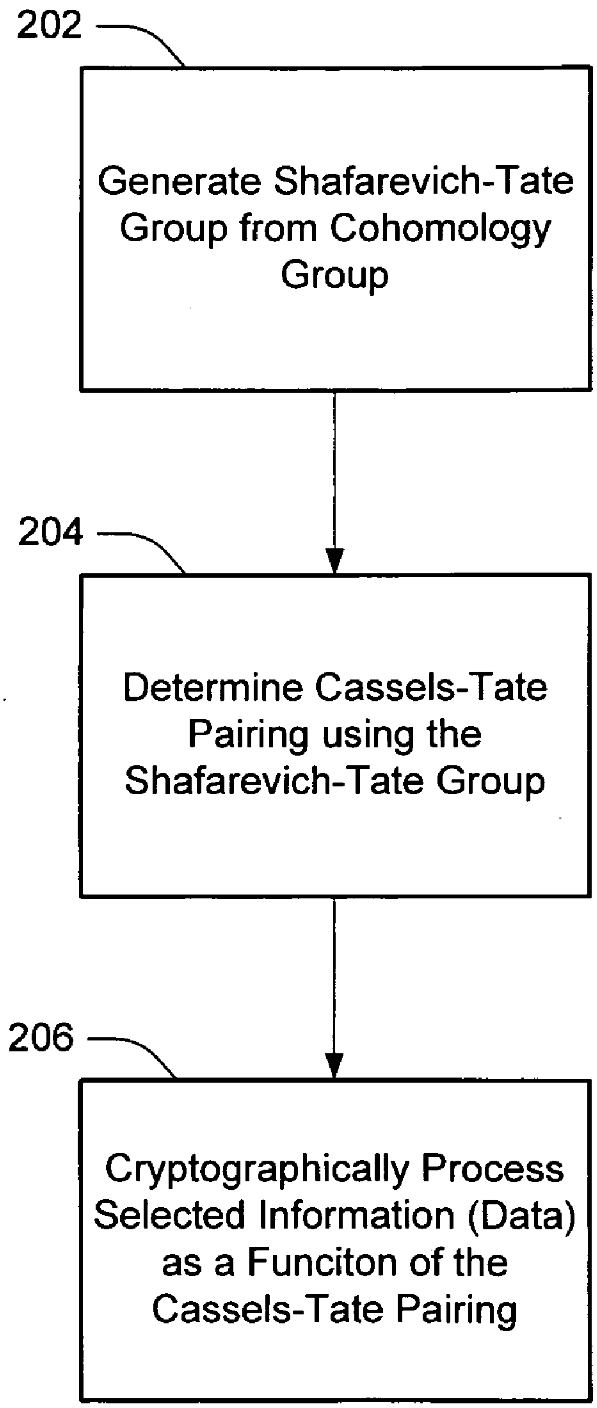

202
Generate Shafarevich-Tate Group from Cohomology Group
204
Determine Cassels-Tate Pairing using the Shafarevich-Tate Group
206
Cryptographically Process Selected Information (Data) as a Funciton of the Cassels-Tate Pairing

Fig. 2

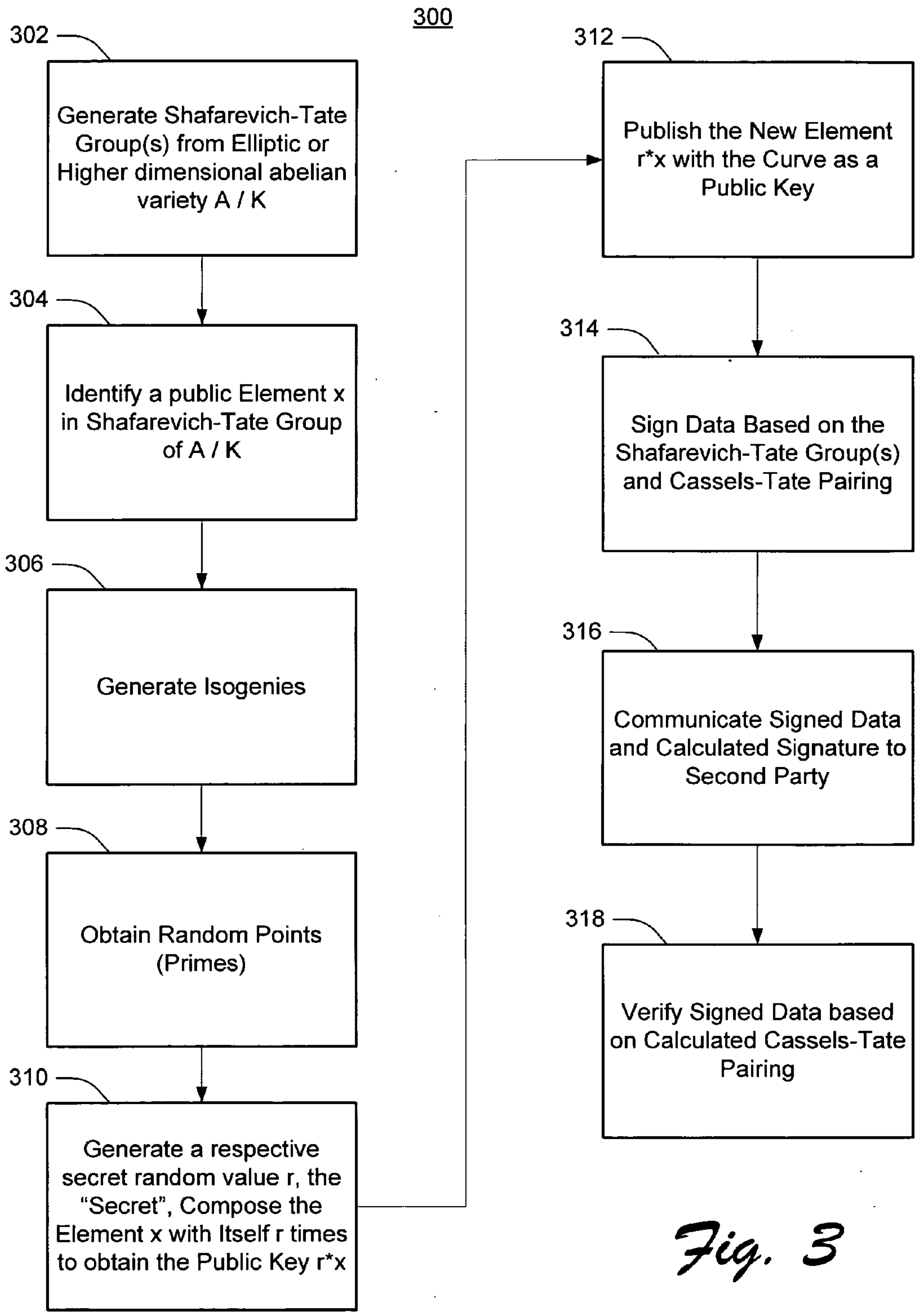

300
302
Generate Shafarevich-Tate Group(s) from Elliptic or Higher dimensional abelian variety A / K
304
Identify a public Element x in Shafarevich-Tate Group of A / K
306
Generate Isogenies
308
Obtain Random Points (Primes)
310
Generate a respective secret random value r, the "Secret", Compose the Element x with Itself r times to obtain the Public Key r*x
312
Publish the New Element r*x with the Curve as a Public Key
314
Sign Data Based on the Shafarevich-Tate Group(s) and Cassels-Tate Pairing
316
Communicate Signed Data and Calculated Signature to Second Party
318
Verify Signed Data based on Calculated Cassels-Tate Pairing

Fig. 3

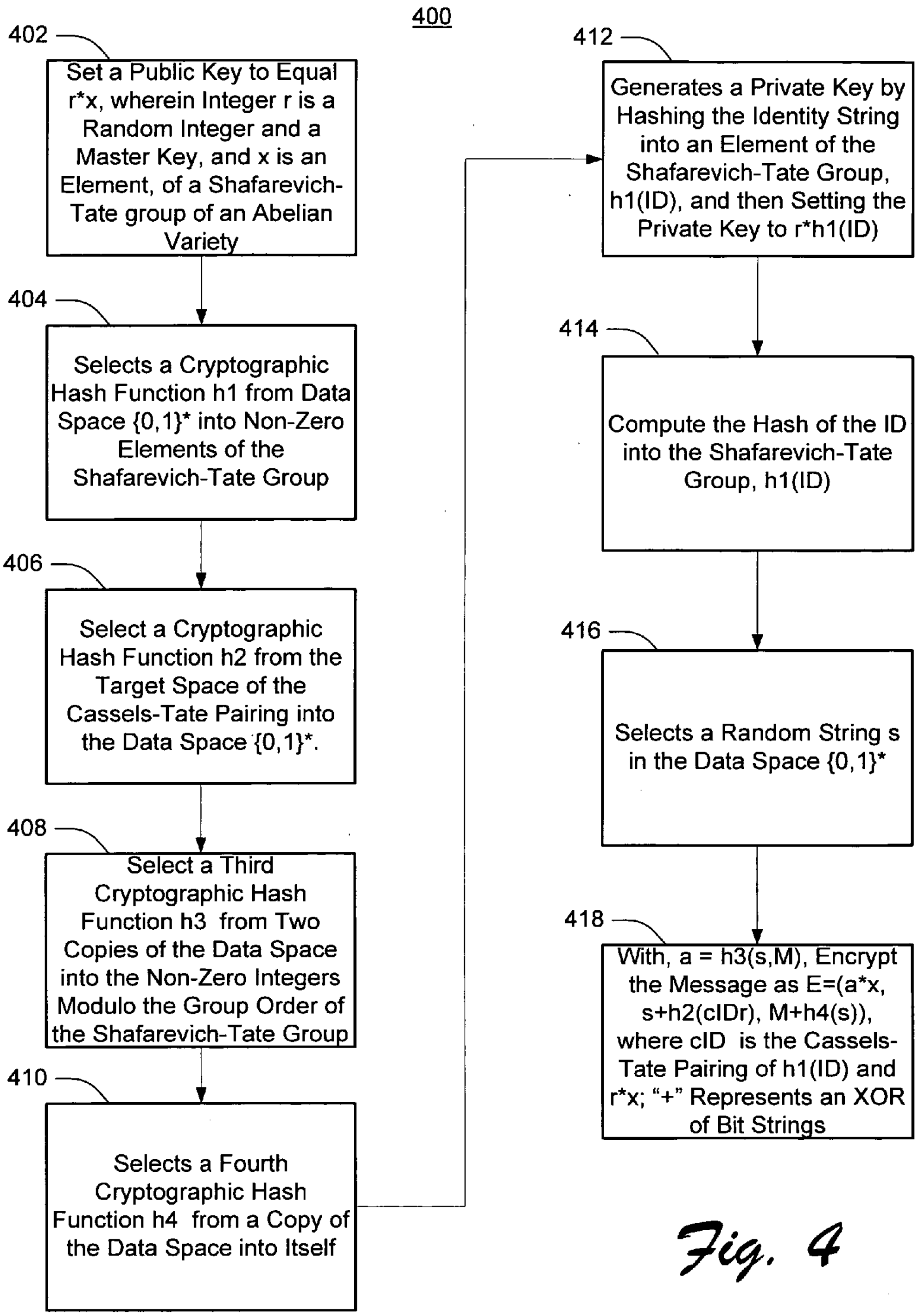

400
402
Set a Public Key to Equal r*x, wherein Integer r is a Random Integer and a Master Key, and x is an Element, of a Shafarevich-Tate group of an Abelian Variety
404
Selects a Cryptographic Hash Function h1 from Data Space {0,1}* into Non-Zero Elements of the Shafarevich-Tate Group
406
Select a Cryptographic Hash Function h2 from the Target Space of the Cassels-Tate Pairing into the Data Space {0,1}*.
408
Select a Third Cryptographic Hash Function h3 from Two Copies of the Data Space into the Non-Zero Integers Modulo the Group Order of the Shafarevich-Tate Group
410
Selects a Fourth Cryptographic Hash Function h4 from a Copy of the Data Space into Itself
412
Generates a Private Key by Hashing the Identity String into an Element of the Shafarevich-Tate Group, h1(ID), and then Setting the Private Key to r*h1(ID)
414
Compute the Hash of the ID into the Shafarevich-Tate Group, h1(ID)
416
Selects a Random String s in the Data Space {0,1}*
418
With, a = h3(s,M), Encrypt the Message as E=(a*x, s+h2(cIDr), M+h4(s)), where cID is the Cassels-Tate Pairing of h1(ID) and r*x; "+" Represents an XOR of Bit Strings

Fig. 4

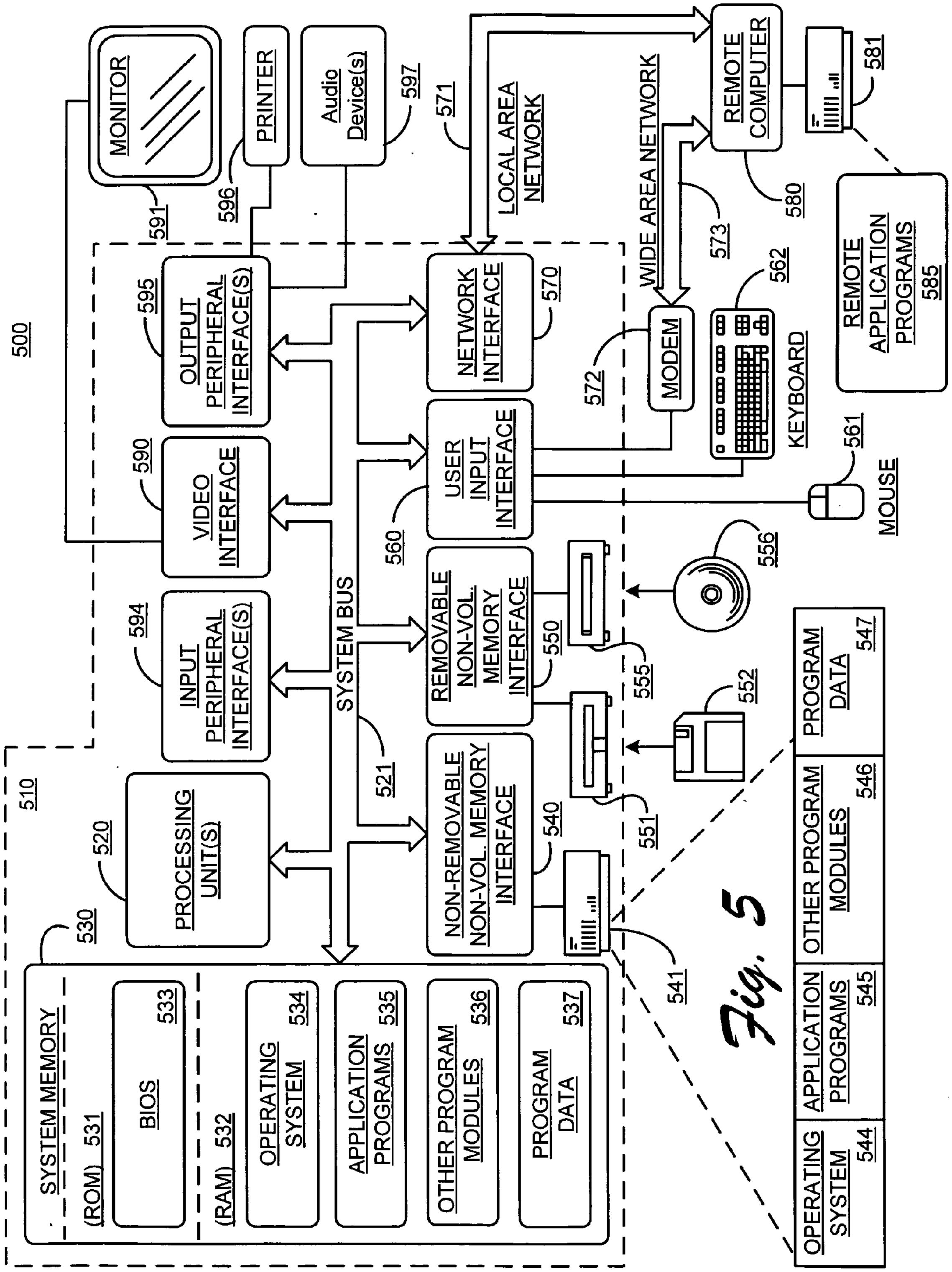

500
510
SYSTEM MEMORY
530
(ROM) 531
BIOS
533
(RAM) 532
OPERATING SYSTEM
534
APPLICATION PROGRAMS
535
OTHER PROGRAM MODULES
536
PROGRAM DATA
537
PROCESSING UNIT(S)
520
SYSTEM BUS
521
INPUT PERIPHERAL INTERFACE(S)
594
VIDEO INTERFACE
590
OUTPUT PERIPHERAL INTERFACE(S)
595
MONITOR
591
PRINTER
596
Audio Device(s)
597
NON-REMOVABLE NON-VOL. MEMORY INTERFACE
540
REMOVABLE NON-VOL. MEMORY INTERFACE
550
USER INPUT INTERFACE
560
NETWORK INTERFACE
570
LOCAL AREA NETWORK
571
WIDE AREA NETWORK
573
MODEM
572
REMOTE COMPUTER
580
581
REMOTE APPLICATION PROGRAMS
585
KEYBOARD
562
MOUSE
561
541
551
552
555
556
OPERATING SYSTEM
544
APPLICATION PROGRAMS
545
OTHER PROGRAM MODULES
546
PROGRAM DATA
547

Fig. 5

CRYPTOGRAPHICALLY PROCESSING DATA BASED ON A CASSELS-TATE PAIRING

TECHNICAL FIELD

[0001] The systems and methods of this specification relate to cryptographic processing.

BACKGROUND

[0002] Existing pairing based cryptographic systems use Weil or Tate pairings evaluated at points on an elliptic curve or abelian variety. For a fixed natural number m, the Weil pairing e_m is a bilinear map that takes as input two m-torsion points on an elliptic curve, and outputs an m th root of unity.

SUMMARY

[0003] Systems and methods for cryptographically processing data based on a Cassels-Tate pairing are described. In one aspect, a Shafarevich-Tate group is generated from a cohomology group. A Cassels-Tate pairing is determined as a function of elements of the Shafarevich-Tate group. Data is then cryptographically processed as a function of the Cassels-Tate pairing.

BRIEF DESCRIPTION OF THE DRAWINGS

[0004] In the Figures, the left-most digit of a component reference number identifies the particular Figure in which the component first appears.

[0005] **FIG. 1** illustrates an exemplary system for cryptographically processing data based on a Cassels-Tate pairing.

[0006] **FIG. 2** shows an exemplary procedure to cryptographically process data based on a Cassels-Tate pairing.

[0007] **FIG. 3** shows an exemplary procedure to digitally sign data using a Cassels-Tate pairing.

[0008] **FIG. 4** shows an exemplary procedure for identity-based encryption using Cassels-Tate pairing.

[0009] **FIG. 5** illustrates an example of a suitable computing environment in which cryptographic processing data based on a Cassels-Tate pairing may be fully or partially implemented.

DETAILED DESCRIPTION

Overview

[0010] The systems and methods for cryptographically processing data based on a Cassels-Tate pairing on a Shafarevich-Tate group provide an alternative to all pairing-based systems that use the Weil or Tate pairings evaluated at points on an elliptic curve or abelian variety. Additionally, the systems and methods have applications in all pairing applications on the Shafarevich-Tate group.

[0011] Although not required, the systems and methods for cryptographically processing data based on a Cassels-Tate pairings are described in the general context of computer-executable instructions (program modules) being executed by a computing device such as a personal computer. Program modules generally include routines, programs, objects, components, data structures, etc., that perform particular tasks or implement particular abstract data types. While the systems and methods are described in the foregoing context, acts and operations described hereinafter may also be implemented in hardware. These and other aspects of the systems and methods for cryptographically processing data based on a Cassels-Tate pairing are now described in greater detail.

An Exemplary System

[0012] **FIG. 1** illustrates an exemplary system **100** for cryptographically processing data based on a Cassels-Tate pairing. System **100** provides an alternative to pairing-based systems based on the Weil or Tate pairings evaluated at points on an elliptic curve or abelian variety. System **100** uses the group of points on the Shafarevich-Tate group of an elliptic curve or abelian variety, combined with the Cassels-Tate pairing on this group. System **100** may implement the operations for Cassels-Tate pairing in a cryptosystem using any one of many known pairing-based cryptographic protocols. For example, in one implementation, system **100** implements protocols based on identity-based cryptographic algorithms such as those directed to signatures (plain, blind, proxy, ring, undeniable, etc), encryption, authenticated encryption, broadcast encryption, encryption with keyword search, batch signatures, key agreement (plain, authenticated, group, etc.), trust authorities and public key certification, hierarchical cryptosystems, threshold cryptosystems and signatures, chameleon hash and signatures, authentication, applications and systems, or the like.

[0013] In other implementation(s), system **100** for cryptographic processing based on a Cassels-Tate pairing implements protocols based on access control, key agreement, non-interactive key distribution, credentials (anonymous, hidden, self-blindable), secret handshakes, provably secure signatures, short signatures, aggregate, ring, and verifiably encrypted signatures, blind and partially blind signatures, proxy signatures, undeniable signatures, signcryption, multisignatures and threshold signatures, limited-verifier and designated-verifier signatures, threshold cryptosystems, hierarchical and role-based cryptosystems, chameleon hash and signatures, verifiable random functions, strongly insulated encryption, intrusion-resilient encryption, certificateless PKC, al, traitor tracing, or the like.

[0014] System **100** includes computing device **102** coupled over a network to a networked computing device **104**. Computing device **102** includes program module(s) **106** and program data **108**. Program modules **106** include, for example, signing/encrypting module **110** to respectively encrypt or sign original data (a respective portion of “other data”**132**) using: (a) a group of points on a Shafarevich-Tate group **114** of an elliptic curve or abelian; and, (b) an associated Cassels-Tate pairing **116**. For purposes of illustration, original data that has respectively been signed or encrypted by signing/encrypting module **110** is shown in the program data portion of computing device **102** as encrypted or signed data **118**. Networked computing device **104** of **FIG. 1** also includes program modules and program data. For example, networked computing device **104** includes verifying/decrypting module **120** to respectively decrypt or verify encrypted or signed data **118** as a function of a Cassels-Tate pairing **122** that is generated by verifying/decrypting module **120** as a function of elements of a Shafarevich-Tate group **114**. These and other aspects of system **100** are now described in greater detail.

Shafarevich-Tate Group

[0015] Shafarevich-Tate group **114** is a set of objects such as elements in a subgroup of a cohomology group **124**. Shafarevich-Tate group **114** provides security to system **100** as a function of the hardness of discrete log in the Shafarevich-Tate group **114**. Shafarevich-Tate group **114** is defined as follows. If K is a number field **124**, denote by M_K the set of nonequivalent valuations on K. Denote by K_v a completion of K with respect to the metric induced by a prime v and by k_v the residue field. In general, if f: G→G' is a morphism of groups denote its kernel by G_f. If ϕ: A→B is an isogeny of abelian varieties, denote by A_ϕ the kernel of ϕ, and by $\hat{\phi}$ the dual isogeny $\hat{B}\to\hat{A}$. For a field K and a smooth commutative K-group scheme G, we write $H^i(K,G)$ to denote the group cohomology $H^i(Gal(K_s/K), G(K_s))$, where K_s is a fixed separable closure of K.

[0016] In view of the above, Shafarevich-Tate group **114** of an abelian variety is defined. Let A be an abelian variety over a number field K. The Shafarevich-Tate group **114** of A, which is defined below, measures the failure of the local-to-global principle for certain torsors. A Shafarevich-Tate group **114** of A over K is

$$III(A/K) := Ker\left(H^1(K, A) \to \prod_{v\in M_k} H^1(K_v, A)\right).$$

Cassels-Tate Pairing

[0017] Let A be an abelian variety defined over a number field K. The Cassels-Tate ("CT") pairing (e.g., **116** or **122**), CT(*,*), is a bilinear, anti-symmetric, non-degenerate pairing (modulo the divisible subgroup) of III(A/K) with $III(\hat{A}/K)$ taking values in Q/Z. The CT pairing is written as a sum of local pairings (e.g., local pairings **126**). Each local pairing is evaluated by a combination of evaluations of the Tate pairing, the Weil pairing, and the Hilbert symbol with respect to m. Special cases of the pairing on A, an elliptic curve, can be evaluated more simply using techniques such as those described in C. Beaver, "5-torsion in the Shafarevich-Tate group of a family of elliptic curves", J. Number Theory, 82(1):25-46, 2000, which is hereby incorporated by reference.

[0018] More particularly, let A be an abelian variety over a number field K with dual $\hat{A}$. The Cassels-Tate pairing CT(*,*) is a pairing such that

$$CT: III(A/K)\times III(\hat{A}/K)\to Q/Z,$$

which is non-degenerate modulo the divisible group. A definition in a special case follows. (For a general definition see William G. McCallum, "On the Shafarevich-Tate group of the Jacobian of a quotient of the Fermat curve", Invent. Math., 93(3):637-666, 1988 (I, Proposition 6.9). For other equivalent definitions see also Bjorn Poonen and Micheal Stoll, "The Cassels-Tate pairing on polarized abelian varieties", Ann. of Math. (2), 150(3):1109-1149, 1999.) Let ϕ,ψ be isogenies of A over K. The restriction of the Cassels-Tate pairing is restricted to the kernels of ϕ and ψ.

[0019] There are exact sequences

$$0 \to A_\psi(\overline{K}) \to A_{\phi\psi}(\overline{K}) \overset{\psi}{\to} A_\phi(\overline{K}) \to 0 \text{ and}$$

$$0 \to A_\phi(\overline{K}) \to A(\overline{K}) \overset{\phi}{\to} A(\overline{K}) \to 0.$$

If * is a global cohomology class, cocycle, or cochain, we write $*_v$ for the corresponding local object. Let a ϵ $III(A/K)_\phi$ and a'ϵ $III(K,\hat{A})_{\hat{\psi}}$. We define CT(a,a'). Choose elements b and b' of $H^1(K,A_\phi)$ and $H^1(K,\hat{A}_{\hat{\psi}})$ mapping to a and a' respectively. For each v, a maps to zero in $H^1(K_v,A)$, and so we can lift b_v to an element $b_{v,1}$ ϵ $H^1(K_v,A_{\phi\psi})$ that is in the image of $A(K_v)$. Suppose that a is divisible by ψ in $H^1(K,A)$, say $a=\psi a_1$, and choose an element b_1 ϵ $H^1(K,A_{\phi\psi})$ mapping to a_1. Then $b_{v,1}-b_{1,v}$ maps to zero under $H^1(K_v,A_{\phi\psi})\to H^1(K_v,A_\phi)$, and so it is the image of an element c_v in $H^1(K_v,A_\psi)$. Then we define CT(a,a') to be

$$CT(a, a') = \sum_{v\in M_k} inv_v(c_v \cup b'_v),$$

where the cup-product is induced by the Weil pairing

$$e_\psi: A_\psi\times\hat{A}_\psi\to G_m.$$

[0020] By the cup-product induced by the Weil pairing we mean the composition

$$H^1(K_v, A_\psi)\times H^1\left(K_v, \hat{A}_{\hat{\psi}}\right) \to H^2\left(K_v, A_\psi \otimes \hat{A}_{\hat{\psi}}\right) \overset{e_\psi}{\to} H^2(K_v, G_m)$$

of the regular cup-product with the map on cohomology induced by the Weil pairing. The map inv_v is the canonical map $H^2(K_v,G_m)\to Q/Z$. (The image of inv_v lies inside $m^{-1}Z/Z$.)

[0021] A Cassels-Tate pairing **122** is described as a sum of local pairings. More particularly, suppose that the map of Galois modules

$$\psi: A_{\phi\psi}(\bar{K})\to A_\phi(\bar{K})$$

has a Galois invariant section

$$s: A_\phi(\bar{K})\to A_{\phi\psi}(\bar{K}).$$

Then we can take $a_1=s_a$. We will now express the Cassels-Tate pairing **122** as a sum of local pairings. Let III:=III(A/K) and III':=$III(\hat{A}/K)$. Let S_ϕ be the Selmer group, which is a subset of $H^1(K,A_\phi)$ defined by the exact sequence

$$0\to A(K)/\phi A(K)\to S_\phi\to III_\phi\to 0.$$

Also, let $S_{\hat{\psi}}$ be the $\hat{\psi}$-Selmer group, which is defined by the corresponding exact sequence for $\hat{\psi}$. We can now lift the Cassels-Tate pairing to $S_\phi\times S_{\hat{\psi}}$. Then the pairing on the Selmer group is described as a sum of local pairings. The motivation for this is the following. We will apply this for ϕ=ψ. There is small chance of computing III_ϕ directly, but we may be able to compute the Selmer group S_ϕ. The lift of the Cassels-Tate pairing to the Selmer group $S_\phi\times S_{\hat{\phi}}$ is trivial on elements coming from A(K)/ϕA(K). So if the Cassels-Tate pairing on $S_\phi\times S_{\hat{\phi}}$ is nontrivial, then we must have nontrivial ϕ-torsion in III.

[0022] By the definition of III, the third vertical map in

$$\begin{tikzcd}
0 \arrow[r] & A(K)/\phi A(K) \arrow[r] \arrow[d] & S_\phi \arrow[r] \arrow[d] & III \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & A(K_v)/\phi A(K_v) \arrow[r] & H^1(K_v, A_\phi) \arrow[r] & 0 &
\end{tikzcd}$$

is 0. Hence we-get a map $l_{v,\phi}$:$S_\phi \rightarrow A(K_v)/\phi A(K_v)$. We use $l_{v,\phi}$ to map the Selmer group into the local groups $A(K_v)/\phi A(K_v)$. We can now define a local pairing

$$<,>_v^{\phi,\psi}:A(K_v)/\phi A(K_v)\times\hat{A}(K_v)/\hat{\psi}\hat{A}(K_v)\rightarrow Q/Z$$

such that for b ϵ S_ϕ and b'ϵ $S'_{\hat{\psi}}$ we have

$$CT(b, b') = \sum_{v\in M_K} \langle l_{v,\phi}(b), l_{v,\hat{p}si}(b')\rangle_v^{\phi,\psi}.$$

Definition of the Local Pairing

[0023] To define the local pairing (e.g., local pairings **126**), "$<,>_v^{\phi,\psi}$", we consider the following diagram

$$\begin{tikzcd}
 & A_\phi(K_v)/\psi \arrow[d] & A_{\phi\psi}(K_v) \arrow[d] \\
 & A(K_v)/\psi A(K_v) \arrow[r, "i_\psi"] \arrow[d, "\phi"] & H^1(K_v, A_\psi) \arrow[d] \\
0 \arrow[r] & A(Kv)/\phi\psi A(K_v) \arrow[r, "i_{\phi\psi}"] \arrow[d] & H^1(K_v, A_{\phi\psi}) \arrow[d, "\psi"] \\
0 \arrow[r] & A(Kv)/\phi A(K_v) \arrow[r, "i_\phi"] & H^1(K_v, A_\phi).
\end{tikzcd}$$

Let x ϵ $A(K_v)/\phi A(K_v)$, x'$\epsilon\hat{A}(K_v)/\hat{\psi}\hat{A}(K_v)$. Let x_1 be a lifting of x to $A(K_v)/\phi\psi A(K_v)$. Then $i_{\phi\psi}(x_1)$ and $s_* i_\phi(x)$ both have the same image in $H^1(K_v,A_\phi)$, hence $(i_{\phi\psi}(x_1)-s_* i_\phi(x))$ is the image of an element c_v ϵ $H^1(K_v,A_\psi)$. Define

$$<x,x'>_v^{\phi,\psi=inv}{}_{v[cv}\cup i_{\hat{\psi}}(x')].$$

[0024] The local pairing $<,>_v^{\phi,\psi}$ is a bilinear pairing of abelian groups. The Cassels-Tate pairing **122** on $S_\phi\times S'_{\hat{\psi}}$ may be expressed as a sum of local pairings as in McCallum (McC88 p. 640)

$$CT(b, b') = \sum_{v\in M_K} \langle l_{v,\phi}(b), l_{v,\phi}(b')\rangle_v^{\phi,\psi},$$

and the following lemma there reduces the above sum to a finite sum: if v is a complex Archimedean valuation, or if v is non-archimedean, A has good reduction modulo the maximal ideal of v, and v(deg(ϕ)deg(ψ))=0, then $<,>_v^{\phi,\psi}$ is trivial.

[0025] We now describe in reference to **FIG. 2**, and then in reference to **FIG. 3**, how Cassels-Tate pairings are used to cryptographically process select data.

An Exemplary Procedure to Cryptographically Process Data

[0026] **FIG. 2** shows an exemplary procedure **200** to cryptographically process data using a Cassels-Tate pairing. The operations of procedure are described with respect to components of **FIG. 1**. The left-most digit of a component reference number identifies the particular figure in which the component first appears.

[0027] At block **202**, signing/encrypting module **110** generates Shafarevich-Tate group **114** (**FIG. 1**) from cohomology group **124** and abelian variety A over a number field K. At block **204**, signing/encrypting module **110** determines a Cassels-Tate pairing (see "other data"**132**) based on the Shafarevich-Tate group **114** and a secret, r, which is the number of times that an element x of the Shafarevich-Tate group **114** was composed with itself to obtain the public key, r*x. At block **206**, the selected information (e.g., original data) is cryptographically processed as a function of the determined Cassels-Tate pairing. For example, signing/encrypting module **110** encrypts or signs the data as a function of the determined Cassels-Tate pairing. Analogously, verifying/decrypting module **120** respectively decrypts or verifies the data as a function of a generated Cassels-Tate pairing. An exemplary procedure for signing data and verifying signed data using a Cassels-Tate pairing is described below in reference to **FIG. 3**.

[0028] The particular pairing-based cryptology algorithm selected at block **206** to process (e.g., sign or encrypt, and analogously verify or decrypt) the data is arbitrary and a function of the particular algorithm selected for implementation. For example, in one implementation, operations of block **206** use an identity-based encryption algorithm as described below in reference to **FIG. 4**, or alternatively, an algorithm based on key issuing, signatures (plain, blind, proxy, ring, undeniable, etc.), encryption, authenticated or broadcast encryption, etc., to cryptographically process the data. In another implementation, block **206** uses an algorithm based on key agreement, key distribution, signatures (e.g., short or group signatures, etc.), etc., to cryptographically process the data. In yet a different implementation, block **206** uses a different pairing-based cryptographic algorithm to cryptographically process the data.

An Exemplary Procedure for Signing Data Using a Cassels-Tate Pairing

[0029] **FIG. 3** shows an exemplary procedure **300** to cryptographically sign data using a Cassels-Tate pairing. The particular pairing-based cryptology algorithm selected to sign the data is arbitrary and a function of the particular cryptology architecture selected for implementation. The operations of procedure **300** are described with respect to components of **FIG. 1**. The left-most digit of a component reference number identifies the particular figure in which the component first appears.

[0030] In this exemplary implementation, signing/encrypting module **110**, which in this implementation is a signing module, and so referred to as such, implements a signature scheme. At block **302**, signing module **110** generates Shafarevich-Tate group **114** from cohomology group **124** an abelian variety A over a number field K. At block

304, signing module **110** selects and makes public an element x in III(A/K), in the Shafarevich-Tate group **114** of A. At block **306**, signing module **110** generates two isogenies, ϕ and ψ, of degree m, from A to A (e.g., via integer multiplication). There are numerous known techniques that can be used to generate the isogenies. At block **308**, signing module **110** obtains two random points, P and P', generators for the kernels of A_ψ and $\hat{A}_{\hat{\psi}}$, where $\hat{\psi}$ is the dual isogeny $\hat{A}\rightarrow\hat{A}$.

[0031] Any two parties (e.g., Alice and Bob) that desire to encrypt or sign original data and/or decrypt or verify associated encrypted or signed data **118**, and/or establish a common secret, generate respective public keys **128**. At block **310**, a party that wants to generate a respective public key **128** generates a respective secret random number, r, and composes x with itself in the Shafarevich-Tate group **114** r times to generate a new element (the r^th multiple of x, r*x). The number r is a user's (e.g., party A or party B) secret **130**. The secret **130** is not shared. At block **312**, signing/encrypting module **110** publishes this new element as a public key **128**.

[0032] At block **314**, signing module **110** signs original data using the Shafarevich-Tate group(s) **114** to generate signed data **118**. For example, in one implementation, when signing module implements a signature scheme, signing module **110** utilizes hash function, h, from the data space $\{0, 1\}^n$ into III($\hat{A}$/K) to sign original data, M (e.g., a plaintext message). The data space $\{0,1\}^n$ is the set of bit-strings of some length n. Similarly, the data space $\{0,1\}^*$ is the set of bit-strings of some length *. This is accomplished by computing the hash of M, h(M) as an element of III($\hat{A}$/K), then taking the r th multiple r*h(M) to obtain the signature σ=r*h(M). For purposes of illustration, the hash of M, represented as M', Cassels-Tate pairing, and the associated signature σ are shown as a respective portion of "other data"**132**.

[0033] At block **316**, signing module **110** sends M together with the signature σ to a target entity such as to an application executing on networked computing device **104**. The application implements or otherwise accesses logic implemented by verifying/decrypting module **120**. At block **318**, and responsive to receiving M and signature σ, verifying/decrypting module **120**, which is this implementation is a verifying module, validates or verfies the signature of M by hashing M, computing Cassels-Tate pairing **122**, CT(r*x, h(M)), and comparing it with CT(x,σ). If they are the same then the signature on the message is deemed valid.

Evaluating a Cassels-Tate Pairing

[0034] This section indicates how, in certain cases, operations of verifying/decrypting module **120** at block **318** can compute a Cassels-Tate pairing **122** explicitly. In this implementation, attention is focused on the special case where an explicit formula is provided for the pairing **122**. Exemplary notation is also provided.

[0035] 1. It is assumed that the abelian variety A is an elliptic curve E defined over a number field K. Then E is canonically isomorphic to its dual $\hat{E}$.

[0036] 2. It is assumed that there exists an isogeny ϕ of E of degree p which is defined over K. We will let ψ be the dual isogeny, $\psi:=\hat{\phi}$. Then $E_\phi\cong Z/pZ$ and $E_\psi\cong Z/pZ$.

[0037] 3. It is assumed that the full p-torsion of E is defined over K. Let P ϵ E(K) be a generator for the kernel of ψ, and let P'ϵ E(K) be a generator for the kernel of $\phi=\hat{\psi}$.

[0038] 4. It is assumed that the map $\psi:E_p\rightarrow E_\phi$ has a Galois invariant section $s:E_\phi\rightarrow E_p$.

[0039] 5. Let s' be the dual section. Then we let Q:=s'P'.

[0040] Since we have fixed ϕ and ψ we will from now on refer to the local pairings simply as $<,>_v$.

Exemplary Local Pairing in Terms of Hilbert Norm Residue Symbol

[0041] Let ϕ be the isogeny of E of degree p. As above, let P ϵ E(K) be a generator for the kernel of $\hat{\phi}$. Let D_P a divisor on E over K which represents P, and let f_P ϵ K(E) be a function satisfying

$$(f_P)=pD_P.$$

We have the following lemma.

[0042] Lemma 3: Let R ϵ E(K), and let D_R be a divisor equivalent to (R)−(O) and not meeting the support of D_P.

[0043] 1. We have $f_P(\phi D_R)$ ϵ $(K^*)^p$.

[0044] 2. Let g be a function whose divisor div g has disjoint support from D_P. We have f_P(div g) ϵ $(K^*)^p$.

[0045] 3. If D'_P is defined over K and linearly equivalent to D_P, and f'_P ϵ E(K) is such that $(f'_P)=mD'_p$, then $f^p\equiv=f'_Pg^p$ mod K* for some g ϵ K(E).

Proof. Statement 2, immediately above, follows from Weil reciprocity. It follows from Lemma 3 that the map f_P gives us a well defined map

$$l_P:E(K)/\phi E(K)\rightarrow K^*/(K^*)^p$$

This is just the Tate pairing. It follows from (3) that this map only depends on P, not on the divisor chosen to represent it. On the other hand, since P is rational over K, we have a Galois map

$$E_\phi\rightarrow\mu_p\ a\mapsto e_\phi(a,P),$$

which induces a map

$$j_P:\ H'(K,E_\phi)\rightarrow H'(K,\mu_p)=K^*/(K^*)^p,$$

where the equality is the map that comes from Kummer theory.

[0046] Lemma 4: We have $j_P\circ i_\phi=l_P$. Here, i_ϕ is the map from the short exact sequence of cohomology $i_\phi:E(K)/\phi E(K)\rightarrow H^1(K,E_\phi)$. The symbol "$\circ$" denotes composition of maps. Now let P,P' and Q be as above, i.e. P is a generator for the kernel of ψ, P' is a generator for the kernel of $\phi=\hat{\psi}$ and Q:=s'P', where s' is the dual section. Before we proceed consider the following two definitions.

[0047] Definition 5. The Hilbert norm residue symbol $(,)_p$ is a map $(,)_p:K_v^*/(K_v^*)^p\times K_v^*(K_v^*)^p\rightarrow\mu_p$. It is defined by $(x,y)_p:=(x^{1/p})^{([\bar{y},K_v]-1)}$. Here $\bar{y}$ is any element of K* mapping to y and $[\bar{y},K_v]$ denotes the Artin symbol.

[0048] Definition 6. Let $\zeta,\zeta'\epsilon\mu_p$. Let $\mathrm{Ind}_\zeta(\zeta')$ be the unique element u ϵ 1/m Z/Z such that $\zeta^{\mu u}=\zeta'$. We can now prove the following proposition.

[0049] Proposition 7: Under the identifications

$$H^1(K_v,\mu_p)=K^*_v/(K^*_v)^p$$

and

$$H^2(K_v,\mu_p\hat{x}\mu_p)=H^2(K_v,\mu_p)\hat{x}\mu_p=(p^{-1}Z/Z)\hat{x}\mu_p=\mu_p$$

the Hilbert norm residue symbol $(,)_p$ may be identified with the cup product pairing

$$H^1(K_v,\mu_p)\times H^1(K_v,\mu_p)\rightarrow H^2(K_v,\mu_p\hat{x}\mu_p).$$

Proof. This follows from the discussion in Serre, Local Fields, Chapter XIV. We can now prove the following theorem that relates the local pairing to the Hilbert symbol.

[0050] Theorem 8: Let x,y ϵ $E(K_v)/\phi E(K_v)$. We have $<x,y>_v=\mathrm{Ind}_{e_\psi(P,P')}[(l_Q(x_1),l_P(y))_p]$, (1), where x_1 is any lifting of x to $E(K_v)/pE(K_v)$. Proof. See, Theorem 2.6 in McCallum.

[0051] Representation of elements in the Selmer group S_ϕ are now described. Let v be the distinguished place as above where the local pairing is nontrivial. To each element $\zeta\epsilon$ S_ϕ, we associate a point T ϵ $E(K_v)/\phi E(K_v)$ by letting $T:=l_{v,\phi}(\zeta)$. Here $l_{v,\phi}$ is as in Theorem 1 and as described with respect to the Cassels-Tate pairing as local pairings. This element T ϵ $E(Q_p)$ uniquely represents the element $\zeta\epsilon$ $S_\phi(E/K)$.

[0052] Evaluating the local pairing $<S,T>_v$ for S,T ϵ $K_v=Q_p$ is now described. By Theorem 8 the local pairing **126** can be evaluated as an application of two Tate pairings **122** and one Hilbert symbol (see, “other data”**132**). In one implementation, projective coordinates are used to avoid divisions, and denominator cancellation techniques are used to evaluate the Tate pairing.

Exemplary Identity-Based Encryption

[0053] **FIG. 4** shows an exemplary procedure **400** of system **100** for identify-based encryption using the Cassels-Tate pairing on the Shafarevich-Tate group of an abelian variety. The operations of procedure **400** are described with respect to components of **FIG. 1**. The left-most digit of a component reference number identifies the particular figure in which the component first appears. Operations of procedure **400** are based on the following: let x be an element, possibly a generator, of the Shafarevich-Tate group **114** of the dual of an abelian variety, and let r be a random integer less than the group order of the Shafarevich-Tate group.

[0054] At block **402**, a program module **106**, for example, signing/encrypting module **110** sets a public key **128** to equal to r*x. In the implementation of exemplary procedure **400**, module **110** is an encryption module and is referred to as such with respect to the procedure. The integer r is the master key. At block **404**, the program module **106** selects a cryptographic hash function h_1 from the data space {0,1}* into the non-zero elements of the Shafarevich-Tate group. At block **406**, the program module **106** selects a cryptographic hash function h_2 from the target space of the Cassels-Tate pairing **116** into the data space {0,1}*.

[0055] At block **408**, the program module **106** selects a third cryptographic hash function h_3 from two copies of the data space {0,1}* into the non-zero integers modulo the group order of the Shafarevich-Tate group **114**. At block **410**, the program module **106** selects a fourth cryptographic hash function h_4 from a copy of the data space {0,1}* into itself. At block **412**, and for a given identity string, ID in {0,1}*, an authority for the system **100** (e.g., a program module **106** such as encrypting module **110**) generates a corresponding private key by hashing the identity string into an element of the Shafarevich-Tate group **114**, h_1(ID) and then setting the private key to be r*h_1(ID). For purposes of illustration, such an identity string and the private key are shown as respective portions of “other data”**132**.

[0056] At block **414**, to encrypt a message M in the data space {0,1}* using the public key ID, the program module **106** computes the hash of ID into the Shafarevich-Tate group, h_1(ID). The message M is a respective portion of “other data”**132**. At block **416**, the program module **106** selects a random string s in the data space {0,1}*. At block **418**, let $a=h_3(s,M)$. The program module **106** encrypts the message **118** as follows $E=(a*x,\ s+h_2(c_{ID}^a),\ M+h_4(s))$, where c_{ID} is the Cassels-Tate pairing **116** of h_1(ID) and r*x; the + symbol represents an XOR operation of bit strings.

[0057] Decryption of the message can be accomplished as follows. The decryptor **120** possesses the private key, $D=r*h_1(ID)$, associated to the identity string ID. The decryptor receives cipher text E=(F,G,H). The decryptor sets $s=G+h_2(CT(D,F))$. Then the decryptor sets the message M equal to $M=H+h_4(s)$. Then the decryptor sets $a=h_3(s,M)$, and tests that the received value F is equal to a*x. If not, the decryptor rejects the message.

An Exemplary Operating Environment

[0058] **FIG. 5** illustrates an example of a suitable computing environment in which cryptographic processing data based on a Cassels-Tate pairing may be fully or partially implemented. Exemplary computing environment **500** is only one example of a suitable computing environment for the exemplary system of **FIG. 1** and exemplary operations of **FIGS. 2-4**, and is not intended to suggest any limitation as to the scope of use or functionality of systems and methods the described herein. Neither should computing environment **500** be interpreted as having any dependency or requirement relating to any one or combination of components illustrated in computing environment **500**.

[0059] The methods and systems described herein are operational with numerous other general purpose or special purpose computing system, environments or configurations. Examples of well-known computing systems, environments, and/or configurations that may be suitable for use include, but are not limited to, personal computers, server computers, multiprocessor systems, microprocessor-based systems, network PCs, minicomputers, mainframe computers, distributed computing environments that include any of the above systems or devices, and so on. Compact or subset versions of the framework may also be implemented in clients of limited resources, such as handheld computers, or other computing devices. The invention is practiced in a distributed computing environment where tasks are performed by remote processing devices that are linked through a communications network. In a distributed computing environment, program modules may be located in both local and remote memory storage devices.

[0060] With reference to **FIG. 5**, an exemplary system for cryptographically processing data based on a Cassels-Tate pairing includes a general purpose computing device in the form of a computer **510** implementing, for example, system **100** of **FIG. 1**. The following described aspects of computer **510** are exemplary implementations of computing devices

102 and/or **104** of **FIG. 1**. Components of computer **510** may include, but are not limited to, processing unit(s) **520**, a system memory **530**, and a system bus **521** that couples various system components including the system memory to the processing unit **520**. The system bus **521** may be any of several types of bus structures including a memory bus or memory controller, a peripheral bus, and a local bus using any of a variety of bus architectures. By way of example and not limitation, such architectures may include Industry Standard Architecture (ISA) bus, Micro Channel Architecture (MCA) bus, Enhanced ISA (EISA) bus, Video Electronics Standards Association (VESA) local bus, and Peripheral Component Interconnect (PCI) bus also known as Mezzanine bus.

[**0061**] A computer **510** typically includes a variety of computer-readable media. Computer-readable media can be any available media that can be accessed by computer **510** and includes both volatile and nonvolatile media, removable and non-removable media. By way of example, and not limitation, computer-readable media may comprise computer storage media and communication media. Computer storage media includes volatile and nonvolatile, removable and non-removable media implemented in any method or technology for storage of information such as computer-readable instructions, data structures, program modules or other data. Computer storage media includes, but is not limited to, RAM, ROM, EEPROM, flash memory or other memory technology, CD-ROM, digital versatile disks (DVD) or other optical disk storage, magnetic cassettes, magnetic tape, magnetic disk storage or other magnetic storage devices, or any other medium which can be used to store the desired information and which can be accessed by computer **510**.

[**0062**] Communication media typically embodies computer-readable instructions, data structures, program modules or other data in a modulated data signal such as a carrier wave or other transport mechanism, and includes any information delivery media. The term "modulated data signal" means a signal that has one or more of its characteristics set or changed in such a manner as to encode information in the signal. By way of example and not limitation, communication media includes wired media such as a wired network or a direct-wired connection, and wireless media such as acoustic, RF, infrared and other wireless media. Combinations of the any of the above should also be included within the scope of computer-readable media.

[**0063**] System memory **530** includes computer storage media in the form of volatile and/or nonvolatile memory such as read only memory (ROM) **531** and random access memory (RAM) **532**. A basic input/output system **533** (BIOS), containing the basic routines that help to transfer information between elements within computer **510**, such as during start-up, is typically stored in ROM **531**. RAM **532** typically contains data and/or program modules that are immediately accessible to and/or presently being operated on by processing unit **520**. By way of example and not limitation, **FIG. 5** illustrates operating system **534**, application programs **535**, other program modules **536**, and program data **537**.

[**0064**] The computer **510** may also include other removable/non-removable, volatile/nonvolatile computer storage media. By way of example only, **FIG. 5** illustrates a hard disk drive **541** that reads from or writes to non-removable, nonvolatile magnetic media, a magnetic disk drive **551** that reads from or writes to a removable, nonvolatile magnetic disk **552**, and an optical disk drive **555** that reads from or writes to a removable, nonvolatile optical disk **556** such as a CD ROM or other optical media. Other removable/non-removable, volatile/nonvolatile computer storage media that can be used in the exemplary operating environment include, but are not limited to, magnetic tape cassettes, flash memory cards, digital versatile disks, digital video tape, solid state RAM, solid state ROM, and the like. The hard disk drive **541** is typically connected to the system bus **521** through a non-removable memory interface such as interface **540**, and magnetic disk drive **551** and optical disk drive **555** are typically connected to the system bus **521** by a removable memory interface, such as interface **550**.

[**0065**] The drives and their associated computer storage media discussed above and illustrated in **FIG. 5**, provide storage of computer-readable instructions, data structures, program modules and other data for the computer **510**. In **FIG. 5**, for example, hard disk drive **541** is illustrated as storing operating system **544**, application programs **545**, other program modules **546**, and program data **547**. Note that these components can either be the same as or different from operating system **534**, application programs **535**, other program modules **536**, and program data **537**. Application programs **535** includes, for example program modules of computing devices **102** or **104** of **FIG. 1**. Program data **537** includes, for example, program data of computing devices **102** or **104** of **FIG. 1**. Operating system **544**, application programs **545**, other program modules **546**, and program data **547** are given different numbers here to illustrate that they are at least different copies.

[**0066**] A user may enter commands and information into the computer **510** through input devices such as a keyboard **562** and pointing device **561**, commonly referred to as a mouse, trackball or touch pad. Other input devices (not shown) may include a microphone, joystick, game pad, satellite dish, scanner, or the like. These and other input devices are often connected to the processing unit **520** through a user input interface **560** that is coupled to the system bus **521**, but may be connected by other interface and bus structures, such as a parallel port, game port or a universal serial bus (USB).

[**0067**] A monitor **591** or other type of display device is also connected to the system bus **521** via an interface, such as a video interface **590**. In addition to the monitor, computers may also include other peripheral output devices such as printer **596** and audio device(s) **597**, which may be connected through an output peripheral interface **595**.

[**0068**] The computer **510** operates in a networked environment using logical connections to one or more remote computers, such as a remote computer **580**. In one implementation, remote computer **580** represents computing device **102** or networked computer **104** of **FIG. 1**. The remote computer **580** may be a personal computer, a server, a router, a network PC, a peer device or other common network node, and as a function of its particular implementation, may include many or all of the elements described above relative to the computer **510**, although only a memory storage device **581** has been illustrated in **FIG. 5**. The logical connections depicted in **FIG. 5** include a local area network (LAN) **581** and a wide area network (WAN) **573**, but may also include other networks. Such networking environments are commonplace in offices, enterprise-wide computer networks, intranets and the Internet.

[**0069**] When used in a LAN networking environment, the computer **510** is connected to the LAN **571** through a network interface or adapter **570**. When used in a WAN networking environment, the computer **510** typically includes a modem **572** or other means for establishing communications over the WAN **573**, such as the Internet. The modem **572**, which may be internal or external, may be connected to the system bus **521** via the user input interface **560**, or other appropriate mechanism. In a networked environment, program modules depicted relative to the computer **510**, or portions thereof, may be stored in the remote memory storage device. By way of example and not limitation, **FIG. 5** illustrates remote application programs **585** as residing on memory device **581**. The network connections shown are exemplary and other means of establishing a communications link between the computers may be used.

CONCLUSION

[**0070**] Although the systems and methods for Cassels-Tate pairing in cryptography have been described in language specific to structural features and/or methodological operations or actions, it is understood that the implementations defined in the appended claims are not necessarily limited to the specific features or actions described. For example, although signing/encryption module **110** (**FIG. 1**) and verifying/decrypting module **120** (**FIG. 1**) are shown on different respective computing devices (i.e., devices **102** and **104**), in another implementation, logic associated with these program modules can be implemented on a single computing device **102**. In another example, and although the systems and methods for Cassels-Tate pairing have been described in exemplary signing and identity-based implementations, the systems and methods are also applicable to all pairings-based applications on the Shafarevich-Tate group such as those indicated above in paragraphs [0012] and [0013].

[**0071**] In yet another alternate implementation, the public element x can be chosen from the Shafarevich-Tate group of the dual of A, and the messages M can be hashed into the Shafarevich-Tate group of A. Similarly for the Identity-Based Encryption and other applications, the roles of A and the dual of A can be switched.

[**0072**] Accordingly, the specific features and operations of system **100** are disclosed as exemplary forms of implementing the claimed subject matter.

1. A method comprising:

generating a Shafarevich-Tate group from a cohomology group;

determining a Cassels-Tate pairing based on elements of the Shafarevich-Tate group; and

cryptographically processing data based on the Cassels-Tate pairing.

2. A method as recited in claim 1, wherein the cohomology group is associated to an abelian variety selected from an elliptic curve or a Jacobian variety of a higher genus curve.

3. A method as recited in claim 1, wherein the Cassels-Tate pairing is a sum of local pairings.

4. A method as recited in claim 1, wherein the cryptographic processing is a signature-based verification scheme or an identity-based encryption scheme.

5. A method as recited in claim 1, wherein the cryptographic processing selects a public element x from the Shafarevich-Tate group of a dual of A, and messages M are hashed into the Shafarevich-Tate group of A.

6. A method as recited in claim 1, wherein the cryptographic processing is based on the Shafarevich-Tate group and a dual of A, wherein roles of A and the dual of A are switched.

7. A method as recited in claim 1, wherein cryptographically processing further comprises signing the data by hashing the data with hashing function h acting on data space $\{0, 1\}^*$, the data being hashed into III(Â/K).

8. A method as recited in claim 1, wherein cryptographically processing further comprises:

selecting an element x from the Shafarevich-Tate group;

selecting a random number r;

composing the element x, r times with itself to generate a public key; and

wherein r is a maintained as a secret.

9. A method as recited in claim 1, wherein cryptographically processing further comprises:

selecting an element x from the Shafarevich-Tate group;

composing the element x, r times with itself to generate a public key, r being the secret; and

publishing the element x, the public key r*x and the abelian variety so that the data can be decrypted or verified by an independent entity.

10. A method as recited in claim 1, wherein the Cassels-Tate pairing is evaluated on different inputs and their values are compared, and wherein the method further comprises:

receiving a public key generated from a secret determined as a function of an element of the Shafarevich group; and

decrypting or verifying the data as a function of the public key and the Cassels-Tate pairing.

11. A method as recited in claim 1, wherein the signature $\sigma=r*h(M)$ and message M are transmitted, and wherein the method further comprises verifying the data by:

(a) hashing the data; and

(b) computing a second Cassels-Tate pairing, CT(r*x, h(M)); and

(c) comparing the second Cassels-Tate pairing with the first Cassels-Tate pairing, $CT(x,\sigma)$.

12. A method as recited in claim 1, wherein cryptographically processing further comprises signing the data by:

computing a hash h of the data such that h(the data) is an element of III(Â/K);

determining an r th multiple r*h(the data), wherein r is a random number; and

evaluating the Cassels-Tate pairing to obtain signature $\sigma=r*h(M)$.

13. A computer-readable medium comprising computer-program instructions executable by a processor for:

generating a Shafarevich-Tate group from a cohomology group;

determining a Cassels-Tate pairing based on elements of the Shafarevich-Tate group; and

cryptographically processing data based on the Cassels-Tate pairing.

14. A computer-readable medium as recited in claim 13, wherein the cohomology group is associated to an abelian variety selected from an elliptic curve or a Jacobian variety of a higher genus curve.

15. A computer-readable medium as recited in claim 13, wherein the Cassels-Tate pairing is a sum of local pairings.

16. A computer-readable medium as recited in claim 13, wherein the cryptographic processing is a signature-based verification scheme or an identity-based encryption scheme.

17. A computer-readable medium as recited in claim 13, wherein the computer-program instructions for cryptographically processing further comprise instructions for signing the data by hashing the data with hashing function h acting on data space {0, 1}*, the data being hashed into III(Â/K).

18. A computer-readable medium as recited in claim 13, wherein the computer-program instructions for cryptographically processing further comprise instructions for:

selecting an element x from the Shafarevich-Tate group;

selecting a random number r;

composing the element x, r times with itself to generate a public key; and

wherein r is a maintained as a secret.

19. A computer-readable medium as recited in claim 13, wherein the computer-program instructions for cryptographically processing further comprise instructions for:

selecting an element x from the Shafarevich-Tate group;

composing the element x, r times with itself to generate a public key, r being the secret; and

publishing the element x, the public key r*x and the abelian variety so that the data can be decrypted or verified by an independent entity.

20. A computer-readable medium as recited in claim 13, wherein the Cassels-Tate pairing is evaluated on different inputs and their values are compared, and wherein the computer-program instructions further comprise instructions for:

receiving a public key generated from a secret determined as a function of an element of the Shafarevich group; and

decrypting or verifying the data as a function of the public key and the Cassels-Tate pairing.

21. A computer-readable medium as recited in claim 13, wherein the signature $\sigma=r*h(M)$ and message M are transmitted, and wherein the computer-program instructions further comprise instructions for verifying the data by:

(a) hashing the data; and

(b) computing a second Cassels-Tate pairing, CT(r*x, h(M)); and

(c) comparing the second Cassels-Tate pairing with the first Cassels-Tate pairing, $CT(x,\sigma)$.

22. A computer-readable medium as recited in claim 13, wherein the computer-program instructions for cryptographically processing further comprise instructions for signing the data by:

computing a hash h of the data such that h(the data) is an element of III(Â/K);

determining an r th multiple r*h(the data), wherein r is a random number; and

evaluating the Cassels-Tate pairing to obtain signature $\sigma=r*h(M)$.

23. A computing device comprising:

a processor; and

a memory comprising computer-program instructions executable by the processor for:

generating a Shafarevich-Tate group from a cohomology group;

determining a Cassels-Tate pairing based on elements of the Shafarevich-Tate group; and

cryptographically processing data based on the Cassels-Tate pairing.

24. A computing device as recited in claim 23, wherein the cohomology group is associated to an abelian variety selected from an elliptic curve or a Jacobian variety of a higher genus curve.

25. A computing device as recited in claim 23, wherein the Cassels-Tate pairing is a sum of local pairings.

26. A computing device as recited in claim 23, wherein the cryptographic processing is a signature-based verification scheme or an identity-based encryption scheme.

27. A computing device as recited in claim 23, wherein the computer-program instructions for cryptographically processing further comprise instructions for signing the data by hashing the data with hashing function h acting on data space {0, 1}*, the data being hashed into III(Â/K).

28. A computing device as recited in claim 23, wherein the computer-program instructions for cryptographically processing further comprise instructions for:

selecting an element x from the Shafarevich-Tate group;

selecting a random number r;

composing the element x, r times with itself to generate a public key; and

wherein r is a maintained as a secret.

29. A computing device as recited in claim 23, wherein the computer-program instructions for cryptographically processing further comprise instructions for:

selecting an element x from the Shafarevich-Tate group;

composing the element x, r times with itself to generate a public key, r being the secret; and

publishing the element x, the public key r*x and the abelian variety so that the data can be decrypted or verified by an independent entity.

30. A computing device as recited in claim 23, wherein the Cassels-Tate pairing is evaluated on different inputs and their values are compared, and wherein the computer-program instructions further comprise instructions for:

receiving a public key generated from a secret determined as a function of an element of the Shafarevich group; and

decrypting or verifying the data as a function of the public key and the Cassels-Tate pairing.

31. A computing device as recited in claim 23, wherein the signature σ=r*h(M) and message M are transmitted, and wherein the computer-program instructions further comprise instructions for verifying the data by:

(a) hashing the data; and

(b) computing a second Cassels-Tate pairing, CT(r*x, h(M)); and

(c) comparing the second Cassels-Tate pairing with the first Cassels-Tate pairing, CT(x,σ)).

32. A computing device as recited in claim 23, wherein the computer-program instructions for cryptographically processing further comprise instructions for signing the data by:

computing a hash h of the data such that h(the data) is an element of III(Â/K);

determining an r th multiple r*h(the data), wherein r is a random number; and

evaluating the Cassels-Tate pairing to obtain signature σ=r*h(M).

33. A computing device comprising:

generating means to generate a Shafarevich-Tate group from a cohomology group;

determining means to determine a Cassels-Tate pairing based on elements of the Shafarevich-Tate group; and

cryptographically processing means to cryptographically process data based on the Cassels-Tate pairing.

* * * * *