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(54)	Title ON-LINE INTELLIGENT CLASSIFICATION MET BASED ON MICRO-MOTION CHARACTERISTIC	THOD OF AERIAL ENTITY TARGETS
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ABSTRACT (Figure 1)

The present disclosure discloses an on-line intelligent classification method of aerial entity targets based on micro-motion characteristics, and the technology belongs to the field of aerial target recognition and classification. Since the micro-motion characteristic is a unique manifestation of the target motion characteristic, this fine motion characteristic can be extracted by using the modern signal processing technology, which provides basis for radar target recognition and classification. The method of the present disclosure includes the following steps: (1) sampling radar receiving signals, and establishing an over-complete atomic library based on the micro-Doppler characteristics of the micro-motion target; (2) using smoothing L0 norm reconstruction method to perform multi-parameter over-complete atomic decomposition on the signals, to obtain the micro-Doppler frequency of the target; (3) classifying the target by a density clustering algorithm by using the micro-Doppler frequency of the target. The present disclosure is not affected by cross terms, high in estimation accuracy, and easy to be realized in engineering, thus having strong engineering application value and promotion prospects.



FIG.1

ON-LINE INTELLIGENT CLASSIFICATION METHOD OF AERIAL ENTITY TARGETS BASED ON MICRO-MOTION CHARACTERISTICS

TECHNICAL FIELD

[01] The present disclosure relates to aerial target recognition and classification field, and more particularly, to an on-line intelligent classification method of aerial entity targets based on micro-motion characteristics.

BACKGROUND ART

[02] With the rapid development of target feature control technologies such as stealth and camouflage and false target imaging deception jamming technologies, various bait interferences with high fidelity with real targets severely affect the performance of radar. Among them, the real target and the bait are very close in size and scattering characteristics, and it is difficult to identify them based on structural features, so it is extremely difficult to classify them. Through research, it is found that certain aerial targets have unique micro-motion characteristics, which enable the radar to identify real targets from the baits and debris. Therefore, extracting corresponding micro-Doppler features based on the micro-motion difference of the target is of vital importance to the radar target recognition and classification. Since the radar echo of the micro-motion target has characteristics of nonlinearity and multi-components, corresponding analysis tools with high resolution, low cross-term, and large detectable dynamic range (DNR) are needed to better reveal the micro-Doppler characteristics of the target.

[03] At present, the generally used extraction and classification algorithm based on micro-Doppler characteristics is to use Wigner-Ville distribution to extract the micro-Doppler characteristics. This method is mainly implemented by the following four steps:

[04] (1) Performing Wigner-Ville transformation on the echo signals to obtain signal discrete W-V distribution W(m, k), $1 \le m \le n$, $1 \le k \le 2N+1$;

[05] (2) Projecting a peak to a time-frequency plane, to obtain an instantaneous frequency $f(m) = \arg \left\{ \max_{1 \le k \le 2N+1} \left[W(m,k) \right] \right\}$, $\arg \{g\}$ is an argument calculation;

[06] (3) Compensating for the frequency shift f_u caused by the target's radial movement;

[07] (4) Getting the signal micro-Doppler frequency according to the equation, $f_{m-D}(m) = f(m) - f_u$ $1 \le l \le M$.

[08] This method has the following defects:

[09] (1) The Wigner-Ville method will be affected by cross terms. When the micro-Doppler frequencies of true and false targets are similar, their spectral features are prone to have overlapping components and become hard to extract, so it is difficult to judge whether the target is interfered by deceit;

[10] (2) The estimation accuracy of the micro-Doppler frequency is affected by the signal observation time. In the case of short observation time, the accuracy is low, so the real-time performance is not high.

SUMMARY

[11] 1. Technical problems to be solved

[12] Embodiments of the present disclosure provide an on-line intelligent classification method of aerial entity targets based on micro-motion characteristics. The method has high frequency domain resolution, good effect in extraction of subtle features of signals, and solves the problem that the existing Wigner-Ville method is affected by cross-term and estimation accuracy of micro-Doppler frequency is not high under short-term conditions.

[13] The technical solution of the on-line intelligent classification method of aerial entity targets based on micro-motion characteristics provided by the present disclosure includes the following steps:

[14] Step 1: the chirp signal s(t) received by the radar receiver is sampled at a sampling interval T_s by a sampler to become a discrete signal $s(nT_s)$, wherein *n* denotes a sequence number of a sampling point; $s(nT_s)$ is sent to the radar signal processing computer;

- [16] Step 2: initialization (set decomposition parameters)
- [17] Setting *T* as the pulse width of the radar;

[18] λ is the radar wavelength;

[19] f_s is the sampling frequency;

[20] f_{mD-u} the micro-Doppler frequency of the micro-motion target;

[21] Setting $G(U \times N)$ as an over-complete atom library, U is the number of atoms in the atom library, $N=T/f_s$;

[22] σ_1 , L, δ , and β are the decomposition parameters of the smoothing L0 norm method respectively;

[23] Step 3: according to the characteristics of the signal echo x(t) of the micro-motion target, establishing a matching atom g_r to construct an over-complete atom library G;

[24] Creating atoms according to the micro-motion characteristics of the target, $g_r = \exp[j2\pi(\sin f_{mD-u}n)]$, r=1, 2, L, N. Set a search precision and range. Assume that the value of the search range fm_{D-u} is, $f_{mD-u} \in [0,U]\Delta f_{mD-u}$, u=1,2,L,U, and U is the number of micro-Doppler frequencies to be searched, and the $U \times N$ matrix of constructed over-complete atom library G is:

[25]
$$G(g_r) = \begin{bmatrix} g_r(f_{mD-1}) & g_r(f_{mD-2}) & \cdots & g_r(f_{mD-U}) \end{bmatrix}$$

[26] $G = [g_1, g_2, \dots, g_N]^T$, and it can be seen that atom g_r in the dictionary matches the micro-Doppler characteristics of the signal containing the micro-motion target; in order to ensure that the decomposition coefficient has sufficient sparsity and matches the tracked reconstruction accuracy, we can increase the redundancy of the transformation matrix by adding the number of the atoms to enhance flexibility of signal approaching, so as to improve the

sparse representation of signals;

[27] Step 4: projecting the echo signal x(t) onto the low-dimensional observation vector y by using the atom library G and a non-coherent measurement matrix Φ , and converting a micro-Doppler frequency extraction problem into an L0 norm optimization problem;

[28] First, projecting the micro-motion echo signal x(t) onto the low-dimensional observation vector y by using the atom library G and a non-coherent measurement matrix Φ , that is,

$$[29] \qquad y = \boldsymbol{\Phi} \boldsymbol{x} = \boldsymbol{\Phi} \boldsymbol{G}^T \boldsymbol{x}$$

[30] Wherein, Φ is a measurement matrix of M×N, M<N, and Φ and redundancy dictionary *G* satisfy incoherence.

[31] Because the actual echo signal is affected by noise, the noisy signals are no longer strictly sparse signals, that is, the decomposition coefficient of the signals in each transform space is not strictly sparse, thus the above equation is transformed into the following form:

$$[32] \qquad y = \boldsymbol{\Phi} \boldsymbol{G}^T \boldsymbol{x} + \boldsymbol{z}$$

[33] Wherein, z is a random noise or other error terms. According to the principles of compressed sensing, the above equitation is transformed into L0 norm optimization problem:

$$[34] \qquad \min \left\| \boldsymbol{G}^{T} \boldsymbol{x} \right\|_{0} s.t. \ \boldsymbol{y} = \boldsymbol{\Phi} \boldsymbol{G}^{T} \boldsymbol{x}$$

[35] Step 5: reconstructing a Doppler frequency of the micro-motion target by using the smoothing L0 norm method:

[36] (1) initializing each parameter in the algorithm:

[37] (1) let
$$s = G^T x = \boldsymbol{\Phi}^T \left(\boldsymbol{\Phi} \boldsymbol{\Phi}^T \right)^{-1} y$$

[38] (2) choosing an appropriate σ descending sequence $\{\sigma_1, \sigma_2, \cdots, \sigma_j\}, \sigma = \beta \sigma_{j-1};$

[39] (2) external circulation: j=1, 2, 3, L, J.

[40] (1) let $\sigma = \sigma_j$;

- [41] ② let $s = \hat{s}_{j-1}$;
- [42] ③ internal circulation: l=1, 2, L, L.

[43] (a) let
$$d = \left[\frac{s_1}{\sigma^2} \exp(-s_1^2/2\sigma^2), \dots, \frac{s_1}{\sigma^2} \exp(-s_n^2/2\sigma^2)\right]^{T}$$

[44] (b) updating the reconstructed signal $s \leftarrow s - \delta \sigma^2 d$;

[45] (c) according to the principle of gradient projection, getting $s \leftarrow s - A^T (AA^T)^{-1} (As - y);$

[46] (4) $\hat{s}_i = s;$

[47] (3) obtaining the reconstructed signal $\mathbf{x} = (\mathbf{G}^T)^{-1} \hat{s}_j$.

[48] (4) density clustering algorithm:

[49] Assume that a data set X_p is a data set that contains micro-motion Doppler frequencies of all signals, assume ε is a search radius, and *Minpts* is a neighborhood density threshold.

[50] (1) marking all objects in X_p as unpassed

[51] (2) randomly selecting one object p in an object X_p , and marking p as passed

[52] (3) if at least one ε neighborhood of p has more or equal *Minpts* objects, creating a new cluster *C* and adding p into *C*.

[53] Let *N* be a collection of objects in ε neighborhood of *P*; for each point *p'* of *N*, if *p'* is unpassed, marking it as passed; if at least one of ε neighborhood of *p'* has more or equal *Minpts* objects, adding these objects to *N*; if *p'* is not a member of any cluster, adding *p'* to *C*

[54] (5) outputting result *C*, which is a classification result of the target group.

[55] Compared with the background technology, the advantageous effects of the present disclosure indicate that: (1) since the Wigner-Ville method belongs to bilinear transformation, when the micro-Doppler frequencies of the micro-motion target are close, the time-frequency result will be easily affected by the cross term and becomes blurred, so it is difficult to distinguish the micro-Doppler frequencies of the micro-motion target. The smoothing L0 norm method used by the present disclosure has high frequency domain resolution, good effect on the extraction of subtle features of signals, and will not be affected by cross-term interference, and can accurately determine various targets. (2) The estimation accuracy of the micro-Doppler frequency is improved. The minimum frequency resolution based on the Wigner-Ville method is limited by signal duration. Under the condition that the sampling frequency f_s is certain, the minimum frequency resolvable unit is inversely proportional to the signal duration. Therefore, in order to improve the micro-Doppler frequency resolution, it needs to prolong the signal observation time; however, the smoothing L0 norm method belongs to non-orthogonal decomposition, and its minimum frequency resolution capability is not limited by the signal duration, but merely depends on the redundancy of the atom library G, so we only need to increase the number of atoms or reduce the micro-Doppler frequency unit to improve the frequency resolution, and then improve the accuracy of micro-Doppler frequency estimation.

[56] In order to illustrate the beneficial effects of the present disclosure and the background technology, two methods are used to estimate the micro-Doppler frequency of the actual micro-moving target echo signal. Based on the Wigner-Ville method, when the micro-Doppler frequencies are similar, the time-frequency result is affected by the cross term and becomes blurred, so the micro-Doppler frequency cannot be accurately extracted; however, the smoothing L0 norm method proposed by the present disclosure is not affected by the cross term, and has high frequency domain resolution capability, and can accurately estimate two micro-Doppler frequencies. It can be seen that compared with the background technology, the present disclosure has the following two beneficial effects: it is not interfered by the cross terms, and the estimation accuracy of the micro-Doppler frequency of the micro-motion target is high.

BRIEFT DESCRIPTION OF THE DRAWINGS

[57] FIG. 1 is a flow chart of the method for extracting micro-Doppler frequency of a micro-motion target based on the smoothing L0 norm method; and

[58] FIG. 2 is a time-frequency projection diagram of the micro-Doppler frequency extraction method of the Wigner-Ville-based micro-motion targets.

DETAILED DESCRIPTION OF THE EMBODIMENTS

[59] The following will describe in detail the online intelligent classification method of aerial entity targets based on micro-motion characteristics of the present disclosure with reference to the accompanying drawings.

[60] Conditions of the embodiment: assume that the radar transmits a single-frequency signal $x'(t) = exp(j2\pi ft)$ of the carrier frequency f=10GHz at the moment t, the radar wavelength $\gamma=3cm$, the radar pulse width T=10ms, and the sampling interval $T_s=5\times10^{-5}$ seconds. It is assumed that the movement states of the two micro-motion targets are precession and swing respectively, and the micro-Doppler frequencies are 7Hz and 8Hz respectively.

[61] In order to facilitate the study of the micro-motion characteristics of the micro-motion target, suppose that the relative translation between the target and the radar has been compensated by the corresponding compensation means, and only the micro-motion characteristics are left. The target signal model that can be established is:

 $[62] \qquad x(t) = \exp(j2\pi \times \sin 5t) + \exp(j\pi \times \sin 6t) + w(t)$

[63] Wherein, w(t) is Gaussian white noise with a mean value of 0 and a variance of 1; the above analog signal is sent to the radar signal processing computer to perform the following steps (refer to FIG. 1 of the specification):

[64] Step 1: the chirp signal x(t) received by the radar receiver is sampled at a sampling interval T_s by a sampler to become a discrete signal $x(nT_s)$, wherein *n* denotes a sequence number of the sampling point; $x(nT_s)$ is sent to the radar signal processing computer;

- [66] Establishing a zero matrix, $G(U \times N)$;
- [67] a number U of searched micro-Doppler frequencies is set as 1600;
- [68] the micro-Doppler unit Δf_{mD-u} is set as 0.003125;

[69]
$$\sigma_1 = 4 \max |s_i|, L=3, \delta=2, \beta=0.8;$$

[70] Step 3: constructing an over-complete atom library

[71] Creating atoms according to the micro-motion characteristics of the true and false warheads, $g_r = \exp[j2\pi(\sin f_{mD-u}n)]$, r=1, 2, L, N. Set a search precision and range. Assume that the value of the search range f_{mD-u} is $f_{mD-u} \in [0,U]\Delta f_{mD-u}$, u=1,2,L,U, and U is the number of micro-Doppler frequencies to be searched, and matrix $U \times N$ of the constructed over-complete atom library G is:

[72]
$$G(g_r) = \begin{bmatrix} g_r(f_{mD-1}) & g_r(f_{mD-2}) & \cdots & g_r(f_{mD-U}) \end{bmatrix}$$

[73] $G = [g_1, g_2, \dots, g_N]^T$, and it can be seen that atom g_r in the dictionary matches the micro-Doppler characteristics of the signal containing the micro-motion target; in order to ensure that the decomposition coefficient has sufficient sparsity and matches the tracked reconstruction accuracy, we can enhance flexibility of signal approaching by adding the number of the atoms to increase the redundancy of the transformation matrix, so as to improve the sparse representation of the signals;

[74] Step 4: projecting the echo signal x(t) onto the low-dimensional observation vector y by using the atom library G and a non-coherent measurement matrix Φ , and converting a micro-Doppler frequency extraction problem into an L0 norm optimization problem;

[75] First, projecting the micro-motion echo signal x(t) onto the low-dimensional observation vector y by using the measurement matrix Φ non-coherent with the atom library G,

namely,

$$[76] y = \boldsymbol{\Phi} \boldsymbol{x} = \boldsymbol{\Phi} \boldsymbol{G}^T \boldsymbol{x}$$

[77] Wherein, Φ is a measurement matrix of $M \times N$, M < N, and Φ and redundancy dictionary *G* satisfy incoherence.

[78] Since the actual echo signal is affected by noise, the noisy signals are no longer strictly sparse signals, that is, the decomposition coefficient of the signals in each transform space is not strictly sparse, thus the above equation is transformed into the following one:

$$[79] \qquad y = \boldsymbol{\Phi} \boldsymbol{G}^T \boldsymbol{x} + \boldsymbol{z}$$

[80] Wherein, z is a random noise or other error items. According to the principles of compressed sensing, the above equitation is transformed into the L0 norm optimization problem:

[81]
$$\min \left\| \boldsymbol{G}^T \boldsymbol{x} \right\|_0 s.t. \boldsymbol{y} = \boldsymbol{\Phi} \boldsymbol{G}^T \boldsymbol{x}$$

[82] Step 5: reconstructing a Doppler frequency of the micro-motion target by using the smoothing L0 norm method:

[83] (1) initializing each parameter in the algorithm:

[84] (1) let
$$s = G^T x = \boldsymbol{\Phi}^T (\boldsymbol{\Phi} \boldsymbol{\Phi}^T)^{-1} y$$

[85] (2) choosing an appropriate σ descending sequence $\{\sigma_1, \sigma_2, \cdots, \sigma_j\}, \sigma = \beta \sigma_{j-1};$

- [86] (2) external circulation: j=1, 2, 3, L, J.
- [87] (1) let $\sigma = \sigma_j$;

[88] 2 let
$$s = \hat{s}_{j-1}$$
;

[89] ③ internal circulation: l=1, 2, L, L.

[90] (a) let
$$d = \left[\frac{s_1}{\sigma^2} \exp(-s_1^2/2\sigma^2), \dots, \frac{s_1}{\sigma^2} \exp(-s_n^2/2\sigma^2)\right]^T$$

[91] (b) updating the reconstructed signal
$$s \leftarrow s - \delta \sigma^2 d$$

[92] (c) according to the principle of gradient projection, getting $s \leftarrow s - A^T (AA^T)^{-1} (As - y);$

[93] (4)
$$\hat{s}_j = s;$$

[94] (3) obtaining the reconstructed signal $\mathbf{x} = (\mathbf{G}^T)^{-1} \hat{s}_j$.

[95] Step 6: density clustering algorithm

[96] assume that a data set X_p is a data set that contains micro-motion Doppler frequencies of all signals, assume ε is a search radius, and *Minpts* is a neighborhood density threshold,

 $\varepsilon = 0.5$, and *Minpts* is a neighborhood density threshold, and *Minpts* = $\frac{2}{3} |X_p(u)|$.

[97] (4) marking all objects in X_p as unpassed

[98] (5) randomly selecting one object p in an object X_p , and marking p as passed

[99] (3) if at least one ε neighborhood of p has more or equal *Minpts* objects, creating a new cluster *C* and adding p into *C*.

[100] (7) Let N be a collection of objects in ε neighborhood of P; for each point p' of N, if p' is unpassed, marking it as passed; if at least one of ε neighborhood of p' has more or equal *Minpts* objects, adding these objects to N; if p' is not a member of any cluster, adding p' to C

[101] (8) outputting result C, until there is no any unpassed object.

[102] Step 7: outputting result C, and C is a classification result of the target cluster, C=2.

[103] In order to compare the prior art Wigner-Ville-based distribution peak detection method, simulation is carried out under the same conditions. FIG. 2 is a projection map on the time-frequency plane during the Wigner-Ville transformation. Due to the influence of cross terms, it is unable to distinguish two micro-motion targets, so the extraction of micro-Doppler frequencies becomes very difficult; however, by using the smooth L0 norm method, it can accurately extract the micro-Doppler frequencies of the micro-moving target, 7*Hz* and 8*Hz*. It can be seen that compared with the background technology, the present disclosure has the advantages of high precision of the target micro-Doppler frequency estimation and no cross-term influence, and it can accurately classify and identify targets.

[104] In a broad format, the present disclosure as claimed herein provides an on-line intelligent classification method of aerial entity targets based on micro-motion characteristics comprising:

(1) establishing a matching atom g_r according to characteristics of a micro-motion target signal echo s(t), and constructing an over-complete atom library G;

(2) projecting an echo signal x(t) to a low-dimensional observation vector y by using the atom library G and a non-coherent measurement matrix Φ , and converting a micro-Doppler frequency extraction problem into an L0 norm optimization problem;

(3) reconstructing a Doppler frequency of the micro-motion target by using a smoothing L0 norm method;

(4) density clustering algorithm:

assume that a data set X_p is a data set that contains micro-motion Doppler frequencies of all signals, assume ε is a search radius, and *Minpts* is a neighborhood density threshold.

(1) marking all objects in X_p as unpassed

(2) randomly selecting one object p in an object X_p , and marking p as passed

(3) if at least one ε neighborhood of *p* has more or equal *Minpts* objects, creating a new cluster *C* and adding *p* into *C*,

let N be a collection of objects in ε neighborhood of P; for each point p' of N, if p' is unpassed, marking it as passed; if at least one of ε neighborhood of p' has more or equal *Minpts* objects, adding these objects to N; if p' is not a member of any cluster, adding p' to C; and (5) outputting result C, which is a classification result of the target group.

[105] The term "comprise" and variants of the term such as "comprises" or "comprising" are used herein to denote the inclusion of a stated integer or stated integers but not to exclude any other integer or any other integers, unless in the context of usage an exclusive interpretation of the term is required.

CLAIMS

1. An on-line intelligent classification method of aerial entity targets based on micro-motion characteristics comprising:

(1) establishing a matching atom g_r according to characteristics of a micro-motion target signal echo s(t), and constructing an over-complete atom library G;

(2) projecting an echo signal x(t) to a low-dimensional observation vector y by using the atom library G and a non-coherent measurement matrix Φ , and converting a micro-Doppler frequency extraction problem into an L0 norm optimization problem;

(3) reconstructing a Doppler frequency of the micro-motion target by using a smoothing L0 norm method;

(4) density clustering algorithm:

assume that a data set X_p is a data set that contains micro-motion Doppler frequencies of all signals, assume ε is a search radius, and *Minpts* is a neighborhood density threshold.

(1) marking all objects in X_p as unpassed

(2) randomly selecting one object p in an object X_p , and marking p as passed

(3) if at least one ε neighborhood of *p* has more or equal *Minpts* objects, creating a new cluster *C* and adding *p* into *C*,

let N be a collection of objects in ε neighborhood of P; for each point p' of N, if p' is unpassed, marking it as passed; if at least one of ε neighborhood of p' has more or equal *Minpts* objects, adding these objects to N; if p' is not a member of any cluster, adding p' to C; and

(5) outputting result *C*, which is a classification result of the target group.

2. The method according to claim 1, wherein the projecting an echo signal x(t) by using the atom library G and a non-coherent measurement matrix Φ to a low-dimensional observation vector y, and converting a micro-Doppler frequency extraction problem into an L0 norm optimization problem has the following technical features:

first, projecting the micro-motion target echo signal x(t) onto the low-dimensional observation vector y by using the measurement matrix Φ non-coherent with the atom library G, namely:

$$y = \boldsymbol{\Phi} \boldsymbol{x} = \boldsymbol{\Phi} \boldsymbol{G}^T \boldsymbol{x},$$

wherein Φ is a measurement matrix of $M \times N$, M < N, and Φ and redundant dictionary G satisfy incoherence; and

since actual echo signals are affected by noise, noisy signals are no longer strictly sparse signals, that is, decomposition coefficients of the signals in each transformation space are not strictly sparse, the above equation is transformed into the following one:

$$y = \boldsymbol{\Phi} \boldsymbol{G}^T \boldsymbol{x} + \boldsymbol{z},$$

wherein z is random noise or other error terms. According to the principle of compressed sensing, the above equation is transformed into an L0 norm optimization problem:

$$\min \left\| \boldsymbol{G}^{T} \boldsymbol{x} \right\|_{0} s.t. \boldsymbol{y} = \boldsymbol{\Phi} \boldsymbol{G}^{T} \boldsymbol{x}.$$

3. The method according to claim 1, wherein the reconstructing a Doppler frequency of the micro-motion target by using the smoothing L0 norm method has the following technical features:

- (1) initializing each parameter in the algorithm:
- (1) let $s = G^T x = \Phi^T (\Phi \Phi^T)^{-1} y$

(2) choosing an appropriate σ descending sequence $\{\sigma_1, \sigma_2, \cdots, \sigma_j\}, \sigma = \beta \sigma_{j-1};$

- (2) external circulation: j=1, 2, 3, L, J.
- (1) let $\sigma = \sigma_j$;
- (2) let $s = \hat{s}_{j-1};$
- ③ internal circulation: l=1, 2, L, L.

(a) let
$$d = \left[\frac{s_1}{\sigma^2} \exp\left(-\frac{s_1^2}{2\sigma^2}\right), \dots, \frac{s_1}{\sigma^2} \exp\left(-\frac{s_n^2}{2\sigma^2}\right)\right]^T$$

(b) updating the reconstructed signal $s \leftarrow s - \delta \sigma^2 d$;

(c) according to the principle of gradient projection, getting $s \leftarrow s - A^T (AA^T)^{-1} (As - y);$

- (4) $\hat{s}_i = s$; and
- (3) obtaining the reconstructed signal $\mathbf{x} = (\mathbf{G}^T)^{-I} \hat{s}_j$.



FIG.1



