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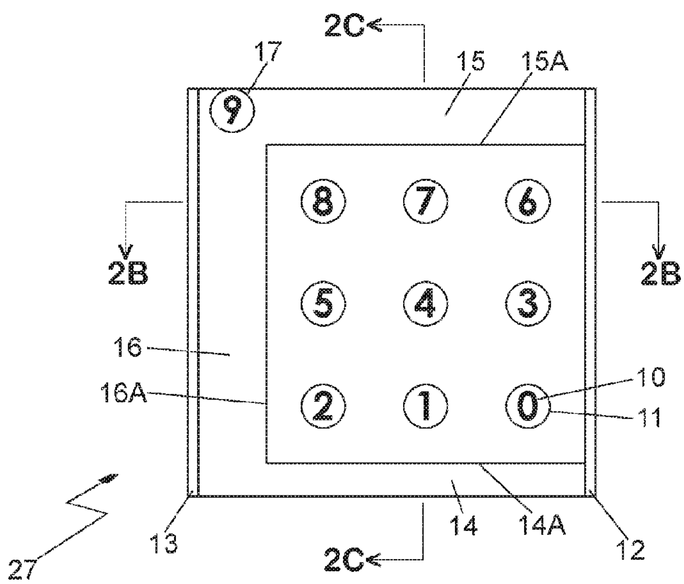

(57) Abrégé/Abstract:
Apparatus and methods for providing instruction include at least one instruction site defining an instruction board and at least one instruction piece configured to be received on the instruction site. A user manipulates the at least one instruction piece to perform a change of state operation relating to the instruction. The apparatus and methods are based on applied cognitive science, where children play the lead role in storylines staged upon a rule-enforcing apparatus and by so doing, become self-enlightened about denumerability, rank-wise denumerability, addition, subtraction, multiplication, division, and other change-of-state processes encountered in mathematics and the quantifiable sciences.

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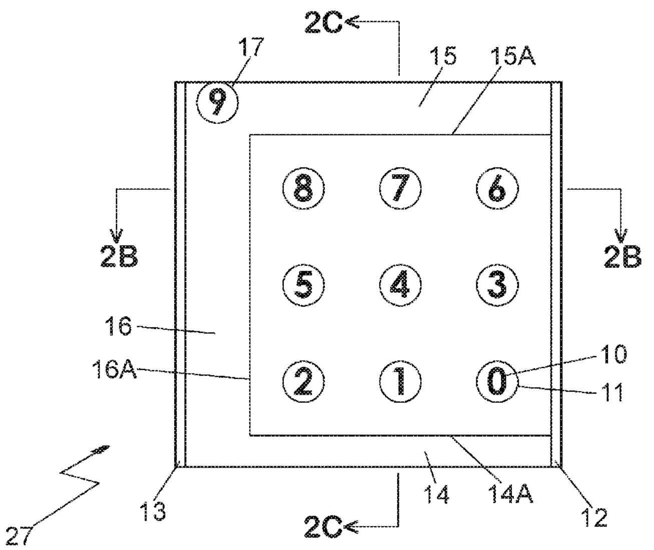

FIG. 2A

(57) Abstract: Apparatus and methods for providing instruction include at least one instruction site defining an instruction board and at least one instruction piece configured to be received on the instruction site. A user manipulates the at least one instruction piece to perform a change of state operation relating to the instruction. The apparatus and methods are based on applied cognitive science, where children play the lead role in storylines staged upon a rule-enforcing apparatus and by so doing, become self-enlightened about denumerability, rank-wise denumerability, addition, subtraction, multiplication, division, and other change-of-state processes encountered in mathematics and the quantifiable sciences.

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Bead-On-Tile Apparatus And Methods

Field of the Invention

[0001] The present invention relates to apparatus and methods designed to provide children grounding, insights, and self-directed instruction in mathematics and the quantifiable sciences.

Background of the Invention and Related Art

[0002] Subitization is about the power of three. Every animal is innately equipped to make three field of view distinctions, namely right, center and left. The present invention taps into this power to subitize. Additionally, in humans there are vertical strata, namely, ground level, eye level and overhead. This overall creates a three-by-three zoned matrix totaling nine zones of alertness. Hence, radix-10 numericity is a natural fit for super-subitized perception in humans. Fingers and finger counting is irrelevant.

[0003] Using modular components, the present invention has broad scope of application to all quantifiable science. However, radix-10 mathematics will be the focus of this disclosure because radix-10 mathematics is the first quantitative science children experience. As long as the invention taps into their subitization arsenal, children are innately equipped to auto-acquire the principles behind mathematics and other quantifiable sciences. The apparatus, on which they play and learn, must reinforce correctness and minimize the potential for goof-ups and self-doubt.

[0004] Apparatus according to the invention, such as depicted in Fig. 1B in one of its many adaptable multi-register forms, in this case a three row/register, five rank, one tray setup, has no prior art. The closest facsimile, merely in terms of possessing a planar layout, was postulated from a hand sketch in the archives of the Royal Danish Library in 1908. Within a solitary, hand-written manuscript dating to 1615 titled “El Primera Nueva Coronica y Buen Gobierno” by its author, Felipe Guaman Poma de Ayala, is a sketch of what modern historians call the Ayala Yupana, an Incan abacus. Ayala’s hand sketch is reproduced in Fig. 1A, rotated 90° counterclockwise. No other sketch like it exists and no physical embodiment of it has ever been unearthed. Nor does anyone know what tokens were used on the Ayala Yupana. Despite the fact that it is naturally designed for a radix-12 number system, several western-centric radix-10 numerical models have been force-fit so the Ayala Yupana functions as a planar, single register, radix-10 abacus. In 2001, Nicolino de Pasquale proposed a radix-40 model.

Summary of the Invention

[0005] In one aspect of the invention, the preferred instruction site for radix-10 numeric state representations is a compact, super-subitized, square tile, on which instruction pieces are moved into instruction locations on an instruction board. In exemplary embodiments, the instruction site is referred to as a "Digit-Square," the instruction pieces are referred to as "beads," the instruction locations are referred to as "bead sites" and the instruction board is referred to as a "Candy Board." Sculpted into the design of the tile is a subitize-informed bead site layout that breathes life into the super-subitization perceptiveness capability of the human brain. A tenth bead site, representing a saturation state, comparable to all ten fingers outstretched, is located in the top left corner of each Digit-Square.

[0006] Preferably, the appropriate cultural and language glyph is printed within the bounds of each bead site and on the tile. For example, the bead site layout of Fig. 2A depicts a right to left magnitude sequence (left is greater) with ascending row/echelons (above is greater), imprinted with conventional Hindu-Arabic digit glyphs, namely "0" through "9". Typographic glyphs act as stepping stones so that, in due course, children self-acquire adult symbol usage. As Figs. 6AA through 6JJ make clear, when beads occupy bead sites on a Digit-Square, the bead count, the bead pattern, and numeric value/state is reinforced by the numeric glyph in the next higher bead site. Fig. 6KK depicts the "TEN" saturation state.

[0007] The invention applies the golden rule: without relatability, learning is imposition not acquisition. On the Digit-Square, starting at "0" incrementing to "TEN" involves eleven states and ten changes of state, as depicted in the eleven Figs. 6AA through 6KK. What children see visually are eleven states. What children don't see visually are the ten changes of state because those are mental constructs called counting, i.e. changes of state via incrementing.

[0008] Another form of instruction site is the "Tray" tile. Compatible with the Digit-Square tile, the Tray tile is depicted in Fig. 3A in a plan view and in section views in Fig. 3B and

Fig. 3C, also showing the preferred bead. Tray tiles serve as bead repository adjuncts to adjoining Digit-Squares.

[0009] One or more tiles, such as Digit-Squares and Trays, may be assembled into a unified tessellation in an embodiment of an instruction board referred to as a "Candy Board" for the parlance of children. Examples are depicted in Fig 2A, being a stand-alone unit tile, and in Fig. 1B, Fig. 4 and Fig. 5, being a unity of plural tiles adhering to a schema of tessellation.

[0010] Candy Boards according to the invention can be custom-module assembled from tile and tile composites interconnected through various interlocking mechanisms including bridging tiles and base mats, to create a desired schema of tessellation. Candy Boards can also be single-molded ready-to-play units with a single row, mimicking an abacus, or two-row, three-row and higher order assemblages, with or without built-in Trays. Figs. 1B, 4, 5, 9A and 9B are examples of such problem-focused layouts.

[0011] Bead sites, preferably bearing printed indicia, are preferably recessed into the tile substrate to create a cavity-mating profile with beads, namely instruction pieces. While all bead sites, such as depicted in Fig. 2A and shown in section in Figs. 2B and 2C, are preferably circular-dimple in form to be compatible with candy beads such as M&Ms, they can take any predetermined form.

[0012] Choking hazards should be avoided at all costs. Because candy is cheap, there is no reason not to use edible beads. Conforming to the dimensions of M&Ms, Skittles and Smarties, the preferred beads are generally round, elliptical or ovulate, finger-friendly candy.

[0013] As depicted in the Digit-Square of Fig. 2A, two horizontal channels and one vertical channel provide bead sliding pathways because sliding is preferred over placement. These channels frame three edges that surround and thus define a bead site plateau region. For other science modeling, such as the electron shells of an atom, channels and plateau regions may number more than one.

[0014] Preferably, the Digit-Square is hemmed in by a right bead-control fence and a left bead-control fence. Such fencing aims to enforce tile grouping, such as the rank system, i.e. numeric order of magnitude. Similar to and compatible with a Digit-Square's enforcement of tile grouping, each Tray has three fences to confine beads to a given rank. One primary objective of the method of plosive-state equilibration is to straddle or to hurdle fencing that divides the Candy Board into ranks or groups.

[0015] Mathematical order of magnitude conventions map directly to the Candy Board's Digit-Square ranking system. For example, in Fig. 4 depicting a three-row, four rank Candy Board, a decal marks the Candy rank. All higher ranks tessellate leftward, such as the Packet rank, depicted in Fig. 4 using an alternative clipart decal, and so on. Mimicking the real world, ranks of candy containment use child-friendly names and images, such as Packets, Bags, Boxes and so on.

[0016] Preferably, each Digit-Square of the same rank is colored and color consistent. Hence, a full-scale Candy Board appears as a series of vertical strips in a light-shade of color that correlate with a set of rank-specific beads in a darker-shade of similar color.

[0017] Preferably, Trays use color to delineate rank that is compatible with the color used by Digit-Squares of the same rank. Preferably, label decals or clipart decals denote the rank to which the Tray pertains. As depicted in Fig. 4, for example, one or more Tray tiles act as bead repositories in conjunction with one or more Digit-Square tiles when setup in a given singular tessellation embodiment. Other such embodiments are depicted in Fig. 1B, Fig. 5, Fig. 9A and Fig. 9B.

[0018] Physical bead movement on structured, rule-enforcing terrain, such as rows and ranks, fences, channels and bead site stamped with a location or number, can be threaded into a storyline and expressed unambiguously via navigation directions. Consequently, storytelling on a physical Candy Board can be a means for demonstrating concepts that are not easily explained. In essence, beads are like pretend cars that drive back and forth between homes, namely bead sites, and color-coordinated parking lots, namely Trays.

[0019] Figs. 6A through 6J along the left-hand margin of the drawings depict stencils optionally bearing indicia, and with optional cutouts. When placed over a Digit-Square, stencils are the preferred means for enforcing the setup of the correct stencil-specific bead count and bead pattern. Cutouts permit the underlying glyph printed in the predetermined bead site on the Digit-Square to show through. This reinforces bead pattern to numeric symbol association. Cutouts can also function as plosive-state bead sites. This emulates radix choking, where the radix of a Digit-Square is reduced, as illustrated in Figs. 8A, 8B and 8E, and as applied to the clock tessellation of Fig. 9A.

[0020] Preferably, indicia bearing chips, as depicted in Fig. 6K and Fig. 7 as a set, act as substitutes for beads laid on the Digit-Square. Chips are the primary means for weaning a child away from bead patterns. Chips are also one means for manifesting algebraic substitution on the Candy Board.

[0021] In another Digit-Square customization, using decals if desired, Fig. 8D depicts how a kludge on the radix-10 Digit-Square can emulate radixes up to hexadecimal, i.e. ounces, and for radix-12, i.e. inches or hours, as depicted in Fig. 8C.

[0022] Although game-play on a physical Candy Board is preferred, especially during a child's earliest learning phases, computer-proctored display devices designed around the layout and techniques of a physical Candy Board provide greater flexibility for dynamically animating storylines in more sophisticated games, or where detection and correction of erroneous game-play is paramount.

[0023] Be it stand-alone intelligent Digit-Squares, computer connected Digit-Squares, or display device Digit-Square analogues, in a computer-proctored embodiment of the apparatus, storylines are preferably presented as text, audio or video, or any combination thereof. Be it localized to a school or distributed via the Internet, a computer-networked embodiment enables an instructor to walk students through a generic problem, but one where each student has a unique instance of the problem on his personal display device to resolve.

[0024] Computer-proctored embodiments are well suited to rigorously enforcing the storyline and the rules of the problem at hand. For example, the computer-proctored device can flag an instructor to intercede, or can handle simple matters on its own. For example, enforcing the order in which bead/icons are placed so the child adheres to "0" followed by "1," followed by "2" and followed by "3", rather than "2", "1", "0" and "3," or any other haphazard bead sequence and placement.

[0025] The computer-proctored embodiment provides enhanced scope for personalized interaction. For example, whenever the child correctly moves a Packet-Rank colored bead/icon to cover-up the "2" bead site in the Packet-Rank on the row of Digit-Squares representing the inventory of candy in some storyline pantry, this change of state triggers the computer-proctored display and voice system to respond, "The new packet added makes three packets of candy in the pantry."

[0026] All other modes of exposition parallel to the tangible and digital game board models and their co-related methods are also contemplated when future technology devises and implements new interaction devices. Such interaction devices include virtual reality 3D configurations, tangible 3D configurations and directly mapping real fingers and finger patterns to virtual digit configurations, along with co-related gestures and words animating the methods by which a game scenario is played out.

Brief Description of the Drawing Figures

[0027] Fig. 1A is a sketch of a single-row, five order of magnitude Ayala Yupana that is presumed to be prior art of ancient Inca.

[0028] Fig. 1B depicts a three-row, five order of magnitude Candy Board.

[0029] Figs. 2A, 2B, 2C depict a radix-10 Digit-Square in plan and elevation views, wherein tessellation interconnects are not shown for purposes of clarity.

[0030] Figs. 3A, 3B, 3C depict a Tray with one bead in plan and elevation views, wherein tessellation interconnects are not shown for purposes of clarity.

[0031] Fig. 4 depicts a Candy Board having three rows of Digit-Squares, with top and bottom Trays.

[0032] Fig. 5 depicts a Candy Board having two rows of Digit-Squares, with top and bottom Trays.

[0033] Figs. 6A to 6K and 6AA to 6KK illustrate ten stencils and a TEN chip along the left margin with their bead pattern counterparts along the right margin.

[0034] Fig. 7 illustrates examples of typographic symbols on a typical set of chips.

[0035] Figs. 8A to 8E illustrate modification means for radix 2, 8, 12, 16 and 60 arithmetic.

[0036] Figs. 9A and 9B illustrate mixed-radix day:hour and min:sec clock tessellation on the Candy Board in a worked example.

[0037] Figs. 10A to 10D illustrate plosive-state equilibration normalizing plosive-state TENs into canonical form in a worked example.

[0038] Fig. 11 is a sample table of the 632M method expressed in pencil-on-paper form showing symmetry and simplicity.

Detailed Description of Exemplary Embodiments of the Invention

[0039] Fig. 1B depicts an exemplary embodiment of a three-row five order of magnitude Candy Board according to the present invention. The Candy Board comprises a plurality of Digit-Squares **27** and Trays **28**. A typical Digit-Square **27** is depicted in a plan view in Fig. 2A and in section views in Fig. 2B and Fig 2C taken along the lines indicated in Fig. 2A. Each Digit-Square **27** has a plurality of bead sites **11**, **17** and an appropriate glyph **10** is imprinted within each bead site **11**, **17** on the tile. Each Digit-Square **27** further has a right bead-control fence **12**, a left bead-control fence **13**, a first horizontal channel **14**, a second horizontal channel **15** and a vertical channel **16**. The channels **14**, **15**, **16** frame three corresponding edges **14A**, **15A**, **16A** that define a bead site plateau region of the Digit Square **27**. A typical Tray **28** is depicted in a plan view in Fig. 3A and in section views in Fig. 3B and Fig 3C taken along the lines indicated in Fig. 3A. Each Tray **28** has three fences **21**, **22**, **23** that enclose the Tray on three sides to confine instruction pieces **24** to a given rank on the Candy Board. The instruction pieces **24** are also referred to herein as "beads" and may comprise relatively small, round, ovulate, finger-friendly edible candy, such as M&Ms, Skittles or the like.

[0040] Fig. 4 depicts another exemplary embodiment of a Candy Board according to the present invention having three rows of Digit-Squares **27**, with top and bottom Trays **28**. The three-row four rank Candy Board depicted in Fig. 4 has an alphanumeric decal **25** indicating the "Candy" rank and an alternative clipart decal **29** indicating the higher "Packet" rank that tessellates leftward from the Candy rank. Fig. 5 depicts another exemplary embodiment of a Candy Board according to the present invention having two rows of Digit-Squares **27**, as depicted in Fig. 2A, with top and bottom Trays **28**, as depicted in Fig. 3A.

[0041] Figs. 6A to 6J illustrate stencils **30**-**39** that may be provided with optional printed indicia and/or optional openings or cutouts **40**-**49**. The bead-pattern counterparts to the stencils **30**-**39** are illustrated in the corresponding Figs. 6AA to 6JJ. When placed over a

Digit-Square **27**, the stencils **30-39** are the preferred means for enforcing the setup of the correct stencil-specific bead count and bead-pattern counterpart. The cutouts **40-49** permit the underlying glyph **10** printed in the corresponding bead site on the Digit-Square **27** to be visible through the stencil, which reinforces bead pattern to numeric symbol association. The cutouts **40-49** may also function as plosive-state bead sites.

[0042] Fig. 6K and Fig. 7 illustrate that one or more indicia bearing chips **26** as a set may act as substitutes for beads laid on a Digit-Square **27**. In one embodiment, chips **26** may serve as the primary means for weaning a child away from bead patterns. In another embodiment, chips **26** may serve as one means for manifesting algebraic substitution on the Candy Board.

[0043] Figs. 8A to 8E illustrate another exemplary embodiment of the invention including modification means for radix 2, 8, 12, 16 and 60 arithmetic. Figs. 9A and 9B illustrate another embodiment of the invention including mixed-radix day:hour and min:sec clock tessellation on the Candy Board in a worked example. Figs. 10A to 10D illustrate another embodiment of the invention including plosive-state equilibration normalizing plosive-state TENs into canonical form in a worked example. Fig. 11 illustrates yet another exemplary embodiment of the invention in a sample table of the 632M method expressed in pencil-on-paper form showing symmetry and simplicity.

[0044] Because modeling and game play on the apparatus is straightforward to anyone knowledgeable in the art, the two essential methods that completely cover the use of the apparatus for doing Addition, Subtraction, Multiplication and Division will provide details on use of the apparatus.

Example 1: The Method of Plosive-State Equilibration

[0045] Plosive-state equilibration is how the Candy Board emulates the pencil-on-paper methods called “Carry” and “Borrow.” Plosive-state equilibration is the preferred method for one tile group/rank to interact with another tile group/rank. On the Digit-Square **27**, a plosive-state lock up occurs when beads occupy every allowable bead site. As depicted in Fig. 2A, the “9” bead site **17** is preferably located on the Digit-Square **27** at the junction of the horizontal channel **15** and the vertical channel **16**. Once a bead occupies this bead site during the operation of “Addition,” namely the amalgamating of two values on two rows on a Candy Board, it physically blockades further bead-in-channel sliding onto the Digit-Square **27**. This physical lock-up manifests what is called a plosive-state TEN, i.e. the bead count and bead pattern depicted in Fig. 6KK.

[0046] More generally described, the method of plosive-state equilibration is triggered whenever a plosive-state bead condition arises on a tile during an operation in progress. Preferred tile designs employ a bead site layout that causes a physical lock-up that arrests further bead play. In order for the operation to proceed further, the method of plosive-state equilibration must resolve the lock-up. Thereafter the operation in progress may resume. Otherwise the operation in progress must abort and perform a related exception state process.

[0047] A cogently designed bead-on-tile model is admirably suited for handling many seemingly complex problems. For example, mixed-radix systems such as days, hours, minutes, and seconds, can be represented and operated on to solve a complex problem. As depicted in Fig. 9A, dual Digit-Squares are used for seconds and minutes. The hours are split into two one-dozen intervals, one for “AM” and one for “PM”. The first rank of the hours uses the radix-12 Digit-Square kludge as depicted in Fig. 8C. The AM/PM rank uses a radix-2, binary stencil adapted from the basic form of Fig. 8A. Days are radix-10. Figs. 9A and 9B illustrate how the Candy Board handles mixed-radix arithmetic when 7 hours, 43 minutes and 38 seconds is added to 1 day, 10 pm, 26 minutes and 12 seconds.

[0048] Rigor makes for relatability. In the clock tessellation, a dual Digit-Square subassembly emulates radix-60 via a specialized stencil. Fig. 8E depicts the use of a “Seconds” stencil in which a cutout set in the “5” location when placed atop a Digit-Square allows the “5” glyph to show through. For example, with five beads on the left Digit-Square and nine on the right, a typographic value “59” is displayed. Add 1 second to “59” and plosive-state TEN lock-up occurs, i.e. “5TEN”. Plosive-state equilibration of TEN causes a sixth bead to socket atop the plosive-state “5” cutout on the stencil, which occludes the “5” glyph printed on the tile bead site, i.e. a second plosive-state lock-up has occurred. Under the rule of rippling, after a second plosive-state equilibration takes place the Candy Board becomes “100”, namely, 1 minute, 00 seconds in canonical form. Rippling is demonstrated in Figs. 10A through 10D where “199” plus 1 ripples via plosive-state equilibration into the canonical form “200”. This might seem tedious overkill, but the apparatus enforces rigor in order to provide a child the visual and tactile means to walk through and demystify quantitative processes step by step.

[0049] During the operation of "Subtraction," the initial setup on a two Digit-Square row Candy Board sites the Subtrahend on the bottom Digit-Square row and the Minuend on the top Digit-Square row. The goal is to completely zero-out the Minuend. Subtraction is the game where a child slides beads from top and bottom Digit-Square rows simultaneously, placing them in the adjacent top and bottom Trays. A "Borrow" lock-up condition arises when the subtrahend in the focus rank runs down to zero beads, but beads still remain in the minuend. In this event, plosive-state equilibration under subtraction, dictates that a bead in the next higher rank of the subtrahend is slid into the Tray, and ten beads in the focus subtrahend rank are slid from the Tray to saturate every bead site in the focus Digit-Square of the subtrahend, forming the TEN bead pattern of Fig 6KK. The method is equivalent to breaking a one dollar bill into ten dimes. With the lock-up resolved, the child resumes simultaneous bead sliding from both minuend and subtrahend Digit-Squares until the minuend is zeroed-out. This process applies generally. Consider the mixed-radix clock problem for subtraction. Starting with the layout of Fig 9B, the child places 7 hours, 43 minutes and 38 seconds in the top Digit-Square row, namely the minuend, as depicted in the top Digit-Square row of Fig. 9A. After subtraction concludes the subtrahend is 1 day, 10 pm, 26 minutes and 12 seconds, as depicted in the bottom Digit-Square row of Fig. 9A, and the top Digit-Square row is now zeroed-out completely, as depicted in the top Digit-Square row of Fig. 9B.

[0050] Plosive-state equilibration is also the means for exploded value representations to normalize into canonical representations and visa-versa. For example, on the Candy Board during addition, a candy packaging operation converts plosive-state TEN Candies into 1 Packet, 0 Candies, namely "10" in the canonical form adults speak aloud as "ten." Figs. 10B through 10D depict "19TEN" resolving to "1TEN0" resolving to the canonical written form "200."

Example 2: The Method of 632M on the Candy Board

[0051] Super-subitization breaks the Digit-Square states "0" through "9" into two components. The "Spine" components 6, 3, 0, (vertical axis) and the "Rib" components 2, 1, 0 (horizontal axis), except that "9" is 6+3. This formulation creates a bi-level tree representing every digit. The Spine+Rib approach gives rise to the 632M-Table which handily slays the so-called complex operations of multiplication and division, as illustrated via pencil-on-paper form in Fig. 11.

[0052] The M in 632M denotes the baseline multiplicand or the divisor value relevant to the problem, also called 1M M-value associated with the 1S S-value. The “632” designates three other S-values, namely 6S, 3S and 2S, being the additional multiples of 1M, calculated via three addition operations.

[0053] The method of 632M multiplication and quotient auto-generation enables children to do multiplication and division without multiplication tables, without the need for memorizing them, without doing single digit multiplication in their heads, and without guess-estimating a candidate quotient digit, rather the quotient is auto-generated as 632M division unfolds. The method requires 1.4 additions or subtractions for each multiplier or quotient digit on average.

[0054] The 632M-Table, manifested in the form of a Candy Board, called the 632M-Board, comprises a column for four S-values with an adjoining column for the four M-Values, where the column is one rank higher than the 1M value, so the highest possible 6M value is accommodated. Illustrating a pencil-on-paper breakdown of the 632M method, Fig. 11 highlights the symmetry inherent to multiplication and division when dissected through the lens of super-subitization. With an M value of 462, the 632M-Table appears as dual, side by side 632M tables in the top/center of Fig. 11. The S-values appearing in the center vertical column 6, 3, 2, 1 (or M), denote multiples of the baseline multiplicand or the divisor values, as the case may be. S-values are used in an automated version of the cascade process, called While-loop Cascading, whereby the 632M method and the 632M-Table can be generalized for operations in radix systems other than radix-10.

[0055] Furthermore, the method of 632M is open to obvious optimization, such as a fall through execution tree requiring at most two M-value operations. Certain digits repeated in a multiplier may give a better M-value selection, such as 532M, for example, whenever 5’s outnumber 6’s by two to one and 9’s are scarce. Similarly, for 742M and 732M, which have an overhead of four additions to setup the M-Table, but otherwise super-subitize over radix-10 as well as 632M does, and are optimal for radix-11, as well. Similar extensions of the method apply to other radixes. For example, using a nine M-value 50/40/30/20/10/632M-

Table with its setup overhead of nine additions, radix-60 arithmetic requires no more than 3 operations per step.

[0056] The setup of a four row 632M-Board executes as follows: Step (A): Setup a series of S-values from top to bottom rows, namely 6, 3, 2, 1 in the S-value field of the 632M-Board. Step (B): Setup the 1M value on both the bottom, next row up and top row (S = 1, 2, 6 rows). Step (C): Add the bottom row into the next row up, which yields 2M in the S=2 row. Step (D): Duplicate the 2M value into the row above it (S=3 row). Step (E): Add the topmost row (S=6) downwards into the row beneath, which yields 3M in the S=3 row. Step (F): Duplicate the 3M value into the topmost row and the bottom row (S=1 and 6 rows). Step (G): Add the bottom row into the topmost row, which yields 6M in the topmost row. As an alternative, double the topmost row in-situ, which makes needless the Step (F) process of duplicating 3M into the bottom row. Step (H): Finally, setup the 1M value in the bottom row (S=1).

[0057] A 632M-Board detached from the Candy Board facilitates both rank shifting and duplication of M-value presets onto the Candy Board in the partial product row during multiplication and the divisor/subtrahend row during division. A child merely needs to replicate the add-shift process for multiplication or the subtract-shift process for division, as illustrated in Fig 11, using the 632M-Board as the template for setting up values on the Candy Board.

WE CLAIM:

1. An apparatus for providing instruction, comprising:

at least one instruction tile having a plurality of instruction sites that are each located at a discrete location within a predefined area on the instruction tile and a saturation state instruction site that is located on the instruction tile remote from the predefined area; and

a plurality of instruction pieces configured to be positioned on the plurality of instruction sites and the saturation state instruction site in a predetermined order; and

a stencil having means for revealing the next instruction site or saturation state instruction site on the instruction tile in the predetermined order.

2. The apparatus of claim 1, wherein each instruction site and the saturation state instruction site has a recess that defines the location of the instruction site and the location of the saturation state instruction site on the instruction tile.

3. The apparatus of claim 1, wherein the instruction pieces are edible.

4. The apparatus of claim 1, wherein the instruction relates to displaying intermediate and final results of state machine emulation of multistate computing for at least one of mathematics and quantifiable sciences.

5. The apparatus of claim 1, wherein each instruction tile has an edge and wherein edges of adjacent instruction tiles are positioned adjacent one another to form an instruction board.

6. The apparatus of claim 5, wherein edges of a plurality of the instruction tiles are positioned adjacent one another to form a tessellation.

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7. The apparatus of claim 5, wherein the edge of a first instruction tile adjoins the edge of a second instruction tile to form a tessellation that defines the instruction board.

8. An apparatus for providing instruction in at least one of mathematics and quantifiable sciences, the apparatus comprising:

an instruction board formed from a plurality of instruction tiles, each of the instruction tiles having a predetermined number of instruction sites that are each located at a predetermined discrete location within an area on the instruction tile defined by a plurality of borders and a single saturation state instruction site that is not located within the area on the instruction tile defined by the plurality of borders; and

a plurality of instruction pieces configured to be received on the instruction tiles at the instruction sites and the saturation state instruction site in a predetermined order;

wherein the instruction pieces are manipulated on the instruction sites and the saturation state instruction site on the instruction tiles to perform a change of state operation that provides the instruction in the at least one of mathematics and quantifiable sciences.

9. The apparatus of claim 8, wherein each of the instruction sites and the saturation state instruction site has a recess formed on the instruction tile.

10. The apparatus of claim 8, wherein each of the instruction tiles has at least one edge, and wherein the edges of adjacent instruction tiles abut to define a tessellation that forms the instruction board.

11. The apparatus of claim 10, wherein the adjacent instruction tiles define a tessellation on the instruction board that is configured to perform at least one of addition, subtraction, multiplication and division.

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12. The apparatus of claim 8, wherein each of the instruction tiles has at least one channel formed thereon for sliding the instruction pieces on the instruction tile, and wherein the channel is disposed between a border of the area and an edge of the instruction tile and the saturation state instruction site is disposed within the channel.

13. The apparatus of claim 8, wherein each of the instruction sites and the saturation state instruction site has printed indicia that corresponds to the location of the respective instruction site and to the location of the respective saturation state instruction site.

14. The apparatus of claim 8, further comprising a stencil having means for revealing the next instruction site or saturation state instruction site on the instruction tile in the predetermined order.

15. A method for providing instruction in at least one of mathematics and quantifiable sciences, the method comprising:

providing an instruction board formed from one or more instruction tiles wherein each instruction tile comprises a plurality of instruction sites at discrete locations within a defined area on the instruction tile and a saturation state instruction site at a location remote from the defined area on the instruction tile;

providing one or more instruction pieces configured to be received on the instruction sites and the saturation state instruction site of the instruction tile in a predetermined order; and

manipulating at least one of the instruction pieces from the saturation state instruction site on a first instruction tile to at least one of the instruction sites on a second instruction tile to perform a change of state operation relating to the at least one of mathematics and quantifiable sciences.

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16. The method of claim 15, wherein the instruction sites and the saturation state instruction site have recesses formed in the instruction tiles.

17. The method of claim 15, wherein the instruction tiles have at least one edge and wherein the edges of adjacent instruction tiles abut one another to form a tessellation that defines the instruction board.

18. A method for SM-table multiplication comprising the steps:

a) providing a multiplier value in canonical form having a tile count;

b) providing a multiplicand value in canonical form having a tile count;

c) providing an instruction board comprising at least a multiplier row having a plurality of tiles and a product row having a plurality of tiles, the multiplier row and the product row each having a tile count equal to the tile count of the multiplier value added to the tile count of the multiplicand value;

d) creating an SM-table having four rows using an S-value list of 6, 3, 2, 1 from a top row to a bottom row, setting a 1M M-value in the bottom row to the multiplicand value, and by using addition, generating a 2M M-value, a 3M M-value and a 6M M-value in a second row, a third row and a fourth row, respectively, above the bottom row;

e) zeroing all of the tiles in the product row;

f) duplicating the multiplier value into the multiplier row;

g) setting a multiplier focus tile at a rightmost tile in the multiplier row;

h) determining whether a value in all of the tiles in the multiplier row at and to the left of the multiplier focus tile are zero and if the value in all of the tiles in the multiplier row at and to the left of the multiplier focus tile are zero terminating the method, otherwise continuing the method with step i);

i) setting a focus row of the SM-table to the top row of the SM-table;

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j) determining whether the value in the multiplier focus tile is less than the S-value in the focus row of the SM-table and if the value in the multiplier focus tile is less than the S-value in the focus row of the SM-table go to step m);

k) reducing the value in the multiplier focus tile by the S-value in the focus row of the SM-table;

l) adding the M-value in the focus row of the SM-table into the product row beginning at the tile in the product row that is aligned with the multiplier focus tile and into all of the product tiles to the left;

m) determining whether the focus row of the SM-table is at the bottom row of the SM-table and if the focus row of the SM-table is at the bottom row of the SM-table go to step o);

n) shifting the focus row of the SM-table down one row in the SM-table and repeating step j);

o) shifting the multiplier focus tile one tile left; and

p) repeating step h);

whereby the product row provides the product value.

19. A method for SM-table quotient auto-generation comprising the steps:

a) providing a divisor value in canonical form;

b) providing a dividend value in canonical form;

c) providing an instruction board comprising at least a quotient row, a dividend row, and a subtrahend row, each row having a plurality of tiles and an additional tile above the order of magnitude of the dividend value;

d) creating an SM-table having four rows using an S-value list of 6, 3, 2, 1 from a top row to a bottom row, setting a 1M M-value in the bottom row to the dividend value, and by using addition, generating a 2M M-value, a 3M

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M-value and a 6M M-value in a second row, a third row and a fourth row, respectively, above the bottom row;

e) zeroing all tiles in the quotient row;

f) duplicating the dividend value into the dividend row;

g) setting a dividend focus tile at a leftmost non-zero valued tile in the dividend row;

h) aligning the tile bearing the leftmost non-zero valued tile of the 1M M-value in the SM-table with the dividend focus tile established in step (g), thereby setting a quotient focus tile to be aligned with a rightmost tile of the 1M M-value in the SM-table, which also establishes a rightmost tile in a partial dividend field of tiles;

i) setting a focus row of the SM-table to the top row of the SM-table;

j) comparing the M-value in the focus row of the SM-table to the value in the partial dividend field of tiles and if the M-value in the focus row is greater go to step n);

k) adding the S-value in the focus row of the SM-table to the value in the quotient focus tile;

l) duplicating the M-value on the focus row of the SM-table from the rightmost tile to a leftmost tile in the subtrahend row, beginning at the subtrahend tile aligned with the quotient focus tile;

m) subtracting the subtrahend row from the partial dividend field within the dividend row, beginning at the subtrahend tile aligned with the quotient focus tile;

n) determining whether the focus row of the SM-table is at the bottom row of the SM-table and if the focus row of the SM-table is at the bottom row of the SM-table go to step p);

o) shifting the focus row of the SM-table down one row in the SM-table and then returning to step j);

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p) determining whether the quotient focus tile is aligned to the rightmost tile of the quotient row and if the quotient focus tile is aligned to the rightmost tile of the quotient row terminating the method, otherwise continuing the method with step q);

q) shifting the dividend focus tile one tile to the right;

r) shifting the quotient focus tile one tile to the right; and

s) repeating step i) to step p);

whereby the dividend row provides a remainder value and the quotient row provides a quotient value.

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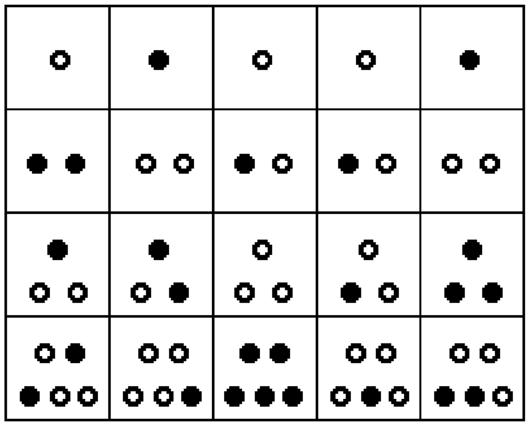


Prior Art – Ayala Yupana, Rotated 90° CCW

FIG. 1A

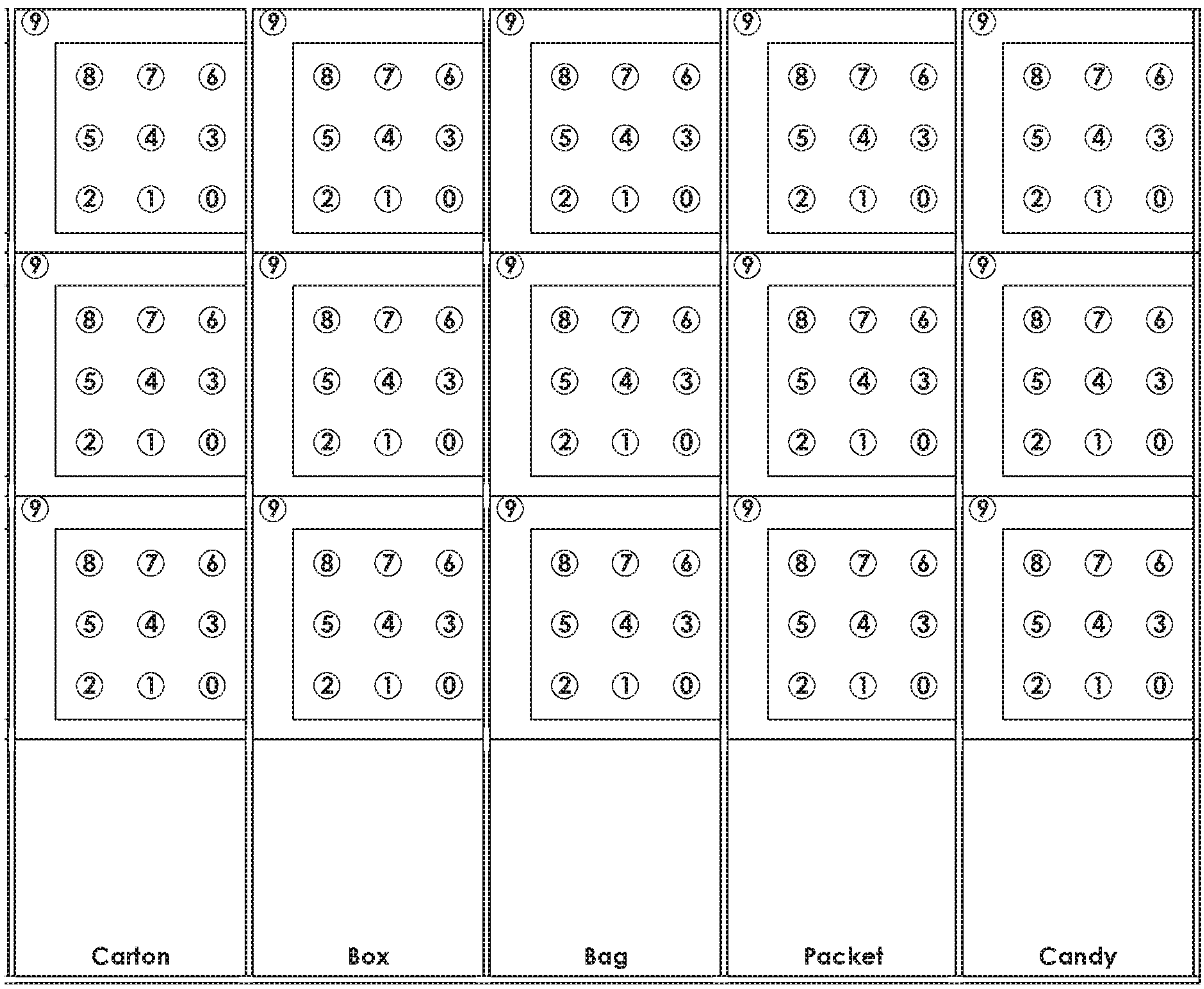


FIG. 1B

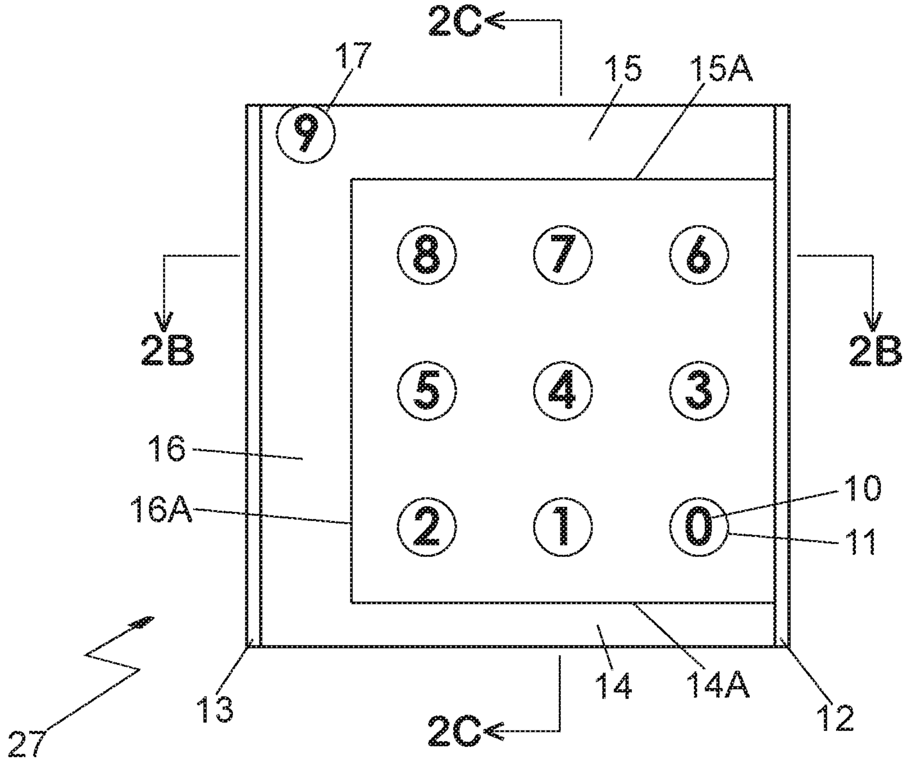


FIG. 2A

FIG. 2B

FIG. 2C

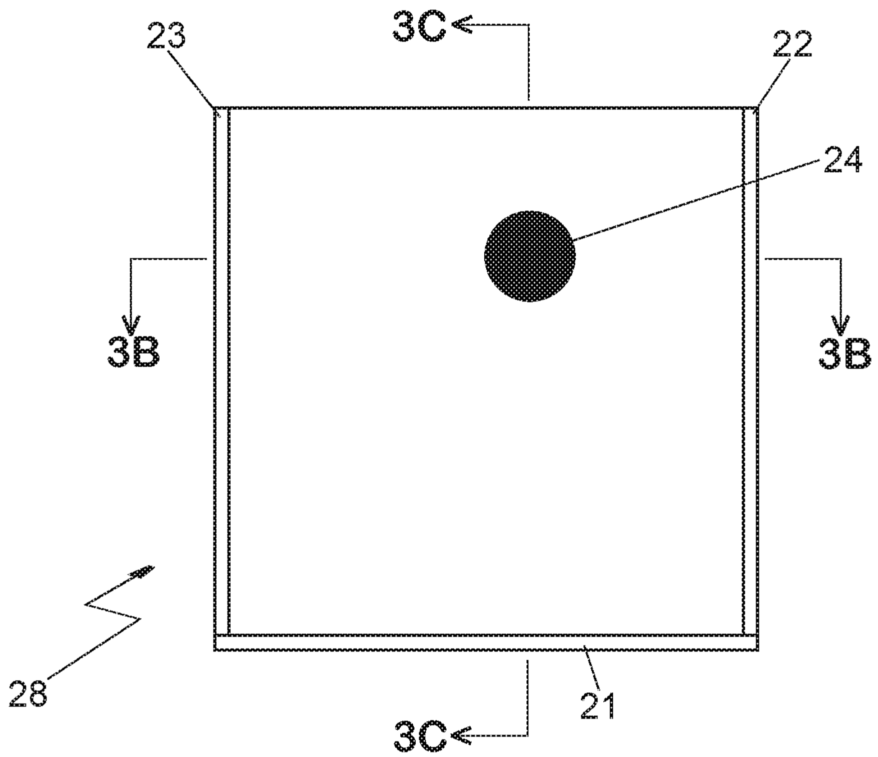


FIG. 3A

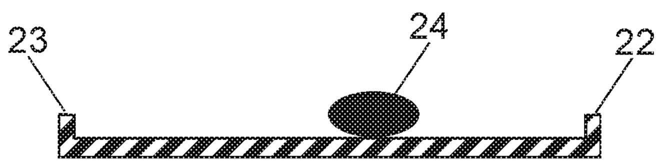


FIG. 3B

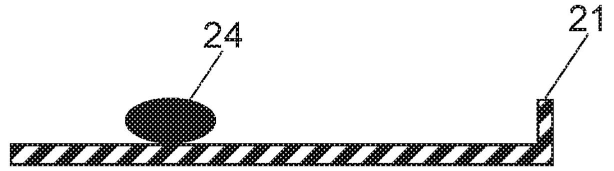


FIG. 3C

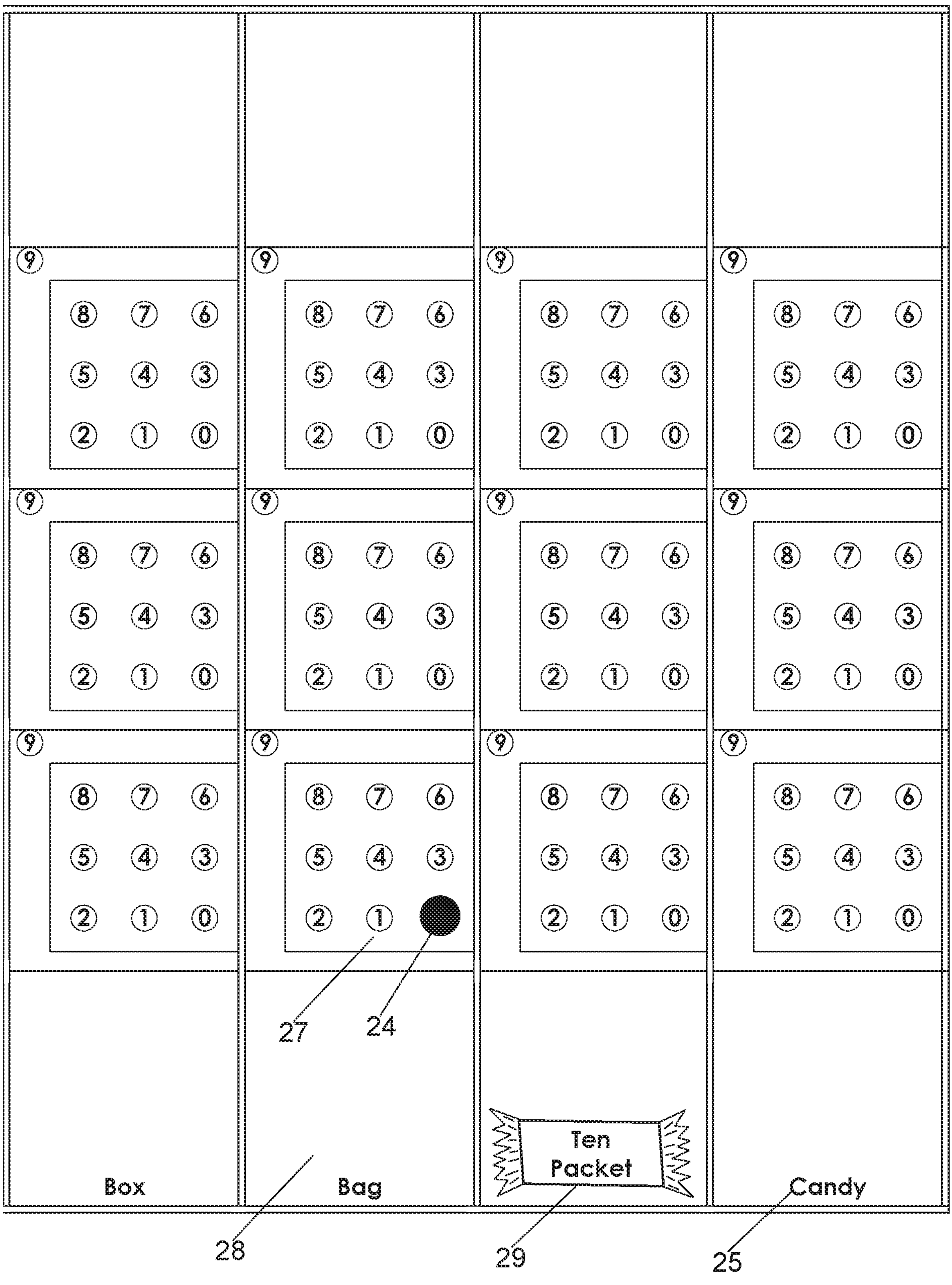


FIG. 4

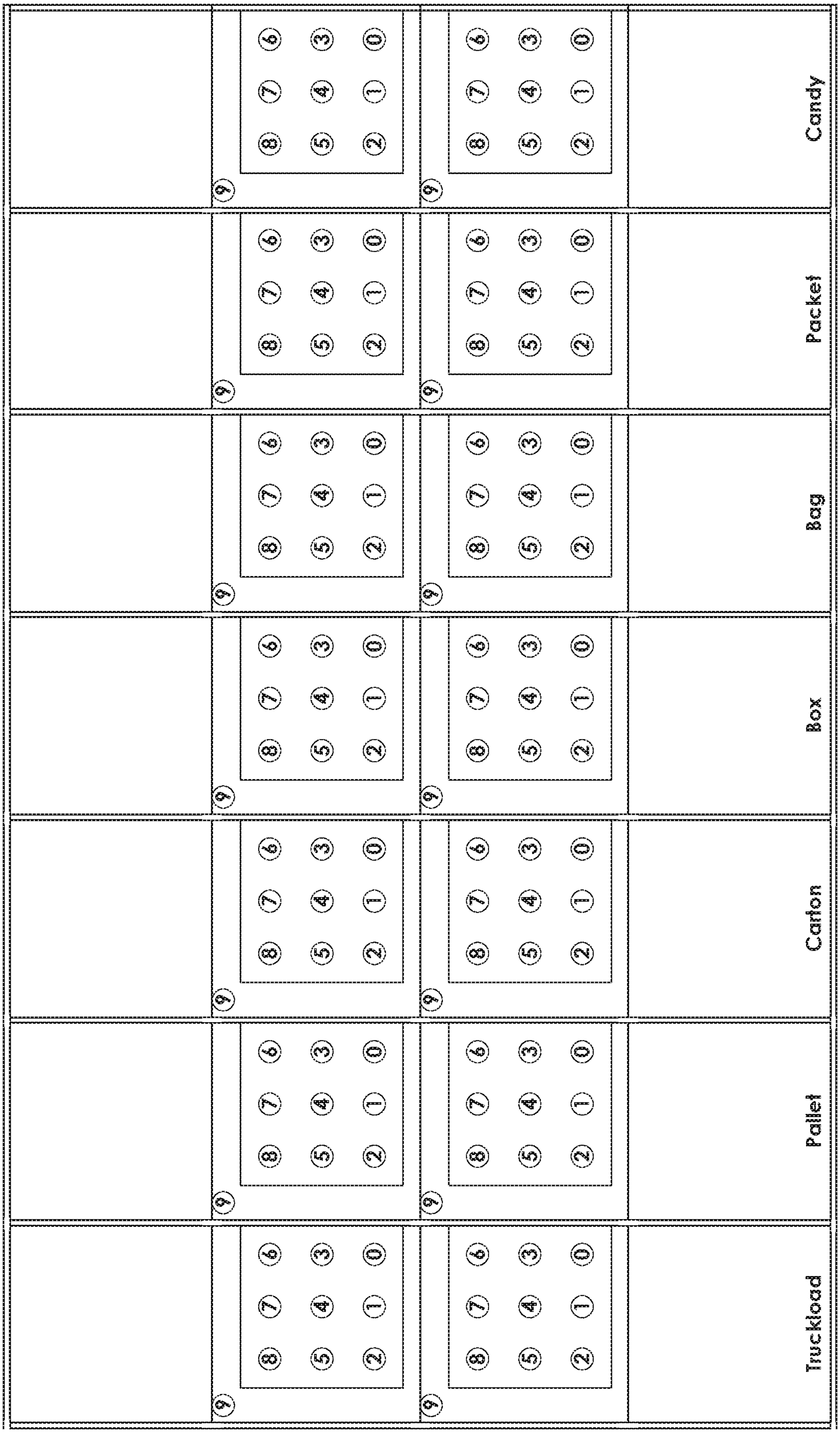


FIG. 5

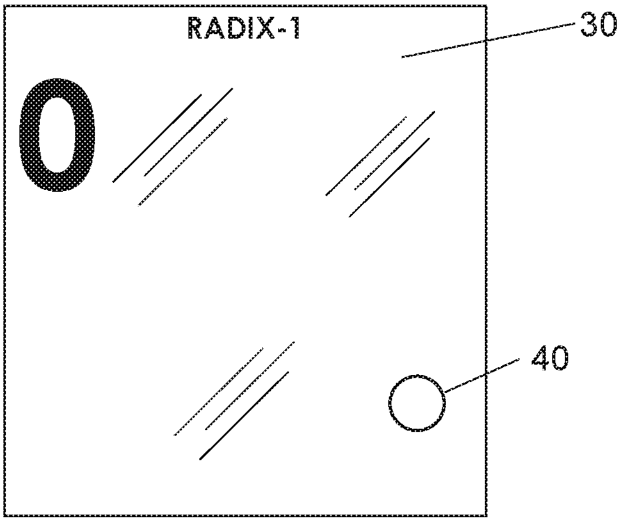


FIG. 6A

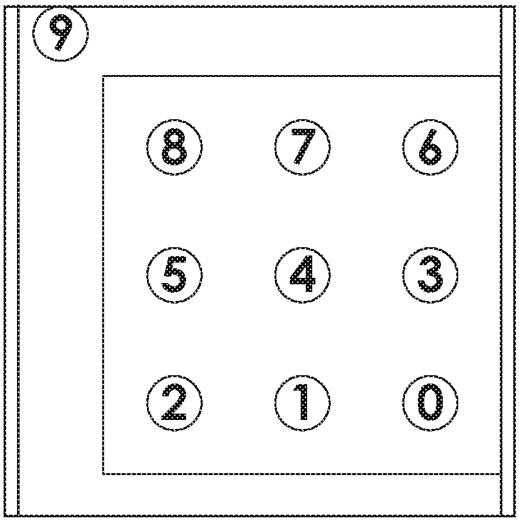


FIG. 6AA

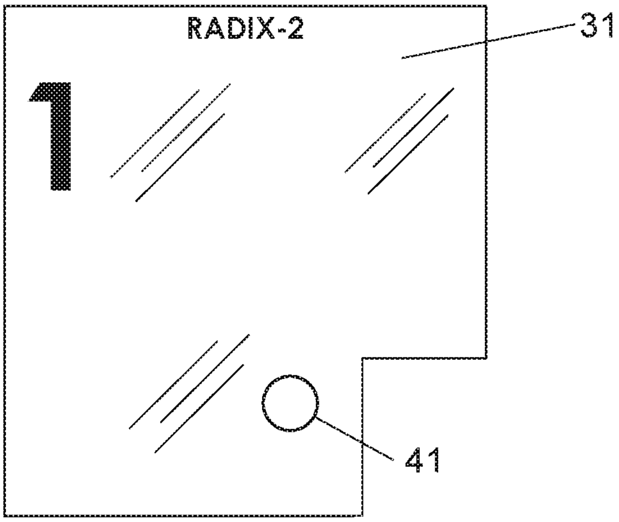


FIG. 6B

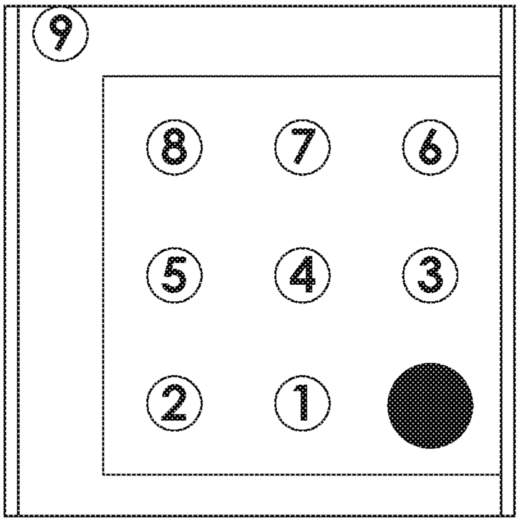


FIG. 6BB

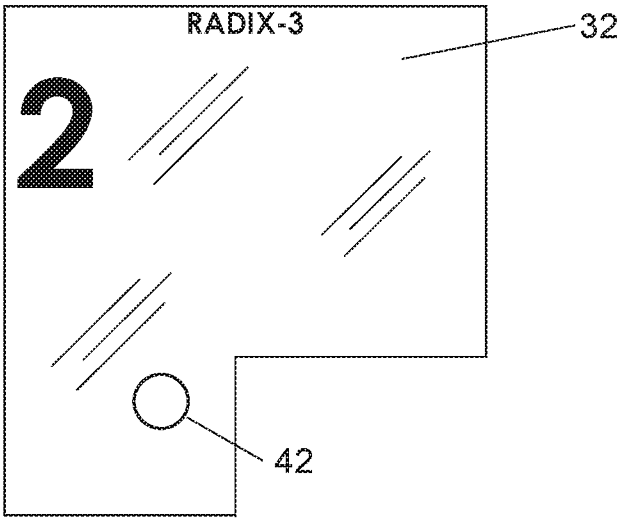


FIG. 6C

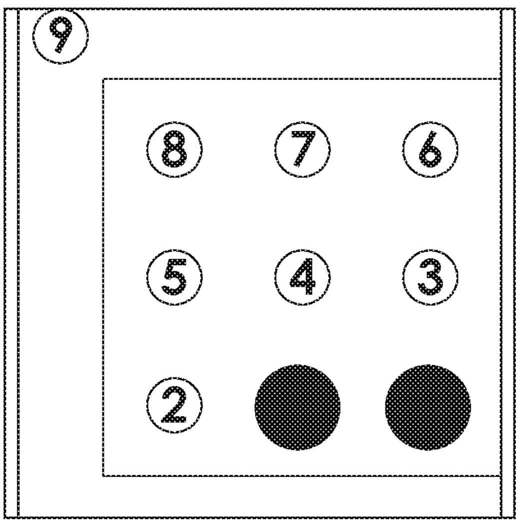


FIG. 6CC

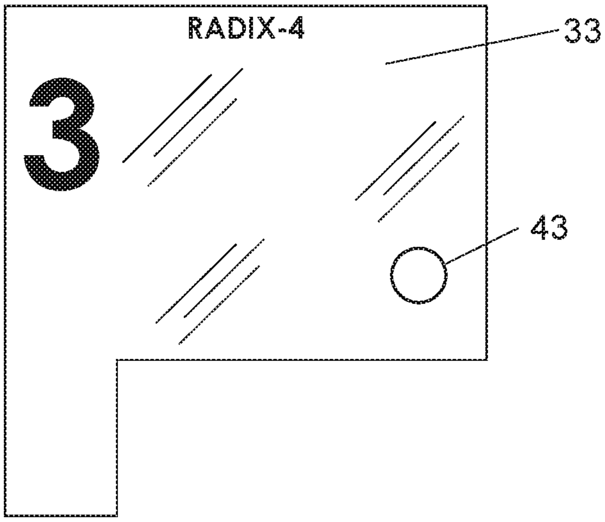


FIG. 6D

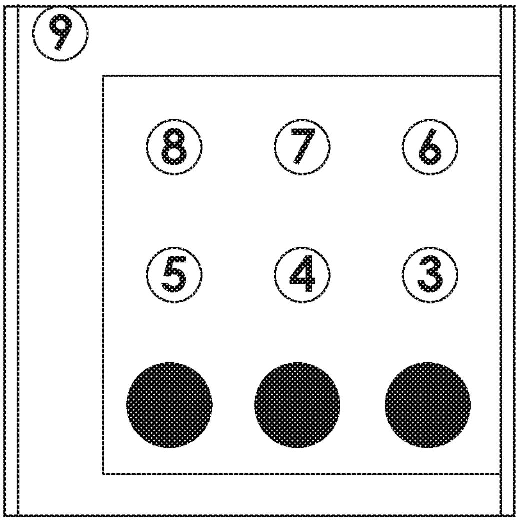


FIG. 6DD

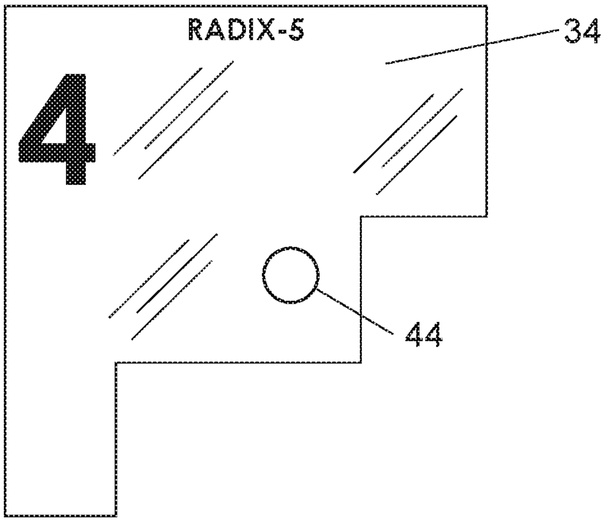


FIG. 6E

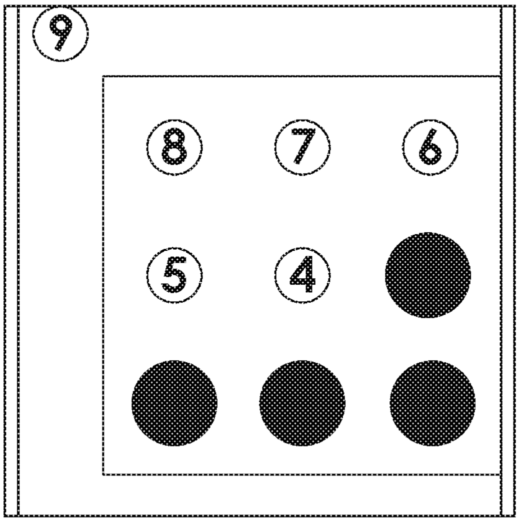


FIG. 6EE

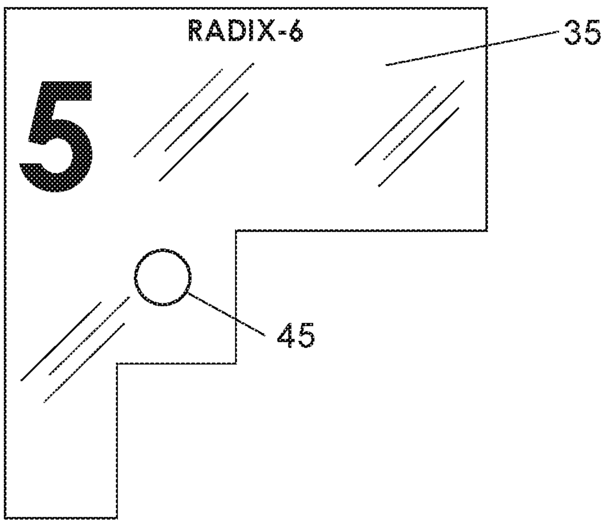


FIG. 6F

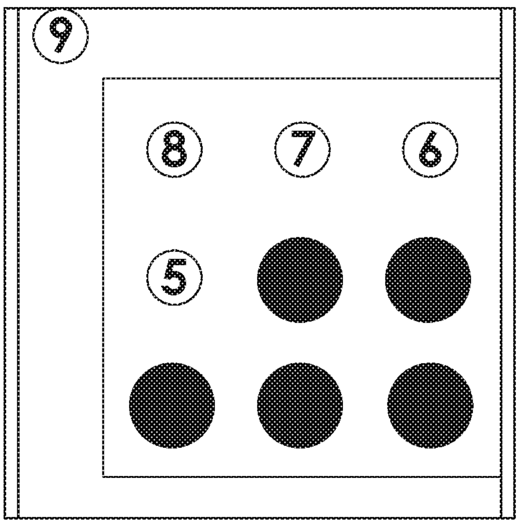


FIG. 6FF

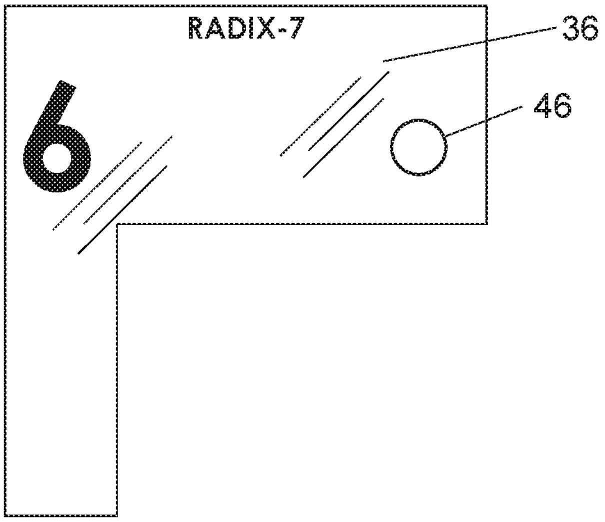


FIG. 6G

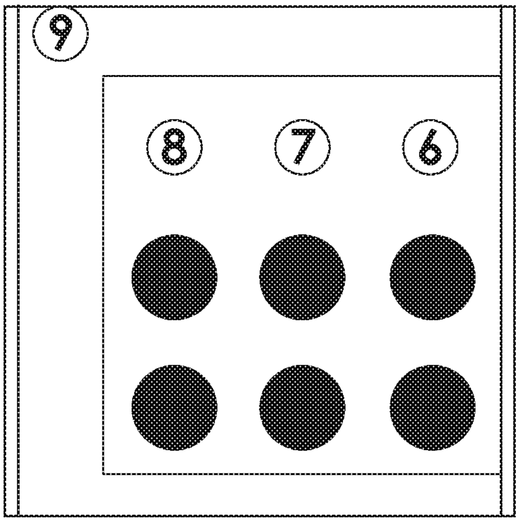


FIG. 6GG

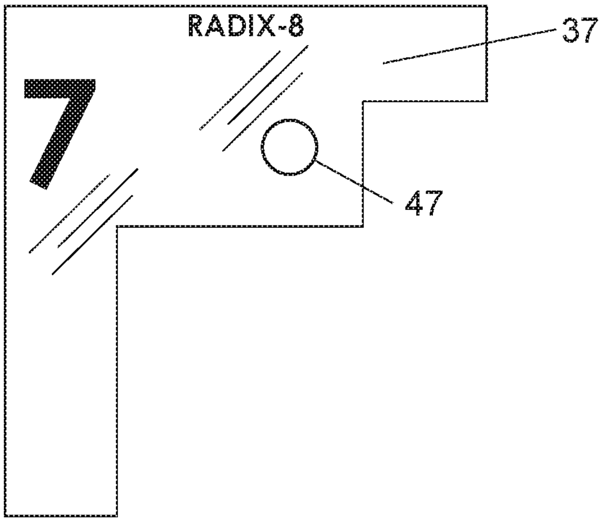


FIG. 6H

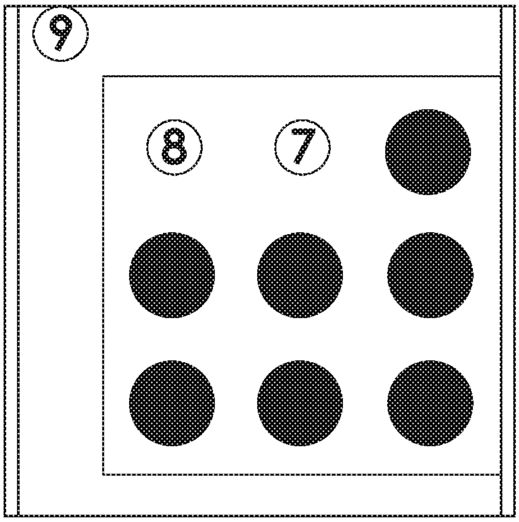


FIG. 6HH

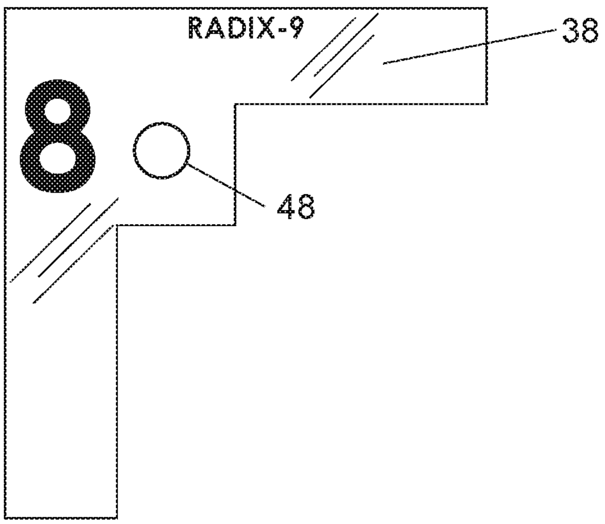


FIG. 6I

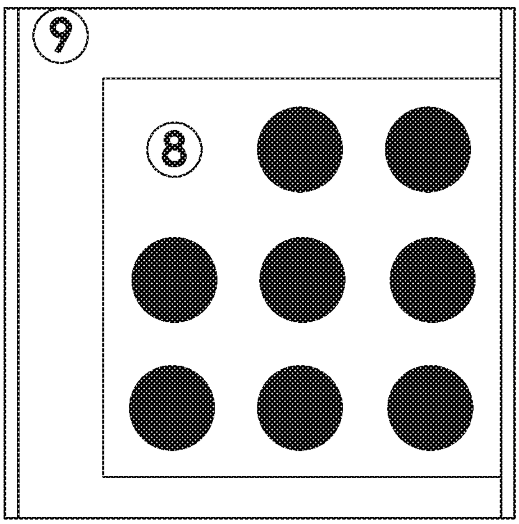


FIG. 6II

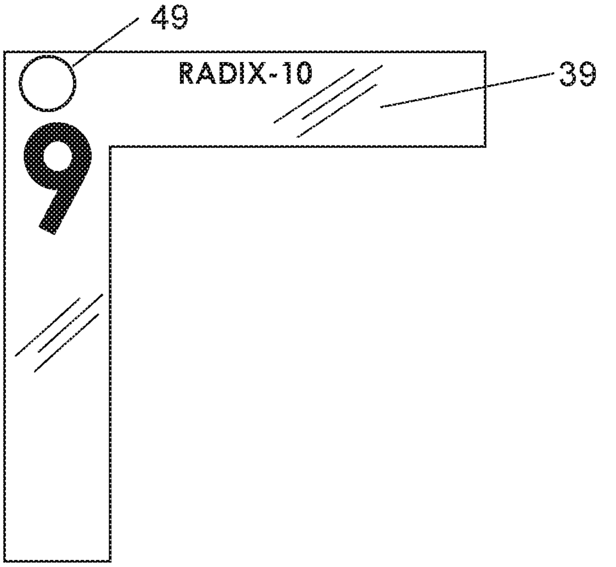


FIG. 6J

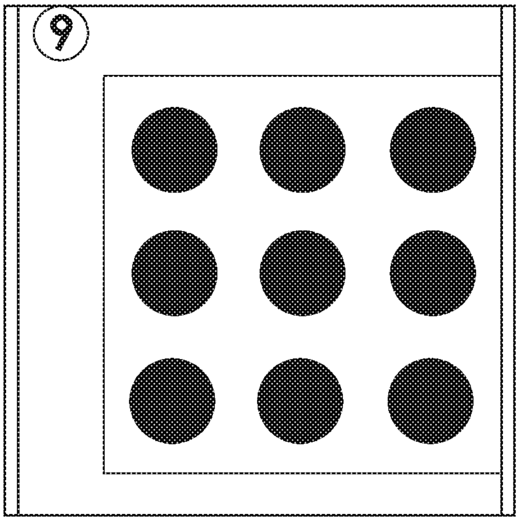


FIG. 6JJ

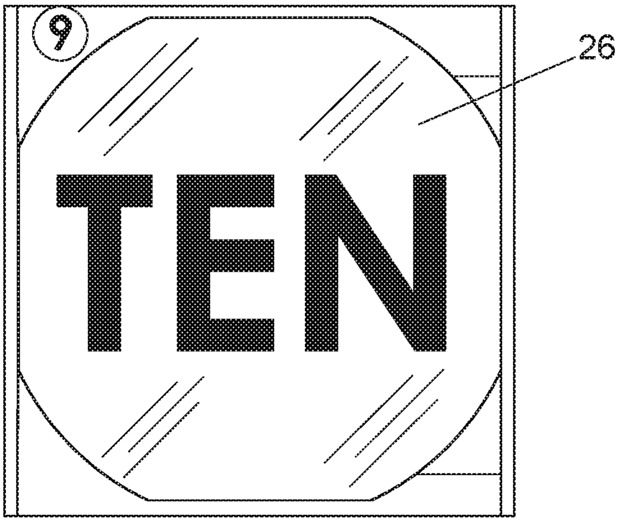


FIG. 6K

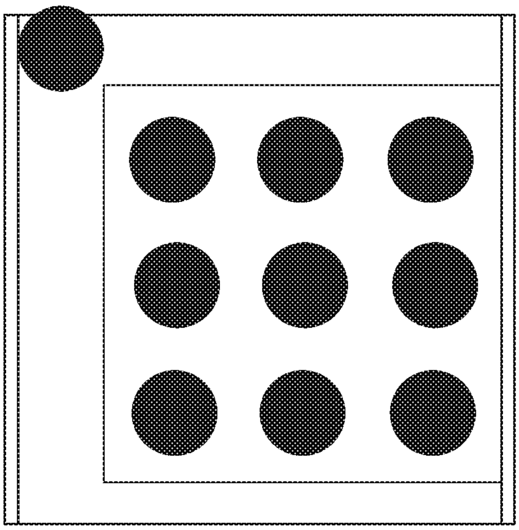


FIG. 6KK

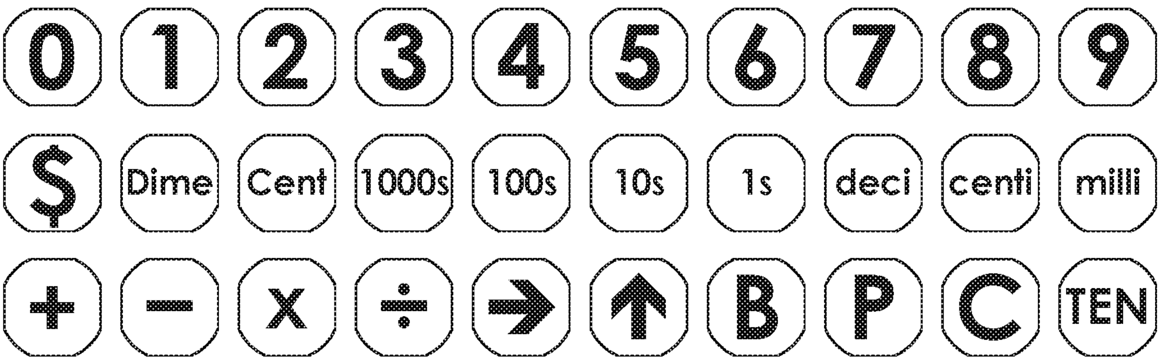


FIG. 7

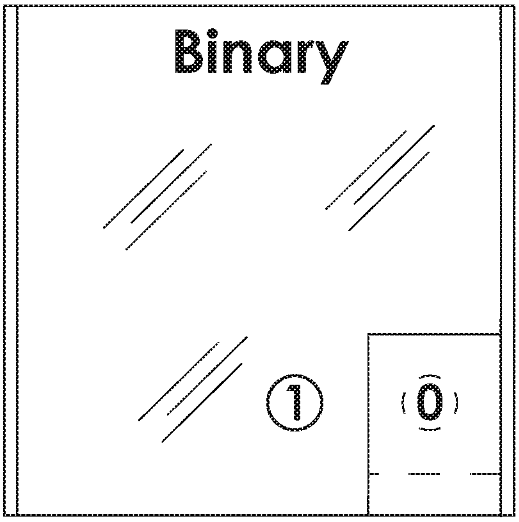


FIG. 8A

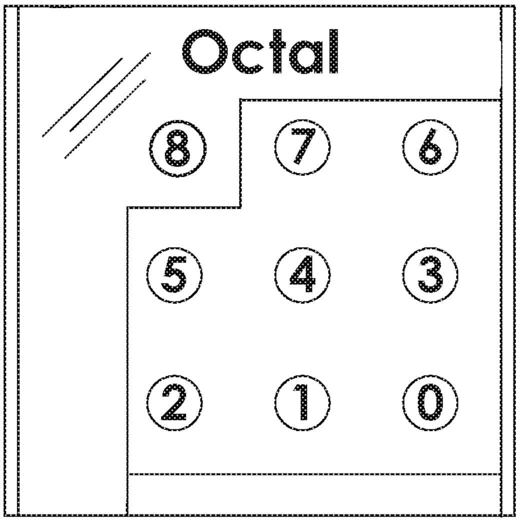


FIG. 8B

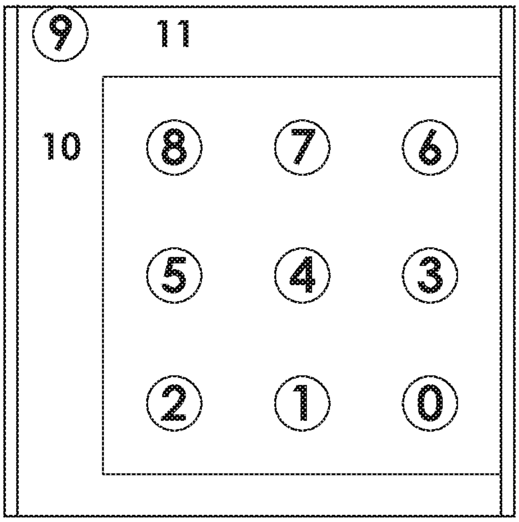


FIG. 8C

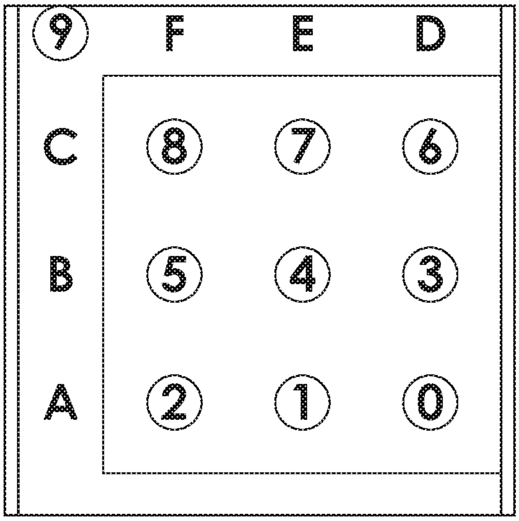


FIG. 8D

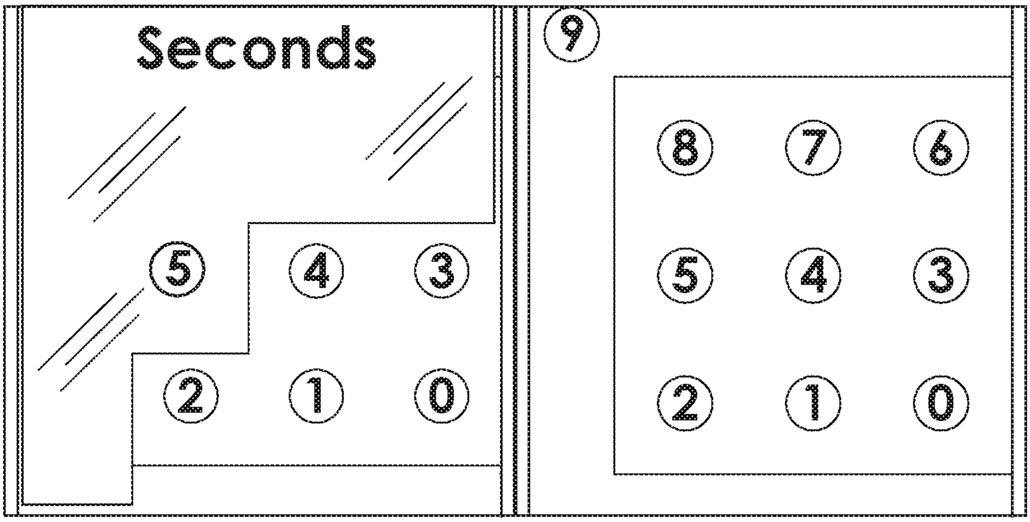


FIG. 8E

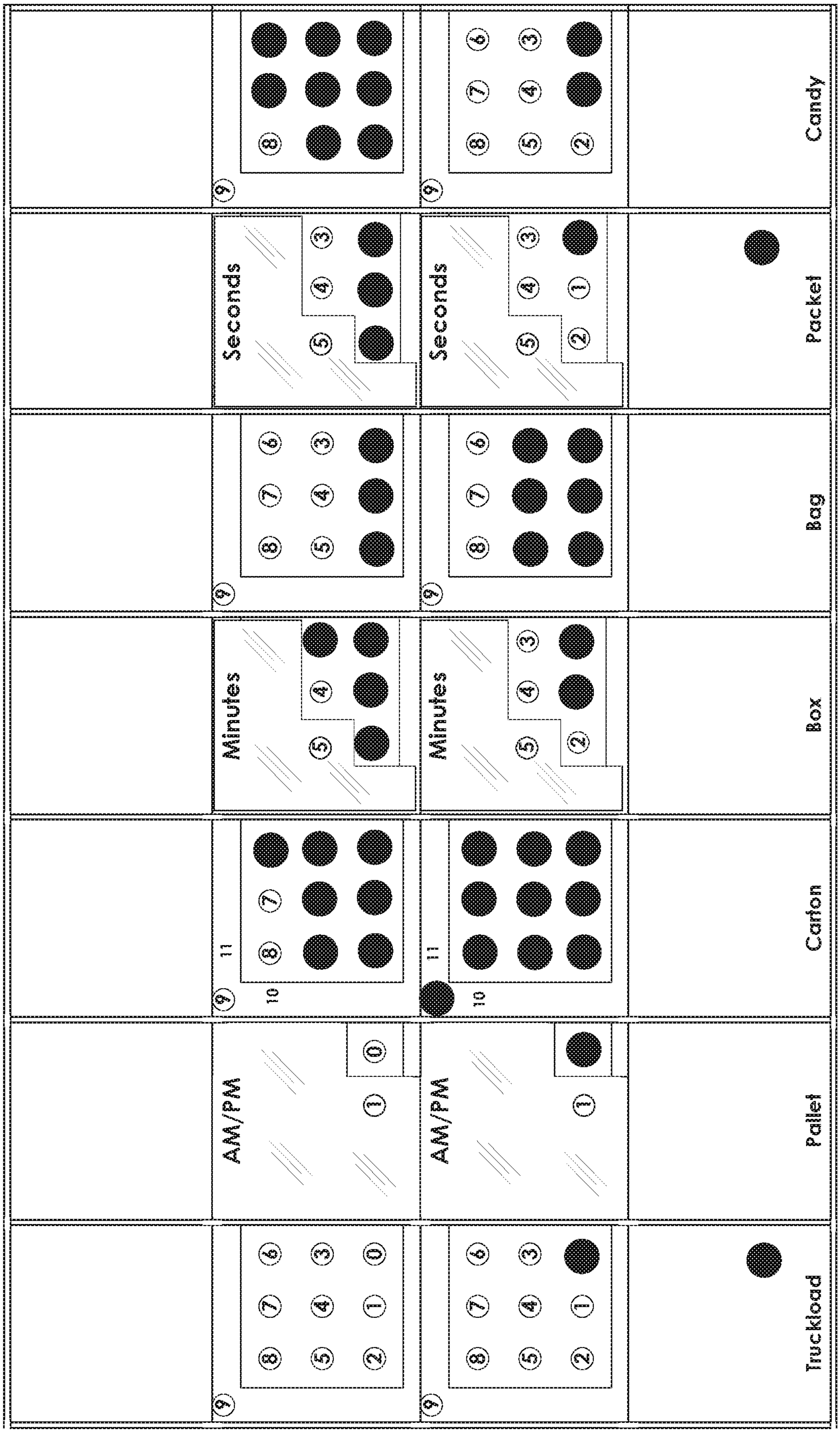

Candy
Packet
Bag
Box
Carton
Pallet
Truckload
Seconds
Seconds
Minutes
Minutes
AM/PM
AM/PM
10
11
10
11

FIG. 9A

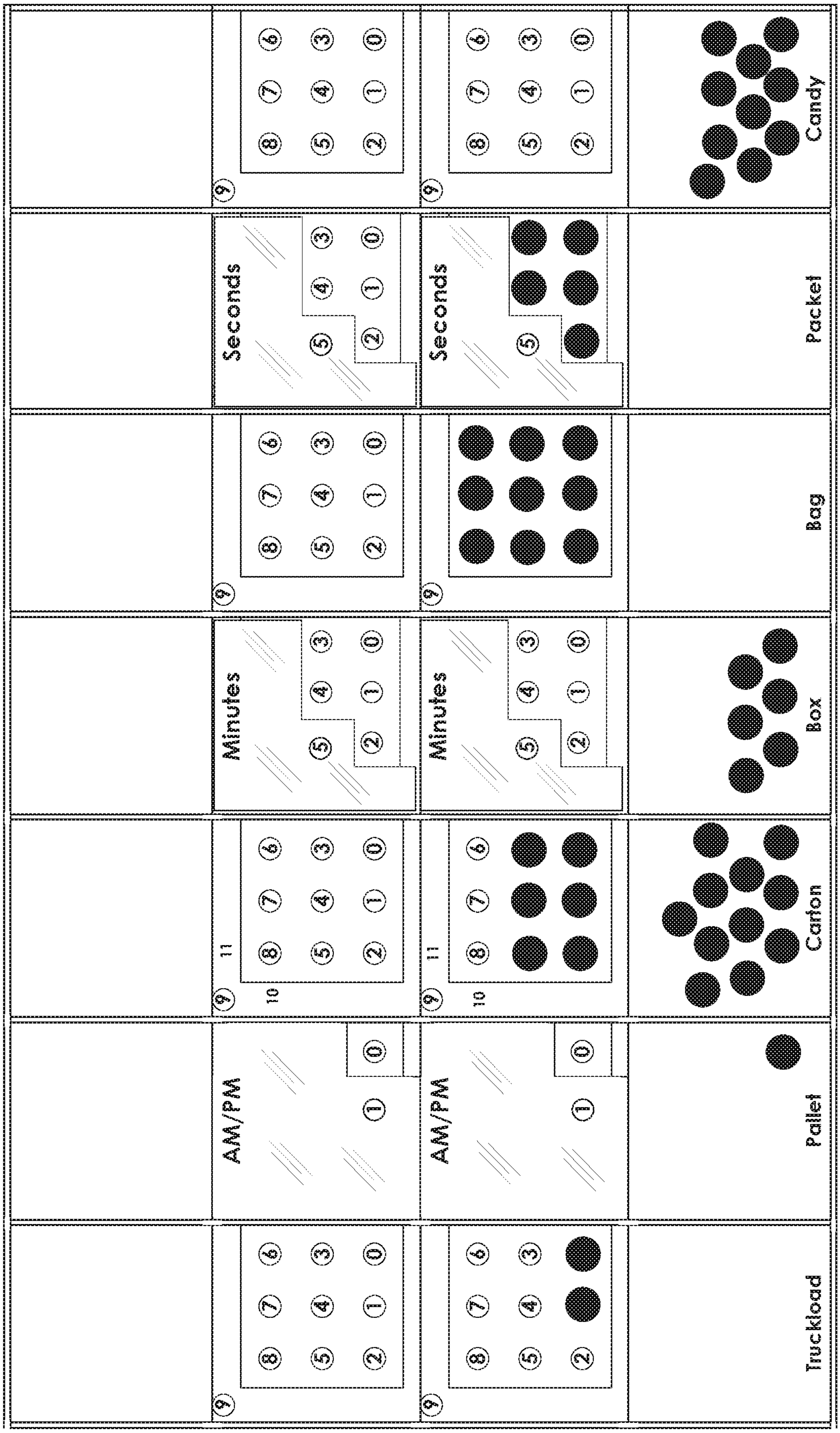

Seconds
Seconds
Minutes
Minutes
AM/PM
AM/PM
10
11
Candy
Packet
Bag
Box
Carton
Pallet
Truckload

FIG. 9B

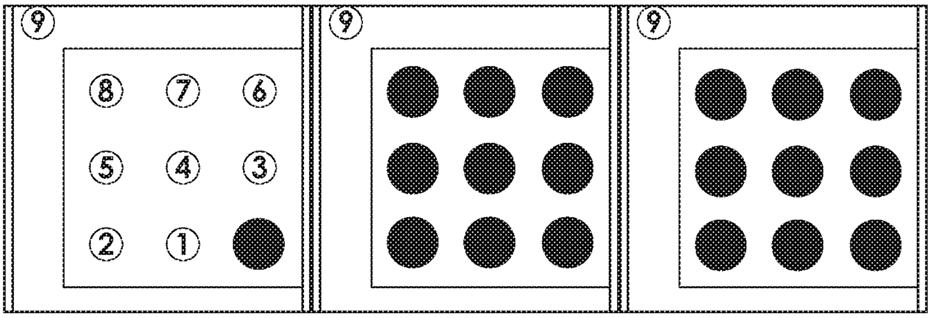


FIG. 10A

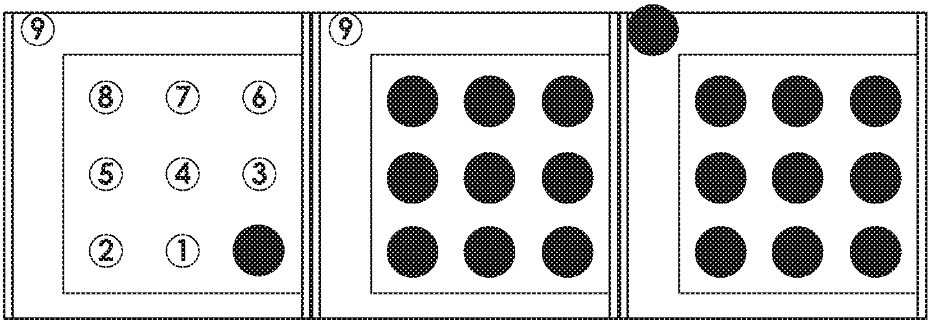


FIG. 10B

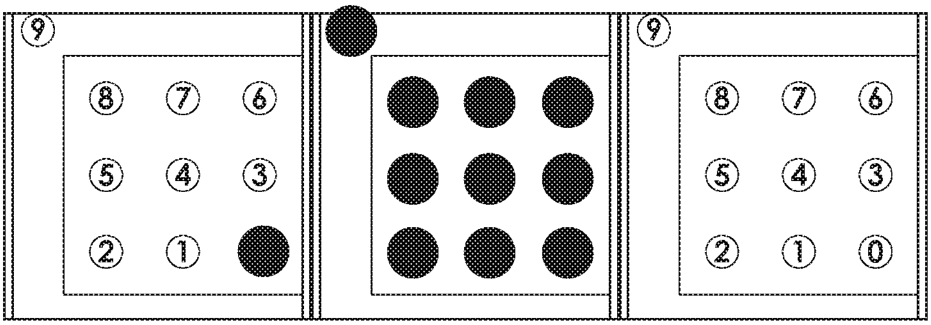


FIG. 10C

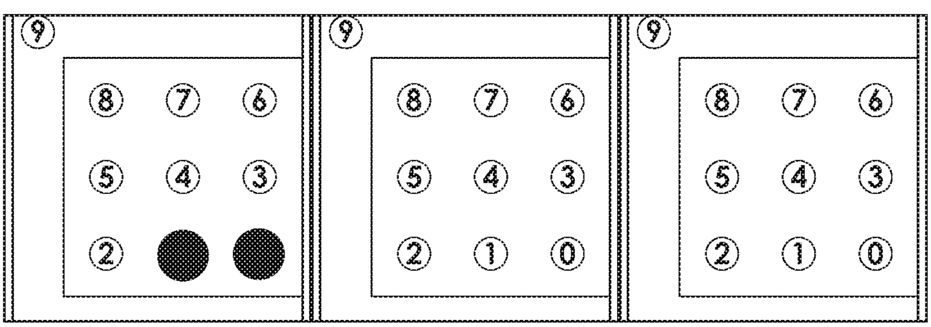
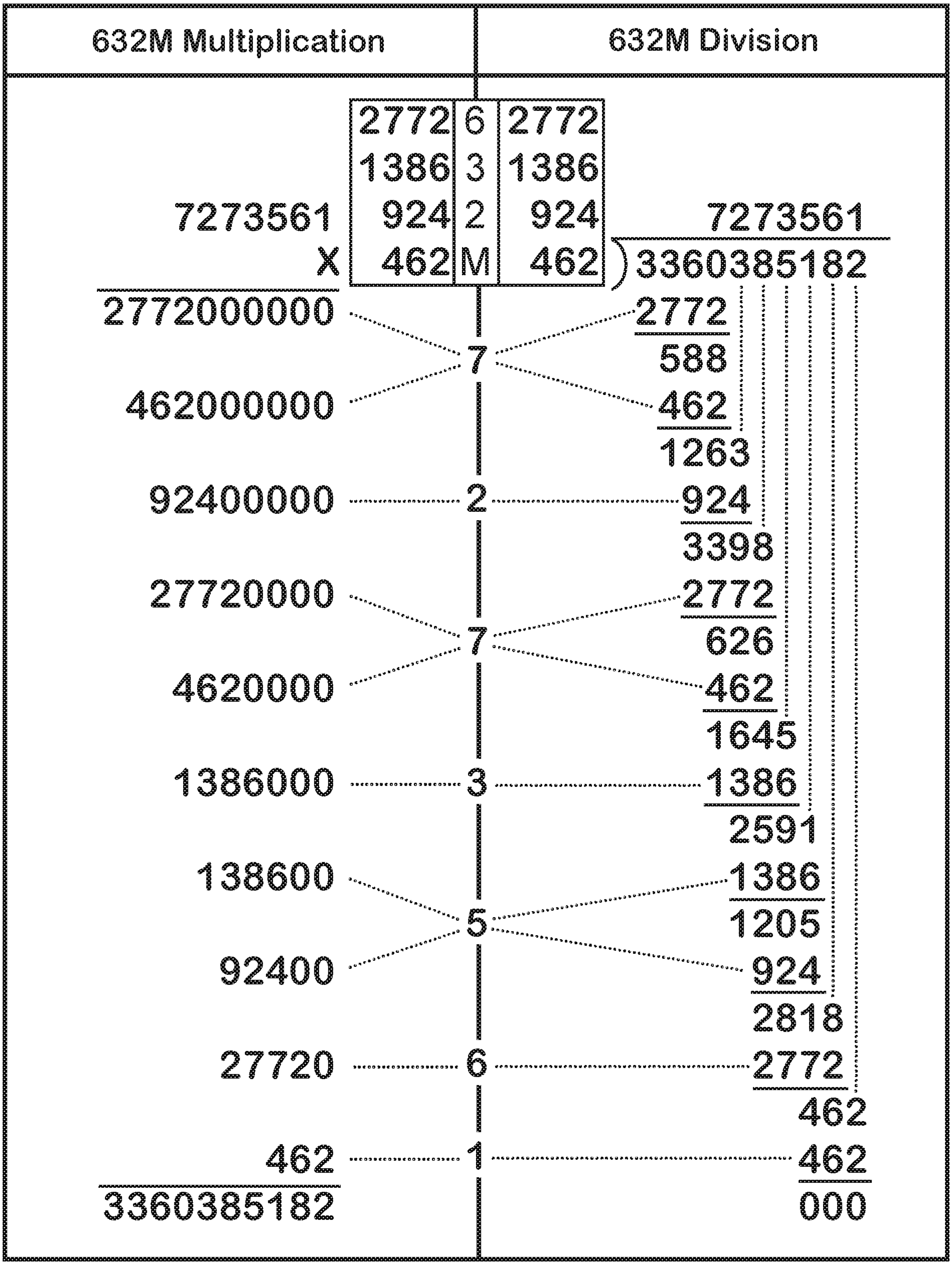


FIG. 10D


632M Multiplication
632M Division
2772 6 2772
1386 3 1386
924 2 924
462 M 462
7273561
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588
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1263
924
3398
2772
626
462
1645
1386
2591
1386
1205
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2818
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462
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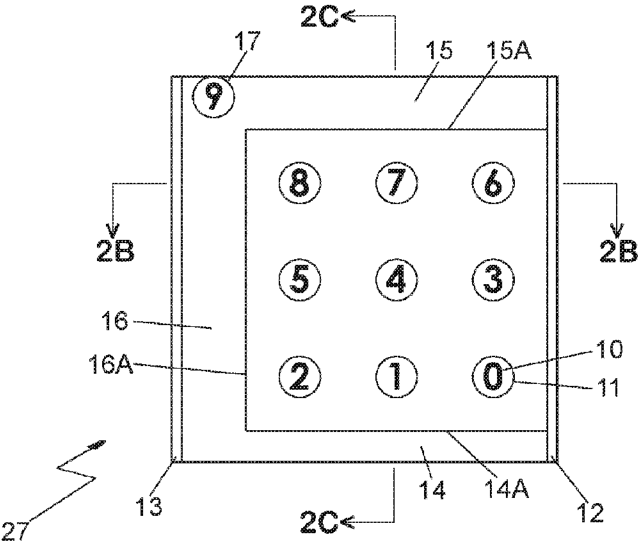
FIG. 11


2C
17
15
15A
9
8
7
6
2B
2B
5
4
3
16
10
16A
2
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11
13
14
14A
12
2C
27