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(54) **FULLY PARALLEL, LOW COMPLEXITY APPROACH TO SOLVING COMPUTER VISION PROBLEMS**

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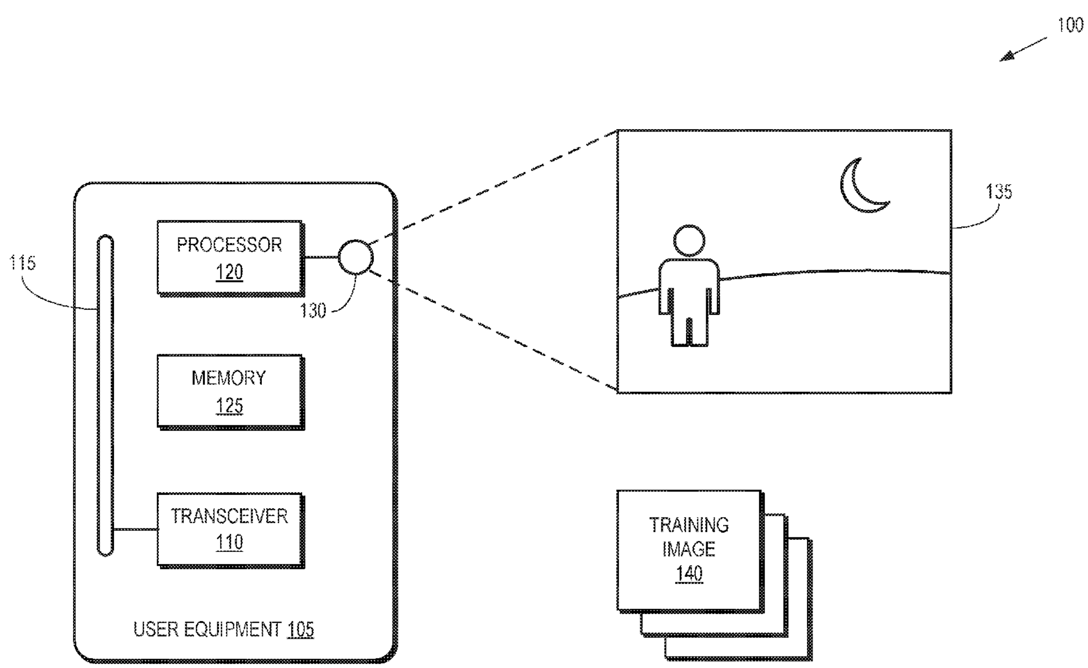
(57) **ABSTRACT**

Values of pixels in an image are mapped to a binary space using a first function that preserves characteristics of values of the pixels. Labels are iteratively assigned to the pixels in the image in parallel based on a second function. The label assigned to each pixel is determined based on values of a set of nearest-neighbor pixels. The first function is trained to map values of pixels in a set of training images to the binary space and the second function is trained to assign labels to the pixels in the set of training images. Considering only the nearest neighbors in the inference scheme results in a computational complexity that is independent of the size of the solution space and produces sufficient approximations of the true distribution when the solution for each pixel is most likely found in a small subset of the set of potential solutions.



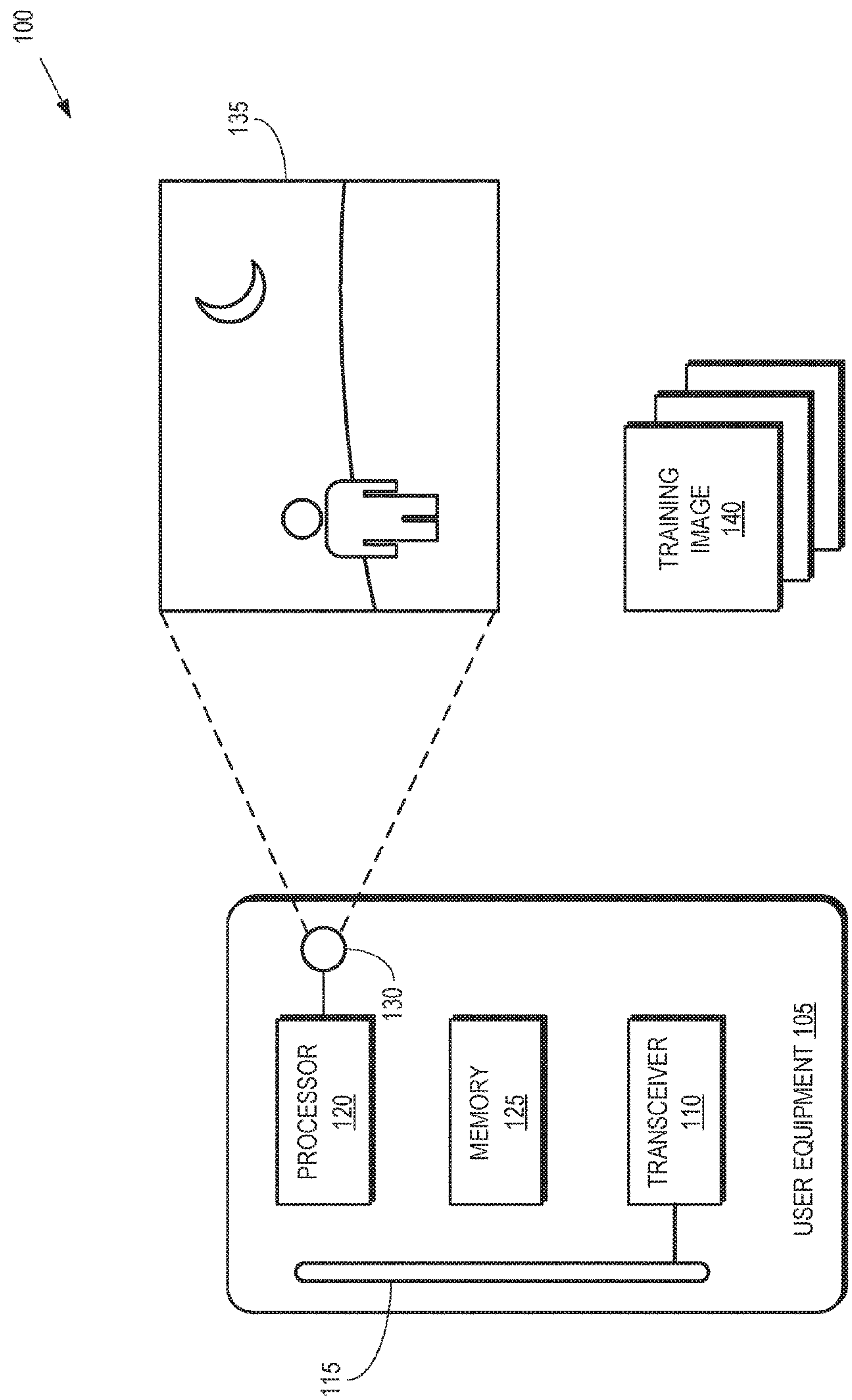

100
135
TRAINING IMAGE 140
130
PROCESSOR 120
MEMORY 125
TRANSCEIVER 110
USER EQUIPMENT 105
115

FIG. 1

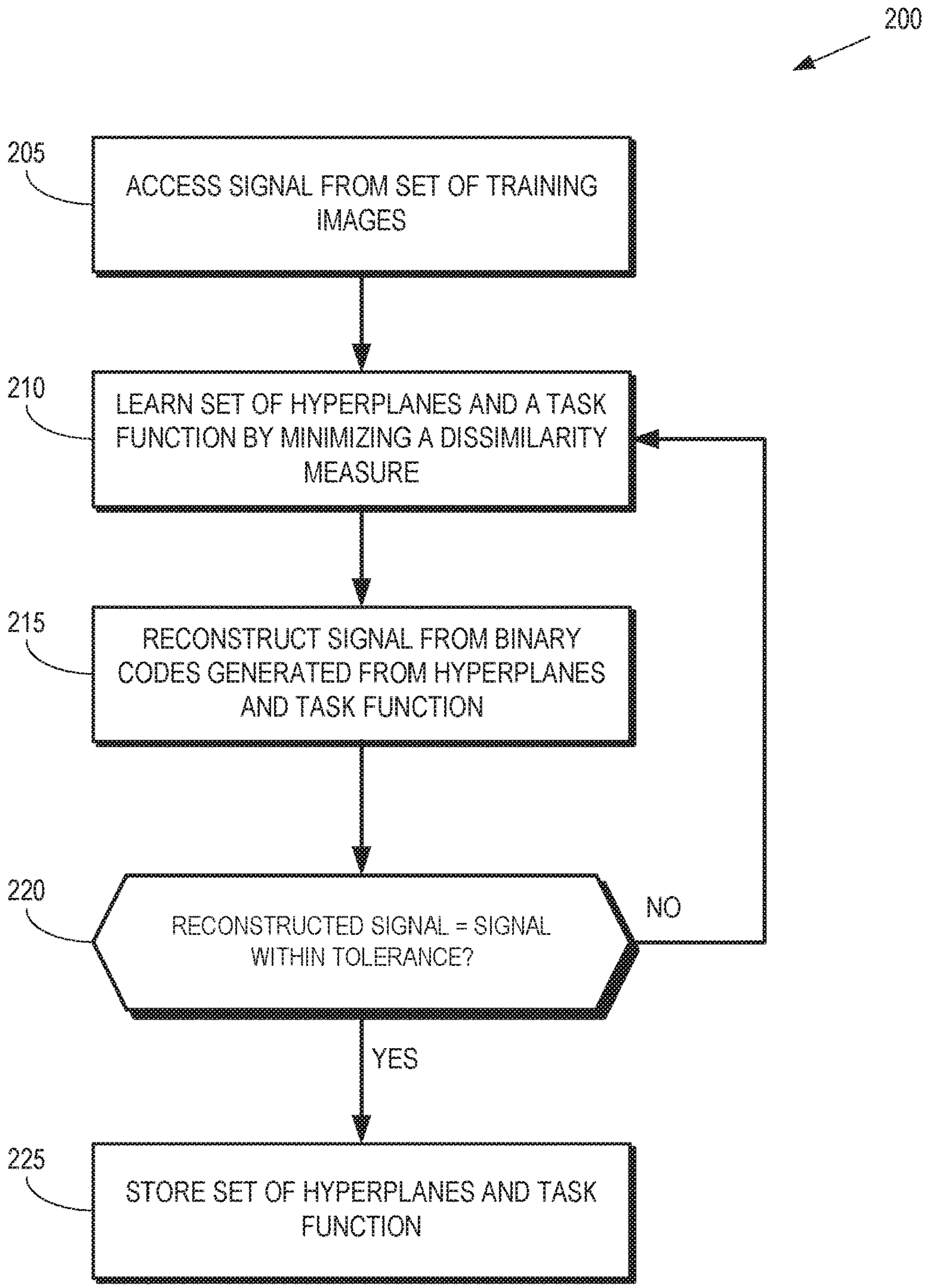

200
205
ACCESS SIGNAL FROM SET OF TRAINING IMAGES
210
LEARN SET OF HYPERPLANES AND A TASK FUNCTION BY MINIMIZING A DISSIMILARITY MEASURE
215
RECONSTRUCT SIGNAL FROM BINARY CODES GENERATED FROM HYPERPLANES AND TASK FUNCTION
220
RECONSTRUCTED SIGNAL = SIGNAL WITHIN TOLERANCE?
NO
YES
225
STORE SET OF HYPERPLANES AND TASK FUNCTION

FIG. 2

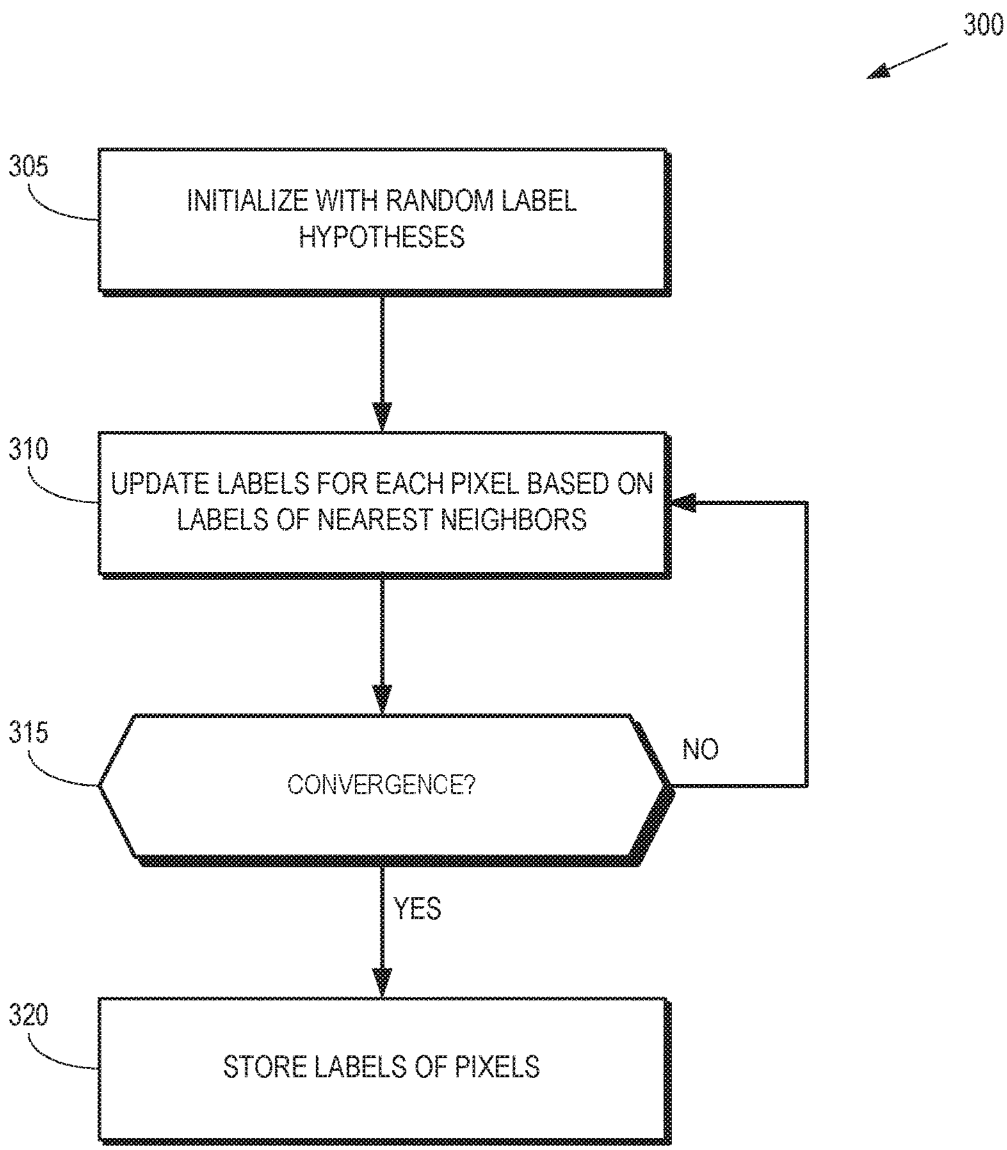

300
305
INITIALIZE WITH RANDOM LABEL HYPOTHESES
310
UPDATE LABELS FOR EACH PIXEL BASED ON LABELS OF NEAREST NEIGHBORS
315
CONVERGENCE?
NO
YES
320
STORE LABELS OF PIXELS

FIG. 3

FULLY PARALLEL, LOW COMPLEXITY APPROACH TO SOLVING COMPUTER VISION PROBLEMS

CROSS REFERENCE TO RELATED APPLICATIONS

[0001] This application claims priority to U.S. Provisional Patent Application 62/473,280, entitled "Low Compute and Fully Parallel Computer Vision with HashMatch" and filed on Mar. 17, 2017, the entirety of which is incorporated by reference herein.

BACKGROUND

[0002] Machine learning is used in many classes of computer vision problems including identification of stereo images, object classification, foreground/background segmentation, disparity estimation, image retrieval, feature approximation, background subtraction, and the like. These problems are typically formulated as a per-pixel image labeling task. For example, pixels in a stereo image are labeled as "left" or "right," to indicate the pixels that are intended to be viewed by the left eye or the right eye, respectively. Computer vision labelling problems are conventionally formulated as conditional random fields (CRFs), which have been shown to provide precise and accurate labeling of the pixels in images. However, the computational complexity of the CRF approach precludes using these approaches in low-compute scenarios such as implementations that solve the computer vision problems in devices such as smart phones, tablet computers, and the like. An alternative approach consists of using deep architectures such as convolutional neural networks (CNNs) to solve general computer vision problems, but these methods also require a considerable amount of computational resources.

BRIEF DESCRIPTION OF THE DRAWINGS

[0003] The present disclosure may be better understood, and its numerous features and advantages made apparent to those skilled in the art by referencing the accompanying drawings. The use of the same reference symbols in different drawings indicates similar or identical items.

[0004] FIG. **1** is a block diagram of a processing system that is configured to solve computer vision problems according to some embodiments.

[0005] FIG. **2** illustrates a method of training first and second functions that are used to solve computer vision problems according to some embodiments.

[0006] FIG. **3** is a flow diagram of a method of generating labels for pixels in an image according to some embodiments.

DETAILED DESCRIPTION

[0007] The efficiency of conventional deep learning techniques can be improved using compression techniques such as the use of binary weights for the input to a convolutional neural network (CNN) and the filters implemented in the CNN, removal of redundant connections and sharing of quantized weights by multiple neurons of a CNN, implementing compact CNN layers that are characterized by a reduced number of parameters, and binarizing the full network. Despite the improved efficiency, these approaches still require multiple convolutions to infer the per-pixel labels. Multiple memory accesses are also required to retrieve stored image patches from memory. The algorithms are therefore both memory and computationally bound. The computational complexity therefore increases in proportion to the sample size, e.g., the number of pixels in an image.

[0008] At least some of the drawbacks in conventional solutions to computer vision problems are reduced or eliminated using a general inference framework (referred to herein as HashMatch) that is performed in parallel on images with a complexity that is independent of size of input, e.g., the number of pixels in a patch, by training a binary unary potential using sparsity and anti-sparsity constraints. The binary unary potential is utilized in an inference scheme that estimates a true distribution of labels that is independent of the solution space. Some embodiments of the binary unary potential are represented by a first function that maps values of the pixels to a binary space that preserves characteristics of the values of the pixels. The binary unary function is also represented by a second function that learns to perform a labeling task to assign labels to the pixels in the image. In some embodiments, the first and second functions are determined based on regularizers that are trained based on sets of training images. A reconstruction function is used to verify that the original data is reconstructed from the binary unary potential. The inference scheme estimates the label for each pixel by selecting a value that is equal to a value of a label of a nearest-neighbor pixel to the pixel based on the corresponding independent per-pixel distribution. The inference scheme is then iterated, e.g., using a coordinate ascent procedure, until values of the labels of the pixels converge. Considering only the nearest neighbors in the inference scheme results in a computational complexity that is independent of the size of the solution space and produces good approximations of the true distribution when the solution for each pixel is most likely found in a small subset of the set of potential solutions, e.g., the entropy of the solution is low.

[0009] FIG. **1** is a block diagram of a processing system **100** that is configured to solve computer vision problems according to some embodiments. The processing system **100** includes a user equipment **105** such as a smart phone, a tablet, or a laptop computer. However, some embodiments of the processing system **100** include other devices that are configured to solve computer vision problems such as desktop computers, servers, and the like. Examples of computer vision problems that are addressed by the processing system **100** include identification of stereo images, object classification, foreground/background segmentation, disparity estimation, image retrieval, feature approximation, background subtraction, and the like.

[0010] The user equipment **105** includes a transceiver **110** for transmitting and receiving signals over an air interface via antenna **115**. The user equipment **105** also includes a processor **120** and a memory **125**. The processor **120** may be used to execute instructions stored in the memory **125** and to store information in the memory **125** such as the results of the executed instructions. The user equipment **105** also implements a camera **130** that is used to acquire images such as the image **135**. The processor **120** is configured to operate on pixels representative of the image **135** and the memory **125** is configured to store values of the pixels of the image **135**. Although the camera **130** is integral to the user equipment **105**, some embodiments of the user equipment **105** (or other processing devices) operate on images acquired by external image acquisition devices.

[0011] The processor **120** maps values of pixels in the image **135** to a binary space using a first function that preserves characteristics of values of the pixels. Labels are iteratively assigned to the pixels in the image **135** in parallel based on a second function. The label assigned to each pixel is determined based on values of a set of nearest-neighbor pixels. In some embodiments, the labels for the pixels in the image **135** are then stored in the memory **125**. The first and second functions are trained prior to using the first and second functions to estimate labels for the pixels in the image **135**. In the illustrated embodiment, the first function is trained to map values of pixels in a set of training images **140** to the binary space and the second function is trained to assign labels to the pixels in the set of training images **140**. Training of the first and second functions is performed by the processor **120** or by an external processor, which then provides information representative of the trained first and second functions to the user equipment **105**.

[0012] Some embodiments of the processor **120** implement a fully parallel and low complexity technique that is based on a pairwise conditional random field (CRF) that is expressed as a probabilistic factorization, P, where:

$$P(Y \mid D) = \frac{1}{Z(D)} e^{-E(Y|D)} \tag{1}$$

$$E(Y \mid D) = \Sigma_i \psi_u(l_i) + \Sigma_i \Sigma_{j \in N_j} \psi_p(l_i, l_j) \tag{2}$$

where E can be interpreted as a measure of error and a data term ψ_u measures how well an inferred solution agrees with input data, e.g., actual values of the pixels in the training images **140** or a cost of assigning a pixel i to a label l_i. The first summation captures the likelihood for a particular solution but does not consider values of neighboring pixels, which can lead to a noisy solution. The second summation considers the nearest neighbors to a pixel and provides regularization of the solution such that a label for a pixel is similar to labels for pixels of the nearest neighbors.

[0013] The implementation of the data term ψ_u depends on the labeling task that is being performed by the processor **120**. For example, if the processor **120** is tasked with finding a nearest neighbor between image patches, the labels l_i correspond to vectors (u, v) that define displacements in the image directions. In that case, the data term ψ_u is represented as:

$$\psi_u(l_i)=|h(x_i)-h(x_{i+l_i})|, \tag{3}$$

which measures the compatibility of two image patches x centered at two-dimensional pixel locations i and i+l_i. The function h(x) is a binary feature that allows efficient computation of $\psi_u(l_i)$ via a Hamming distance. For another example, other classification or regression problems are addressed by representing the data term ψ_u as:

$$\psi_u(l_i)=-\log(g(l_i,h(x_i))), \tag{4}$$

where g is a learned classifier or aggressor that evaluates the likelihood of label l_i given the binary code $h(x_i)$ of an image patch x_i.

[0014] The smoothness cost ψ_p is represented as

$$\psi_p(x_i=l_i,x_j=l_j)=\max(\tau,|l_i-l_j|). \tag{5}$$

The smoothness function encourages neighboring pixels to be assigned similar labels and the value τ is a truncation threshold.

[0015] FIG. **2** illustrates a method **200** of training the first and second functions that are used to solve computer vision problems according to some embodiments. Some embodiments of the method **200** are implemented in the processor **120** shown in FIG. **1**. Other embodiments of the method **200** are implemented in other processors and information representing the trained first and second functions is provided to the processor **120** shown in FIG. **1** so that the processor **120** is able to use the trained first and second functions to perform computer vision tasks, as discussed herein.

[0016] At block **205**, the processor accesses a signal from a set of training images, e.g., values that represent the pixels in the training images. In some embodiments, the processor trains the function $h(x_i)$ to map a signal $x \in \mathbb{R}^n$ in a binary space $b \in \{0,1\}^k$, which preserves characteristics of the original signal.

[0017] At block **210**, the processor learns a set of hyperplanes and a task function by minimizing a dissimilarity measure. In some embodiments, the processor learns a set of hyperplanes $W \in \mathbb{R}^{n \times k}$ and a task function $Z \in \mathbb{R}^{k \times d}$ that minimizes a loss function:

$$\min_{W,Z} \mathcal{L}(\text{sign}(XW)Z, Y) + \Gamma(W) + \Omega(Z) \tag{6}$$

where sign( ) is a function that returns a sign of the operand and $X \in \mathbb{R}^{m \times n}$ and $Y \in \mathbb{R}_{m \times d}$ are matrices whose i-th row corresponds respectively to x_i and y_i. The terms Γ(W) and Ω(Z) are regularizers that encourage particular structures on the predictors W and Z. In some embodiments, the regularizer Γ(W) is chosen to induce sparse solutions in the set of hyperplanes. Optimization of the loss function cannot be performed using first-order methods such as back propagation because the functions are piece-wise constant and the sub-gradient with respect to W is zero almost everywhere. Instead, a dissimilarity measure, D, is introduced, which modifies the problem to:

$$\min_{W,Z,B} \mathcal{L}(B, Z, Y) + \Gamma(W) + \Omega(Z) + \gamma \mathcal{D}(XW, B) \tag{7}$$

subject to the constraint that:

$$\|B\|_\infty \le \mu$$

where

$$\|B\|_\infty = \max_{i,j} |B_{ij}|$$

denotes the l_∞ norm of B and μ>0 is a scalar hyperparameter. This constraint is referred to as an anti-sparsity constraint.

[0018] At block **215**, the processor reconstructs an estimate of the original signal using the hyperplanes and task function generated in block **210**. In some embodiments, a reconstruction function is used to generate the estimate of the original signal based on the hyperplanes and the task function. The estimated signal is then used as a feedback signal to evaluate the quality of the hyperplanes and the task function.

[0019] At decision block **220**, the processor determines whether the reconstructed signal is equal to the actual signal within a predetermined tolerance. If not, the method **200** flows to block **210** and iteratively updates the estimate of the hyperplanes and the task function. In some embodiments, the values of the task function are iteratively updated using a gradient descent technique. If the processor determines that the reconstructed signal is within the predetermined tolerance of the actual signal, the method **200** flows to block **225** and the set of hyperplanes and the task function are stored in a memory, such as the memory **125** shown in FIG. **1**.

[0020] FIG. **3** is a flow diagram of a method **300** of generating labels for pixels in an image according to some embodiments. The method **300** is implemented in some embodiments of the processor **120** shown in FIG. **1**. In order to generate labels for the pixels in parallel using a parallel inference technique, a true distribution (e.g., the distribution P in equation 1) of labels over the pixels is approximated by a distribution Q within a class of distributions that is factorized as a product of independent marginals:

$$Q(Y)=\Pi_i Q(Y_i) \tag{8}$$

This approximation is expected to provide a good approximation of the true distribution in cases when the unary potentials that represent the actual solutions have strong peaks at the actual values of the labels, e.g., the solutions have low entropy.

[0021] At block **305**, values of the labels are initialized using a random label hypothesis to assign random labels to each pixel. A coordinate ascent procedure is then used to update the values of the labels. Using the coordinate ascent procedure guarantees that the iterative method **300** will converge on a solution.

[0022] At block **310**, values of the labels are updated. An update for the label l_i in the marginal of random variable x_i can be generated according to:

$$Q_i^t(l_i) = \frac{1}{Z_i} e^{-M_i(l_i)} \tag{9}$$

$$M_i = \psi_u(l_i) + \sum_{j \in N_i} \sum_{l_i \in \mathcal{L}} Q_j^{t-1} \psi_p(l_i, l_j) \tag{10}$$

$$Z_i = \sum_{l_i \in \mathcal{L}} e^{-M_i(l_i)} \tag{11}$$

However, the complexity of evaluating the updated values according to the equations (9), (10), and (11) is $O(|Y||\mathcal{L}|(|\mathcal{N}||\mathcal{L}|+1))$, which is quadratic in $\mathcal{L}$. Consequently, this approach becomes computationally slow as the size of the label space increases and is therefore impractical for implementation on devices with limited resources such as smart phones, tablets, and the like. In some embodiments, this drawback is addressed by only considering values of labels of nearest neighbor pixels. The solution is further approximated by assuming that the distribution Q has low entropy and is therefore reasonably well approximated by a Dirac δ function. In this approximation, equation (10) is rewritten as:

$$M_i = \psi_u(l_i) + \sum_{j \in N_i} \psi_p\left(l_i, \arg\max_{l_j} Q_j\right) \tag{12}$$

This is equivalent to updating the labels of the pixels i to a maximal value of the marginal functions of the nearest neighbor pixels. The compute complexity of the modified problem is $0(|Y||\mathcal{N}|(1+|\mathcal{N}|))$, which is independent of the size of the label space $|\mathcal{L}|$. In practice the value of $|\mathcal{N}|$ is small, e.g., on the order of four or eight, and in most problems $|\mathcal{L}|>|\mathcal{N}|$. For example, when estimating disparities in an image, the size of the label space $|\mathcal{L}|$ is typically in the hundreds.

[0023] At decision block **315**, the processor determines whether the updating procedure has converged. If not, the method **300** flows back to block **310** to update labels for the pixels in the image. If the updating procedure has converged, the method **300** flows to block **320** and stores the labels of the pixels, e.g., in a memory such as the memory **125** shown in FIG. **1**.

[0024] In some embodiments, certain aspects of the techniques described above may implemented by one or more processors of a processing system executing software. The software comprises one or more sets of executable instructions stored or otherwise tangibly embodied on a non-transitory computer readable storage medium. The software can include the instructions and certain data that, when executed by the one or more processors, manipulate the one or more processors to perform one or more aspects of the techniques described above. The non-transitory computer readable storage medium can include, for example, a magnetic or optical disk storage device, solid state storage devices such as Flash memory, a cache, random access memory (RAM) or other non-volatile memory device or devices, and the like. The executable instructions stored on the non-transitory computer readable storage medium may be in source code, assembly language code, object code, or other instruction format that is interpreted or otherwise executable by one or more processors.

[0025] A computer readable storage medium may include any storage medium, or combination of storage media, accessible by a computer system during use to provide instructions and/or data to the computer system. Such storage media can include, but is not limited to, optical media (e.g., compact disc (CD), digital versatile disc (DVD), Blu-Ray disc), magnetic media (e.g., floppy disc, magnetic tape, or magnetic hard drive), volatile memory (e.g., random access memory (RAM) or cache), non-volatile memory (e.g., read-only memory (ROM) or Flash memory), or microelectromechanical systems (MEMS)-based storage media. The computer readable storage medium may be embedded in the computing system (e.g., system RAM or ROM), fixedly attached to the computing system (e.g., a magnetic hard drive), removably attached to the computing system (e.g., an optical disc or Universal Serial Bus (USB)-based Flash memory), or coupled to the computer system via a wired or wireless network (e.g., network accessible storage (NAS)).

[0026] Note that not all of the activities or elements described above in the general description are required, that a portion of a specific activity or device may not be required, and that one or more further activities may be performed, or elements included, in addition to those described. Still further, the order in which activities are listed are not necessarily the order in which they are performed. Also, the concepts have been described with reference to specific embodiments. However, one of ordinary skill in the art appreciates that various modifications and changes can be made without departing from the scope of the present disclosure as set forth in the claims below. Accordingly, the specification and figures are to be regarded in an illustrative rather than a restrictive sense, and all such modifications are intended to be included within the scope of the present disclosure.

[0027] Benefits, other advantages, and solutions to problems have been described above with regard to specific embodiments. However, the benefits, advantages, solutions to problems, and any feature(s) that may cause any benefit, advantage, or solution to occur or become more pronounced are not to be construed as a critical, required, or essential feature of any or all the claims. Moreover, the particular embodiments disclosed above are illustrative only, as the disclosed subject matter may be modified and practiced in different but equivalent manners apparent to those skilled in the art having the benefit of the teachings herein. No limitations are intended to the details of construction or design herein shown, other than as described in the claims below. It is therefore evident that the particular embodiments disclosed above may be altered or modified and all such variations are considered within the scope of the disclosed subject matter. Accordingly, the protection sought herein is as set forth in the claims below.

What is claimed is:

1. A method comprising:

mapping values of pixels in an image to a binary space using a first function that preserves characteristics of values of the pixels;

iteratively assigning labels to the pixels in the image in parallel based on a second function, wherein the label assigned to each pixel is determined based on values of a set of nearest-neighbor pixels.

2. The method of claim **1**, further comprising:

training the first function to map values of pixels in a set of training images to the binary space; and

training the second function to assign labels to the pixels in the set of training images.

3. The method of claim **2**, wherein training the first and second functions comprises reconstructing estimated values of pixels in the set of training images from the values of the pixels in the binary space and iteratively updating the first and second functions based on a comparison of actual values of the pixels in the set of training images and the estimated values.

4. The method of claim **3**, wherein training the first and second functions comprises learning a set of hyperplanes and a task function that minimize a loss function subject to an anti-sparsity constraint.

5. The method of claim **4**, wherein training the first and second functions comprises training first and second regularizer functions using the set of training images, wherein the first and second regularizer functions encourage corresponding structures for the set of hyperplanes and the task function.

6. The method of claim **5**, wherein learning the set of hyperplanes comprises iteratively updating values of the set of hyperplanes based on the first regularizer, wherein the first regularizer is chosen to induce sparse solutions in the set of hyperplanes.

7. The method of claim **6**, wherein learning the task function comprises iteratively updating values of the task function using a gradient descent technique.

8. The method of claim **1**, wherein iteratively assigning the labels to the pixels in parallel comprises estimating distributions of labels of the pixels as independent marginal functions.

9. The method of claim **8**, wherein estimating the distributions of the labels of the pixels as independent marginal functions comprises estimating the distributions of the labels of the pixels as Dirac δ functions.

10. The method of claim **9**, wherein iteratively assigning the labels to the pixels in parallel comprises assigning, during an iteration, the labels of the pixels to a maximal value of the marginal functions of the nearest neighbor pixels.

11. The method of claim **8**, wherein iteratively assigning the labels to the pixels in parallel comprises iteratively assigning the labels to the pixels in parallel using a coordinate ascent procedure until convergence.

12. An apparatus comprising:

a processor configured to map values of pixels in an image to a binary space using a first function that preserves characteristics of values of the pixels and iteratively assign labels to the pixels in the image in parallel based on a second function, wherein the label assigned to each pixel is determined based on values of a set of nearest-neighbor pixels; and

a memory to store the labels of the pixels.

13. The apparatus of claim **12**, wherein the first function is trained to map values of pixels in a set of training images to the binary space and the second function is trained to assign labels to the pixels in the set of training images.

14. The apparatus of claim **13**, wherein training the first and second functions comprises reconstructing estimated values of pixels in the set of training images from the values of the pixels in the binary space and iteratively updating the first and second functions based on a comparison of actual values of the pixels in the set of training images and the estimated values.

15. The apparatus of claim **14**, wherein training the first and second functions comprises learning a set of hyperplanes and a task function that minimize a loss function subject to an anti-sparsity constraint.

16. The apparatus of claim **15**, wherein training the first and second functions comprises training first and second regularizer functions using the set of training images, wherein the first and second regularizer functions encourage corresponding structures for the set of hyperplanes and the task function.

17. The apparatus of claim **16**, wherein learning the set of hyperplanes comprises iteratively updating values of the set of hyperplanes based on the first regularizer, wherein the first regularizer is chosen to induce sparse solutions in the set of hyperplanes.

18. The apparatus of claim **17**, wherein learning the task function comprises iteratively updating values of the task function using a gradient descent technique.

19. The apparatus of claim **12**, wherein the processor is configured to estimate distributions of labels of the pixels as independent marginal functions.

20. The apparatus of claim **19**, wherein the processor is configured to estimate the distributions of the labels of the pixels as Dirac δ functions.

21. The apparatus of claim **20**, wherein the processor is configured to assign, during an iteration, the labels of the pixels to a maximal value of the marginal functions of the nearest neighbor pixels.

22. The apparatus of claim **19**, wherein the processor is configured to iteratively assign the labels to the pixels in parallel using a coordinate ascent procedure until convergence.

23. A non-transitory computer readable medium embodying a set of executable instructions, the set of executable instructions to manipulate at least one processor to:

map values of pixels in an image to a binary space using a first function that preserves characteristics of values of the pixels;

iteratively assign labels to the pixels in the image in parallel based on a second function, wherein the label assigned to each pixel is determined based on values of a set of nearest-neighbor pixels.

24. The non-transitory computer readable medium of claim **23**, wherein the set of executable instructions is to manipulate the at least one processor to:

train the first function to map values of pixels in a set of training images to the binary space; and

train the second function to assign labels to the pixels in the set of training images.

25. The non-transitory computer readable medium of claim **24**, wherein the set of executable instructions is to manipulate the at least one processor to reconstruct estimated values of pixels in the set of training images from the values of the pixels in the binary space and iteratively updating the first and second functions based on a comparison of actual values of the pixels in the set of training images and the estimated values.

26. The non-transitory computer readable medium of claim **25**, wherein the set of executable instructions is to manipulate the at least one processor to learn a set of hyperplanes and a task function that minimize a loss function subject to an anti-sparsity constraint.

27. The non-transitory computer readable medium of claim **26**, wherein the set of executable instructions is to manipulate the at least one processor to train first and second regularizer functions using the set of training images, wherein the first and second regularizer functions encourage corresponding structures for the set of hyperplanes and the task function.

28. The non-transitory computer readable medium of claim **27**, wherein the set of executable instructions is to manipulate the at least one processor to iteratively update values of the set of hyperplanes based on the first regularizer, wherein the first regularizer is chosen to induce sparse solutions in the set of hyperplanes.

29. The non-transitory computer readable medium of claim **28**, wherein the set of executable instructions is to manipulate the at least one processor to iteratively update values of the task function using a gradient descent technique.

30. The non-transitory computer readable medium of claim **23**, wherein the set of executable instructions is to manipulate the at least one processor to estimate distributions of labels of the pixels as independent marginal functions.

31. The non-transitory computer readable medium of claim **30**, wherein the set of executable instructions is to manipulate the at least one processor to estimate the distributions of the labels of the pixels as Dirac δ functions.

32. The non-transitory computer readable medium of claim **31**, wherein the set of executable instructions is to manipulate the at least one processor to assign, during an iteration, the labels of the pixels to a maximal value of the marginal functions of the nearest neighbor pixels.

33. The non-transitory computer readable medium of claim **30**, wherein the set of executable instructions is to manipulate the at least one processor to iteratively assign the labels to the pixels in parallel using a coordinate ascent procedure until convergence.

* * * * *