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(54) **Title:** COMPRESSED SENSING BASED ADAPTIVE DIRECTION OF ARRIVAL ESTIMATION TECHNIQUE FOR DYNAMIC TARGET ENVIRONMENTS

(57) **Abstract:** With the invention, a direction of arrival estimation method is proposed based on compressed sensing, which is adaptive and also considers hardware constraints. Since it is based on compressed sensing, it is possible to perform reliable and super-resolving estimations with compressed measurements. Within the scope of the invention, a non-heuristic adaptive measurement matrix design method best suited for reconstruction with re-weighted $\ell_1$norm minimization, a method for the projection of any designed measurement matrix in accordance with the constraints of grouped sensor subarrays, and a particle filter method special to compressed sensing based direction of arrival estimation for tracking dynamic targets are proposed.

COMPRESSED SENSING BASED ADAPTIVE DIRECTION OF ARRIVAL ESTIMATION TECHNIQUE FOR DYNAMIC TARGET ENVIRONMENTS

Technical Field

The present invention is about a compressed sensing based, adaptive direction of arrival estimation technique for dynamic target environments.

Related Art

Direction of arrival estimation is a wide research topic having applications in many areas such as radar, sonar, and wireless communications. Direction of arrival estimation is typically performed using sensors arrays. For instance, these sensors can be antennas in RF (radio frequency) applications, and hydrophones in sonar applications. Bartlett (Conventional) Beamforming is the most widely used direction of arrival estimation technique that is being used for many years. The output of this technique is the direction along which the emitted (or reflected) target signal having the maximum power impinges on the sensor array. Capon's Minimum Variance Distortionless Response (MVDR) [1] and Multiple Signal Classification (MUSIC) [2] are other common techniques using the covariance matrix of the data received by the sensor array. These three techniques can be named "classical techniques" since they are used for many years.

Compressed sensing uses the assumption that most signals in nature are sparse in some domain. For direction of arrival estimation, this assumption is equivalent to the existence of just a few targets in the environment. In sparse target scenarios, compressed sensing is used successfully in direction of arrival estimation. Thanks to the sparsity assumption, compressed sensing based direction of arrival estimation techniques enable highly accurate estimations using a few measurements. In a sensor array, the equivalent is that the linear combinations of the analog measurements taken by the sensor elements are digitized in fewer channels. In other words, it is possible to perform highly accurate estimations without digitizing analog outputs of all sensors. Hence, both hardware and software load of the system can be decreased. Mathematically, how the analog sensor outputs are linearly combined is determined by matrices called measurement matrices. The entries of these matrices are complex numbers, and they can be realized in hardware using phase shifters, variable attenuators, and low noise amplifiers. Since measurement matrices directly determine the amount of information received by the system, they have a significant effect on the performance of compressed sensing based techniques. Conventionally, measurement matrices whose entries are drawn from a random Gaussian distribution (Gaussian random matrices) are used in the literature [3].

The algorithms changing (adapting / adapting itself to the changing environment) their behaviors temporally with respect to the prior information in the environment, are called “adaptive algorithms”. Adaptive algorithms have been studied for both classical and compressed sensing based techniques. For compressed sensing based techniques, adaptation is generally provided in two ways. First, by using adaptive measurement matrices, it is ensured that the system takes measurements by considering the regions where the probability of target existence is high [4], [5]. Second, higher weights are assigned to the angles of arrival having higher probability such that the system is tended to choose those directions of arrival during reconstruction. This is typically called “reweighted ℓ_1 minimization” [6].

In direction of arrival estimation, especially in radar and sonar applications, typically the target scene is dynamic, i.e., the targets are in motion. The estimation performed in any time is highly correlated with the estimation performed just before (or the estimation to be performed shortly after) the current estimation. Therefore, a prior information about direction of arrival estimations is available. Adaptive algorithms are especially meaningful in dynamic environments since the temporal correlation between the measurements is high. For target tracking in dynamic scenarios; Kalman filter and particle filter algorithms, which are commonly used in the literature, aim to increase estimation accuracy by considering the temporal correlation of the desired states (e.g., directions of arrival) with the previous (or next) states, as well as their noisy and indirect measurements. Kalman filter provides the optimal solution for models with linear and Gaussian noise. For more general models, e.g., when the model is non-linear and/or the noise is non-Gaussian, particle filter is among the common solutions.

The main disadvantage of Bartlett beamformer is that it cannot provide super-resolving estimations. MUSIC and MVDR can provide super-resolution by using the covariance matrix of the data received by the sensor array. However, the accurate estimation of covariance matrix greatly affects the performance of MUSIC and MVDR. Covariance matrix estimation requires multiple snapshots; therefore, MUSIC and MVDR are not able to operate on single-snapshot data. Furthermore, the covariance matrix estimation requires that the target signals must be uncorrelated in order to prevent rank deficiency in the matrix, and these techniques cannot provide reliable estimations, for example in typical direction of arrival estimation scenarios where multipath fading is present. Apart from all, MUSIC and MVDR techniques are designed for static scenarios and their performances are low in dynamic target scenarios. One of the main reasons is that these techniques require multiple snapshots to generate the covariance matrix, while the dynamic environment changes during this process. Another reason is that these techniques do not consider the temporal correlation between the states unlike Kalman or particle filter.

Compressed sensing based direction of arrival estimation techniques can provide high-resolution estimations using a single snapshot and in environments with correlated sources, unlike classical techniques. Therefore, compressed sensing based techniques are more eligible for dynamic target scenarios compared to classical techniques. Typically, the performance provided by Gaussian random matrices can be surpassed using certain measurement matrix design methods [7]. In adaptive measurement matrix design literature, Gaussian random matrices have been outperformed in many areas such as image processing [8], and direction of arrival estimation [4], [5]. Such measurement matrix design techniques can be examined under three main headings depending on their design criteria, namely “mutual information”, “Cramer-Rao lower bound”, and “mutual coherence”:

1. Mutual Information Based Techniques: These techniques design the measurement matrix such that the mutual information (or conditional mutual information) between the received measurements and directions of arrival is maximized [9], [10]. The main drawback of these techniques is that they require solving non-convex, complicated optimization problems. Hence, these techniques are not suitable for dynamic applications where the measurement matrices are required to be updated adaptively in real-time.
2. Cramer-Rao Lower Bound Based Techniques: Cramer-Rao lower bound provides the minimum mean squared error that can be achieved by an unbiased estimator. The techniques under this heading [11]-[13] design the measurement

matrix minimizing the Cramer-Rao lower bound. In [11], the measurement matrix is designed in closed-form such that it can be obtained rapidly. However, this design is suitable only for a customized problem model and is not applicable, for example, in direction of arrival estimation. The main reason is that the desired signal is assumed to be sparse in its own domain. However, in direction of arrival estimation using sensors arrays, there is a signal coming at each sensor. In [12], again, a closed-form measurement matrix that can be found rapidly is obtained. However, in the problem model, a single target is assumed and the signal characteristics of this target is assumed to be known. Therefore, this problem model also highly restricts the practical applicability of this study. In the study done in [13], a problem model for multiple targets with unknown signal characteristics is tackled. However, the proposed design requires solving a non-convex, complicated optimization problem. This is a highly restrictive design for dynamic scenarios and not applicable when real-time adaptive design is required.

3. Mutual Coherence Based Techniques: Mutual coherence is the maximum value of the normalized inner products of two different columns of a matrix. Mutual correlation based techniques aim to minimize the mutual correlation of effective dictionaries, thereby trying to ensure that the compressed measurements taken are independent of each other. The effective dictionary is achieved by multiplying the measurement matrix with the dictionary. In the first measurement matrix design study [7], Gaussian random matrices have been outperformed in the area of image processing where the measurement matrix is designed such that it minimizes the mutual coherence of the effective dictionary for a particular dictionary. However, this study proposes a slow/iterative design methodology, and does not mention prior information (adaptiveness). Again, in the image processing field, this time an adaptive measurement matrix design has been proposed in [8]. However, in [8], an iterative design methodology which cannot provide the rapid measurement matrix updates in dynamic systems has been provided, although it could have been obtained in closed-form. As a result of the necessity of rapid design for adaptive techniques, in [4], for direction of arrival estimation problems, a design enabling rapid measurement matrix updates has been proposed. However, the solution achieved in this study requires some unrealistic restrictions such as the sensor array used in direction of arrival estimation must be a uniform linear array consisting of isotropic sensors.

To summarize briefly, the main problems of the techniques in the literature are listed below:

- As they offer complex solutions, they cannot provide rapid updates in systems where the measured environment changes rapidly and requires the use of adaptive measurement matrix [9]-[13].
- Since they provide iterative solutions to problems with closed-form solutions, they are insufficient in fast measurement matrix updates in adaptive systems and they need additional convergence checks [8].
- The solutions obtained in systems where closed-form, rapid measurement matrix update is possible can be obtained under many assumptions, thus limiting the flexibility of the system to a great extent, and even offering solutions that are not applicable in practice [4].

In a recent study we were involved in [5], an adaptive measurement matrix design methodology has been proposed by addressing the above problems, and the performance improvement of the proposed method has been demonstrated against many alternative measurement matrix designs, including the Gaussian random matrix. However, the design given in this study is largely based on heuristic techniques and is not based on strong justification. In addition, adaptability is only achieved with the adaptive measurement matrix. Reweighted ℓ_1 minimization is not mentioned and the relationship between the reweighted ℓ_1 minimization and adaptive measurement matrix design is not established. In addition, in other studies on adaptive measurement matrix design, including [5], it is assumed that prior information is already known, and no tracking algorithm is used when proposing adaptive techniques. However, how a prior information is provided is a critical issue for adaptive techniques and cannot be ignored.

Tracking algorithms such as Kalman filter and particle filter typically use maximum likelihood to perform direction of arrival estimation [14]. In addition, compressed sensing based direction of arrival estimation using Kalman filter [15], [18] and particle filter algorithms [16], [17] have been studied in the literature. In the application of the compressed sensing based Kalman filter given in [15], it is assumed that the effective dictionary in the problem model is a Gaussian random matrix, and the dictionary is an orthonormal matrix. This problem model is not suitable for the direction of arrival estimation problems, also the dynamic state model examined in this study is different from the model of the invention. Moreover, since the Kalman filter is designed for linear systems with Gaussian noise, the application area of this study is limited. In the study given in [16], it is assumed that the effective dictionary, as in [15], is a Gaussian random matrix. The dynamic state model is also different from the model of the invention, as in [15]. Also, the particle filter algorithm used is substantially different from that used in the present invention. In the study given in [17], unlike [15], there is no assumption that the dictionary is orthonormal. However, the problem model examined is different from the problem model we examine in this invention, and the measurement matrix is assumed to be a Gaussian random. In [18], similar to other studies, the measurement matrix is assumed to be Gaussian random. In short, how the adaptive measurement matrix design can be adapted to particle filter applications has not been covered in the literature so far.

Finally, the hardware realization of measurement matrices is often overlooked. Most of the designs in the literature propose a fully filled measurement matrix with all elements nonzero. What this means for sensor arrays is that each sensor element is connected individually to all digital channels. For this reason, studies in the literature suggest designs that are very difficult to implement in hardware. This issue has been handled in a study including our research group, and a block diagonal measurement matrix design has been proposed [19]. However, this solution is only valid if the sensor array is uniformly linear and the sensor elements are isotropic. This is both restrictive and unrealistic conditioning.

The prior art application CN107037392B relates to a compressed sensing based direction of arrival estimation technique with limited degrees of freedom and high computational complexity. However, there is no mention of a method developed for obtaining high estimation performance in environments containing correlated sources, thanks to the reconstruction algorithm used based on compressed sensing, and for tracking dynamic targets with particle filter specific to the compressed sensing based direction of arrival estimation problem. Furthermore, an adaptive measurement matrix design is not mentioned in the mentioned application, random matrices are used as measurement matrices.

As a result, due to the problems described above and the inadequacy of existing solutions on the subject, it is deemed necessary to make an improvement in the relevant technical field.

Purpose of the Invention

With the invention, a direction of arrival estimation method is proposed based on compressed sensing, which is adaptive and also considers hardware constraints. Since it is based on compressed sensing, it also enables reliable and super-resolution estimations with compressed measurements. Since compressed measurements are obtained by analog processes and digitized in far fewer channels than the number of sensor elements, the hardware and software load of the system is greatly reduced. Classical techniques (Barttlet Beamforming, MUSIC, MVDR) require digitization of each sensor element in order to make reliable predictions.

Again, thanks to the reconstruction algorithm used based on compressed sensing, high estimation performance is also achieved in environments containing correlated sources (for example, environments with multipath fading), unlike MUSIC and MVDR. At the same time, since it can work with a single snapshot, the overall performance in dynamic scenarios is higher than MUSIC and MVDR.

The problems mentioned in the previous section are encountered with the measurement matrix designs studied in the literature for the compressed sensing based direction of arrival estimation. Within the scope of the invention, three methods are proposed to outperform the aforementioned methods:

- A non-heuristic adaptive measurement matrix design method best suited for reconstruction with re-weighted ℓ_1-norm minimization
- A method for the projection of any designed measurement matrix in accordance with the constraints of grouped sub-arrays
- A method for tracking dynamic targets with a particle filter, specific to the compressed sensing based direction of arrival estimation problem.

The proposed matrix design method is the solution of an optimization problem with a closed-form solution. Since it does not use an iterative algorithm, it is suitable for real-time application and does not impose any constraints on the direction of arrival estimation system. Since it is based on the re-weighted ℓ_1-norm minimization reconstruction, it provides a solution due to the nature of the problem and guarantees that the proposed solution is the most appropriate solution to the relevant prior.

Moreover, the proposed measurement matrix design method is resistant to known distortions such as mutual coupling.

Regardless of the matrix design method, it provides the flexibility of the system and offers practical solutions thanks to the projection method suitable for the constraints of grouped sub-arrays. In addition, the method is suitable for real-time application since it is the solution of an optimization problem with a closed-form solution.

Independently of previous contributions, it is possible to generate a prior information specific to the compressed sensing based direction of arrival estimation systems, thanks to the method developed for tracking dynamic targets with a particle filter. In this way, by using the adaptive measurement matrix design and particle filter concepts in an integrated way; all parts of an adaptive, compressed sensing based direction of arrival estimation system are complete.

The structural and characteristic features of the invention and all its advantages will be understood more clearly thanks to the figures given below and the detailed description written with references to these figures.

Brief Description of the Drawings

Figure 1 is a simplified direction of arrival estimation diagram for a linear array of sensors and a single emitting target.

Figure 2 is the schematic of the hardware implementation of an example system where analog measurements of 4 sensor elements are reduced to 2 digital channels for RF applications.

Figure 3 is the flow chart of an example hardware design showing the digital implementation of the measurement matrix for RF applications.

Figure 4 illustrates how an adaptive direction of arrival estimation system (right) differs from a typical non-adaptive direction of arrival estimation system (left).

Figure 5 shows the schematic of the proposed closed-loop direction of arrival estimation system, assuming that the $\boldsymbol{P}$ matrix containing the prior information is available through a tracking algorithm within a system.

Figure 6 shows a simplified version of the hardware implementation of the RF system in Figure 2 when only right singular vectors are available for the design.

Drawings do not necessarily need to be scaled, and details not necessary for understanding the present invention may be omitted.

Description of Part References

1. Low noise amplifier
2. Phase shifter
3. Variable attenuator
4. ADC (Analog-to-Digital Converter)
5. Digital channel

Detailed Description of the Invention

In this detailed description, preferred embodiments of the invention are explained only for a better understanding of the subject and without any limiting effect.

Direction of arrival (DOA) estimation with sensor arrays is a common field of array signal processing and has wide-ranging applications in areas such as sonar, radar, electronic warfare systems, and wireless communications. Sensor arrays used in DOA estimation problems are generally considered passive, and the following assumptions are made about the environment and the targets being tracked:
Isotropic medium: Every region of the environment has the same properties. For this reason, the media properties do not change depending on the directions of arrival and there is no harm in applying the same mathematical formulations for all angles.

Linear Environment: Superposition can be applied for the system. Therefore, signals in certain regions of the environment can be calculated using linear operations.

Unit-Gain and Unfaded Environment: Without loss of generality, it can be assumed that the signals propagating in the medium do not suffer any fading or gain.

Narrowband Signals with the Same Carrier Frequency: The signals in the environment are considered to be narrowband signals with the same carrier frequency. The mathematical expression for the signal can be written as:

$$s_i(t) = \alpha_i(t)e^{j\beta_i(t)};\ \text{i: target index}$$

When the carrier frequency is included in this equality:

$$s_i(t) = \alpha_i(t)\cos[2\pi f_c t + \beta_i(t)]$$

is found. The signal is assumed to change slowly with time compared to f_c. This assumption can only be considered true when the signals are assumed to be narrowband.

Channel with Additive White Gaussian Noise: The dominant source of the noise in the system is the receivers in the passive sensor elements, and the noise in each receiver is independent of each other, with zero mean and equal standard deviation.

Target signals reach each sensor element with a different time shift. In fact, DOA estimation can be performed thanks to the phase shift created by this time shift. When we select the first sensor element from the sensor array as the reference sensor element and call the distance between this sensor element and the target as r, the signal that the first sensor element receives can be written as:

$$x_{1i}(t) = s_i(t - \tau_{1i}), \tau_{1i} = \frac{r}{c}.$$

Without loss of generality, $\tau_{1i} = 0$ can be assumed. Here, the statements are written under the far-field assumption, without losing the feature of the method. As a result of the far field assumption, the signals impinging on the sensor array elements can be considered parallel. Although this simplifies the equations mentioned below, the method does not lose its advantages in the absence of this assumption. The time shifts for other sensor elements are not the same. For example, the signal received by the pth sensor element is:

$$x_{pi}(t) = s_i(t - \tau_p) = \alpha_i(t - \tau_{pi})\cos[2\pi f_c(t - \tau_{pi}) + \beta_i(t - \tau_{pi})].$$

The delay τ_{pi} depends on the sensor array geometry and directions of arrival. For example, for uniform linear sensors arrays where d denotes the distance between consecutive sensors and θ denotes the direction of arrival of ith target:

$$\tau_{pi} = \frac{(p-1)dcos(\theta)}{c}$$

is found. Since α and β terms change slowly:

$$x_{pi}(t) \cong \alpha_i(t)\cos[2\pi f_c(t) + \beta_i(t) - 2\pi f_c\tau_{pi}] = \mathrm{Re}\{s_i(t)e^{-j2\pi f_c\tau_{pi}}e^{j2\pi f_c t}\}$$

and in phasor domain:

$$x_{pi}(t) \cong s_i(t)e^{-j2\pi f_c \tau_{pi}}$$

is written. Then, if it is assumed that there are K targets in the environment, including noise, the data received by the pth sensor element is written as follows using the superposition feature of the system:

$$x_p(t) = \sum_i x_{pi}(t) + n_p(t) = \sum_{i=1}^{K} s_i(t)e^{-j2\pi f_c \tau_{pi}} + n_p(t)$$

In matrix form, if the number of sensor elements in the used sensor array is M, this equation is written as follows:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} 1 & \cdots & \cdots & 1 \\ e^{-j2\pi f_c \tau_{21}} & e^{-j2\pi f_c \tau_{22}} & \cdots & e^{-j2\pi f_c \tau_{2K}} \\ \vdots & \cdots & \cdots & \vdots \\ e^{-j2\pi f_c \tau_{M1}} & e^{-j2\pi f_c \tau_{M2}} & \cdots & e^{-j2\pi f_c \tau_{MK}} \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_K(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{bmatrix}$$

In matrix-vector multiplication form, we can express this equality as follows:

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n}. \qquad (1)$$

A simplified DOA estimation diagram for a linear array of sensors and a single emitting target is shown in Figure 1. The matrix $\boldsymbol{A}$ in equation (1) is often called the steering matrix. The reason why $\boldsymbol{A}$ is called the steering matrix is because, as we showed earlier, the columns of $\boldsymbol{A}$ (steering vectors) depend on the geometry of the sensor array as well as the directions of arrival of targets. In equation (1), the dimensions are $\boldsymbol{A} \in \mathbb{C}^{M\times K}, \boldsymbol{x} \in \mathbb{C}^{M\times 1}, \boldsymbol{s} \in \mathbb{C}^{K\times 1}, \boldsymbol{n} \in \mathbb{C}^{M\times 1}$. The classical techniques MVDR [1] and MUSIC [2] perform their estimations using the covariance matrix of $\boldsymbol{x}$, i.e., $\mathrm{E}\{\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}\} \approx \sum_\tau \boldsymbol{x}(\tau)\boldsymbol{x}^{\mathrm{H}}(\tau)$, instead of directly using equation (1). Here, τ denotes the snapshot index. The proposed measurement matrix design method is not specific to the mentioned matrix $\boldsymbol{A}$ and can be used with any measurement equation. The method can also work without any problems under the effects such as mutual coupling, which only affects the matrix linearly.

Adaptive Measurement Matrix Design for DOA Estimation

In sparse target scenarios, compressed sensing (CS) based approaches can be used. In the CS based DOA model, a dictionary matrix $\boldsymbol{D}$ is constructed whose columns contain the responses of the sensor array to targets (i.e. steering vectors) at possible directions of arrival. The identified possible directions of arrival are often called the grid points. The angles corresponding to the grid points are determined by the system operator or designer. Ideally, the matrix $\boldsymbol{D}$ would be expected to contain the columns of $\boldsymbol{A}$. However, the targets may not be on the grid points specified by the user. In other words, since the number of columns of $\boldsymbol{D}$ is finite, the specified grid is discrete. Therefore, the vector $\boldsymbol{x}$ given in equation (1) may not be obtained by linear combinations of the columns of $\boldsymbol{D}$. For this reason, to emphasize that the CS based DOA estimation model is an approximation; substituting $\overline{\boldsymbol{x}}, \overline{\boldsymbol{s}}$ for $\boldsymbol{x}, \boldsymbol{s}$ gives the following model:

$$\overline{\boldsymbol{x}} = \boldsymbol{D}\overline{\boldsymbol{s}} + \boldsymbol{n} \quad (2)$$

If the number of grid points is L, the dimensions are $\boldsymbol{D} \in \mathbb{C}^{M\times L}, \overline{\boldsymbol{x}} \in \mathbb{C}^{M\times 1}$, and $\overline{\boldsymbol{s}} \in \mathbb{C}^{L\times 1}$. In equation (2), there are M equations and L unknowns, usually $L > M$. Therefore, the system given in equation (2) has infinite number of solutions. One of the basic assumptions of CS is that the vector $\overline{\boldsymbol{s}}$ is sparse. CS based approaches aim to select the sparsest possible solution. When $||\overline{\boldsymbol{s}}||_0$ represents the number of non-zero elements in the vector $\overline{\boldsymbol{s}}$, the directions of arrival can be found by solving the following optimization problem:

$$\hat{\overline{\boldsymbol{s}}} = \arg\min_{\overline{\boldsymbol{s}}} ||\overline{\boldsymbol{s}}||_0 \text{ subject to } ||\boldsymbol{x} - \boldsymbol{D}\overline{\boldsymbol{s}}||_2^2 \leq \xi. \quad (3)$$

Here ξ is the trade-off parameter between sparsity and data fidelity. Finding the solution that minimizes $||\overline{\boldsymbol{s}}||_0$ requires a combinatorial search. Therefore, instead of minimizing the expression $||\overline{\boldsymbol{s}}||_0$; since the ℓ_1 norm is the convex relaxation of ℓ_0, generally $||\overline{\boldsymbol{s}}||_1$ is minimized. Typically, after $\hat{\overline{\boldsymbol{s}}}$ is found, the grid points corresponding to the largest K element are determined as DOA estimations.

Another advantage of CS based techniques is that they can perform reliable estimations using compressed measurements. This compression is done by measurement matrices. The hardware implementation of the measurement matrices is done with variable attenuators (3), phase shifters (2) and low-noise amplifiers (1). With measurement matrices, the analog measurements of the sensor elements are reduced to a smaller number of digital channels (5). For example, for RF applications, the hardware implementation of a system where analog measurements of 4 sensor elements are reduced to 2 digital channels (5) is shown in Figure 2. In addition to realizability with analog elements, measurement matrices can also be implemented in digital environment. However, for this, all sensor element outputs must be digitized. In this case, there is no gain in hardware compared to classical techniques. However, since the number of measurements can be reduced even in the digital environment, software gain is expected. Again, as an example, the hardware design showing the application of the measurement matrix for RF applications in the digital environment is shown in Figure 3. In addition, the technique to be presented can also be used to ensure adaptability, even if the number of measurements is preserved (not compressed). The most frequently used measurement matrices in the literature are Gaussian random matrices [3].

If the measurement matrix is denoted by $\boldsymbol{\Phi}$ and included in equation (1), the measurements $\boldsymbol{x}$ are compressed and the compressed measurements $\boldsymbol{y}$ are achieved:

$$\boldsymbol{y} = \boldsymbol{\Phi x} = \boldsymbol{\Phi A s} + \boldsymbol{\Phi n}. \tag{4}$$

If m denotes the number of digital channels (5), the dimensions are $\boldsymbol{\Phi} \in \mathbb{C}^{m \times M}$ and $\boldsymbol{y} \in \mathbb{C}^{m \times 1}$. The noise vector $\boldsymbol{n}$ given in equation (1) is generally assumed to be spatially white. However, after the multiplication with $\boldsymbol{\Phi}$, the noise $\boldsymbol{\Phi n}$ does not become white anymore unless $\boldsymbol{\Phi\Phi}^{\mathrm{H}}$ is an identity matrix. Hence, $\boldsymbol{W} = \left(\boldsymbol{\Phi\Phi}^{\mathrm{H}}\right)^{-0.5}$ denoting the whitening matrix, the model with whitened noise is given below:

$$\boldsymbol{Wy} = \boldsymbol{W\Phi x} = \boldsymbol{W\Phi A s} + \boldsymbol{W\Phi n}. \tag{5}$$

By including $\boldsymbol{\Phi}$ in equation (3) and replacing ℓ_0 with ℓ_1 norm, the following problem is solved for DOA estimation:

$$\hat{\boldsymbol{s}} = \arg\min_{\bar{\boldsymbol{s}}} \|\bar{\boldsymbol{s}}\|_1 \ \text{ subject to } \|\boldsymbol{Wy} - \boldsymbol{W\Phi D}\bar{\boldsymbol{s}}\|_2^2 \le \beta \tag{6}$$

The most typical way to design adaptive DOA estimation systems in CS based techniques is the adaptive measurement matrix design [4], [5]. In addition to adaptive measurement matrix design, another method of ensuring adaptability in CS based techniques is to convert the reconstruction algorithm given in equation (6) to reweighted ℓ_1 minimization [6]. If the matrix containing prior information about the targets is called $\boldsymbol{P}$, whose diagonal elements contain the probability of finding the target at each grid point, the problem in equation (6) can be written as follows with the re-weighted ℓ_1 minimization approach:

$$\hat{\boldsymbol{s}} = \arg\min_{\bar{\boldsymbol{s}}} \|\boldsymbol{P}^{-1}\bar{\boldsymbol{s}}\|_1 \ \text{ subject to } \|\boldsymbol{Wy} - \boldsymbol{W\Phi D}\bar{\boldsymbol{s}}\|_2^2 \le \beta \tag{7}$$

In this way, it is ensured that the reconstruction system puts more cost on the places where the target is less likely to be found, thus focusing more on the places where the target is more likely to be found. Defined as $\bar{\bar{\boldsymbol{s}}} \equiv \boldsymbol{P}^{-1}\bar{\boldsymbol{s}}$, the problem in equation (7) can be written as:

$$\hat{\bar{\boldsymbol{s}}} = \arg\min_{\bar{\bar{\boldsymbol{s}}}} \|\bar{\bar{\boldsymbol{s}}}\|_1 \ \text{ subject to } \|\boldsymbol{Wy} - \boldsymbol{W\Phi D P}\bar{\bar{\boldsymbol{s}}}\|_2^2 \le \beta \tag{8}$$

After the solution of equation (8) is found, $\hat{\boldsymbol{s}} = \boldsymbol{P}\hat{\bar{\boldsymbol{s}}}$ is written and K peaks with the highest absolute value of $\hat{\boldsymbol{s}}$ are reported as directions of arrival. The main difference between equation (7) and (8) is that in equation (7) the effective dictionary is $\boldsymbol{\Phi D}$, whereas in equation (8) it is $\boldsymbol{\Phi DP}$. One of the commonly used measurement matrix design techniques in the literature is to approximate the Gram matrix of the effective dictionary to a target matrix. The main purpose of this approach is to reduce the correlation between the columns of the effective dictionary. Thus, in scenarios where re-weighted ℓ_1 minimization is not considered, the measurement matrix design is done as follows:

$$\widehat{\boldsymbol{\Phi}} = \arg\min_{\boldsymbol{\Phi}} \left\| (\boldsymbol{\Phi D})^{\mathrm{H}} (\boldsymbol{\Phi D}) - \boldsymbol{T} \right\|_{\mathrm{F}}^{2} \qquad (9)$$

In the present invention, the efficient dictionary has evolved from $\boldsymbol{\Phi D}$ to $\boldsymbol{\Phi DP}$. So instead of equation (9), the following problem should be solved:

$$\widehat{\boldsymbol{\Phi}} = \arg\min_{\boldsymbol{\Phi}} \left\| (\boldsymbol{\Phi DP})^{\mathrm{H}} (\boldsymbol{\Phi DP}) - \boldsymbol{T} \right\|_{\mathrm{F}}^{2} \qquad (10)$$

In this invention, for arbitrary full-rank $\boldsymbol{T}, \boldsymbol{D}, \boldsymbol{P}$ matrices, the solution of equation (10) will be provided. To achieve that solution, the singular value decomposition of $\boldsymbol{DP}$ matrix is written as $\boldsymbol{DP} = \boldsymbol{U}_{DP} \boldsymbol{\Sigma}_{DP} \boldsymbol{V}_{DP}^{\mathrm{H}}$; and the eigenvalue decomposition of $\boldsymbol{Z} \equiv \boldsymbol{V}_{DP}^{\mathrm{H}} \boldsymbol{T} \boldsymbol{V}_{DP}$ is written as $\boldsymbol{Z} = \boldsymbol{Q \Lambda Q}^{\mathrm{H}} = \sum_i \lambda_i \boldsymbol{q}_i \boldsymbol{q}_i^{\mathrm{H}}, \lambda_1 \geq \lambda_2 \geq \cdots \lambda_M$. If z denotes the number of eigenvalues of $\boldsymbol{Z}$ that are greater than zero, the solution of (10) is given below:

$$\widehat{\boldsymbol{\Phi}} = \begin{cases} \boldsymbol{\Lambda}_m^{1/2} \boldsymbol{Q}_m^{\mathrm{H}} \boldsymbol{\Sigma}_{DP}^{-1} \boldsymbol{U}_{DP}^{\mathrm{H}}, & if\ m \leq z \\ \begin{bmatrix} \boldsymbol{\Lambda}_z^{1/2} \boldsymbol{Q}_z^{\mathrm{H}} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\Sigma}_{DP}^{-1} \boldsymbol{U}_{DP}^{\mathrm{H}}, & if\ m > z \end{cases}$$

Here, $\boldsymbol{\Lambda}_m \equiv \boldsymbol{\Lambda}(1\!:\!m, 1\!:\!m)$, $\boldsymbol{\Lambda}_z \equiv \boldsymbol{\Lambda}(1\!:\!z, 1\!:\!z), \boldsymbol{Q}_m \equiv \boldsymbol{Q}(:, 1\!:\!m), \boldsymbol{Q}_z \equiv \boldsymbol{Q}(:, 1\!:\!z)$ are defined using MATLAB notation [20]. The difference between an adaptive DOA estimation system and a typical, non-adaptive DOA estimation system is shown in Figure 4. The system on the left in Figure 4 represents the non-adaptive DOA estimation system, which assigns equal importance to each angle, while the system on the right represents the adaptive DOA estimation system. For ease of illustration, it is assumed that the angle space is divided into only 4 sectors. It can be assumed that the radii of the circle segments given in Figure 4 are proportional to the diagonal elements of the $\boldsymbol{P}$ matrix.

So far, the relationship between re-weighted ℓ_1 minimization and adaptive measurement matrix design has been demonstrated by the prior information matrix $\boldsymbol{P}$, and it has been demonstrated how to use these two adaptive techniques in an integrated manner. If we assume that the $\boldsymbol{P}$ matrix is available through a tracking algorithm inside the system, the proposed DOA estimation system is as in Figure 5. In Figure 5, the DOA estimation system is shown as a closed-loop system. However, the DOA estimation system can also receive prior information from a tracker outside the system. Whichever way the $\boldsymbol{P}$ matrix is obtained, the presented measurement matrix design methodology is still valid.

In addition, the proposed adaptive measurement matrix design method is not specific to the DOA estimation problems and can be applied to any sparsity based measurement matrix compression problem where the prior information is available. That is, the proposed method does not use a property of $\boldsymbol{A}$ or $\boldsymbol{D}$ matrix specific to the DOA estimation problem.

A Method for the Appropriate Projection of the Proposed Measurement Matrix Design onto a Grouped Sensor Array

The measurement matrix $\boldsymbol{\Phi}$ designed by any method is usually a fully-filled matrix. This means that there must be a connection between all sensor elements and all analog-to-digital converter (ADC) (4) elements. For example, in a system with 64 sensors and 16 ADCs (4), there must be a total of 1024 connections, 1024 phase shifters (2) and 1024 attenuators (3). However, this is not feasible from a practical point of view. For this reason, it has been proposed to group some sensor elements before [19]. In this case, the designed $\boldsymbol{\Phi}$ matrix must have a block-diagonal structure. However, there is no such restriction on the measurement matrix design demonstrated in the previous title. A method suitable for real-time application in the literature and applicable in practice can only work under the condition $\boldsymbol{D}\boldsymbol{D}^H = \boldsymbol{I}_M$ (unit matrix) [19]. In this section, a design method that can be generalized even if this condition is not met, will be described. For this purpose, a measurement matrix is designed by any method, including the method described in the previous section. Then the problem in equation (11) is solved:

$$\arg\min_{\boldsymbol{B},\widehat{\boldsymbol{\Phi}}} \left\|\boldsymbol{\Phi} - \boldsymbol{B}\widehat{\boldsymbol{\Phi}}\right\|_F^2, \qquad \text{subject to } \widehat{\boldsymbol{\Phi}} \in \text{block diagonal matrices} \tag{11}$$

As can be noticed from the previous section, multiplying the measurement matrix with an invertible matrix from left does not actually change the measured information or the noise distribution. For this reason, it is desired to decompose the designed measurement matrix by expressing it as the product of a block diagonal matrix and an invertible matrix. In equation (11), the dimensions are $\boldsymbol{B} \in C^{m\times m}, \widehat{\boldsymbol{\Phi}} \in C^{m\times M}$. Any $\widehat{\boldsymbol{\Phi}}$, which is the solution of the above equation, should carry similar information as $\boldsymbol{\Phi}$.

Let J denote the number of sensor groups. If we assume that there are N_j sensors and $N_{C,j}$ channels in each sensor group, then the problem can be expressed by using the equations below:

$$\boldsymbol{\Phi} = [\boldsymbol{\Phi}_1 \quad \cdots \quad \boldsymbol{\Phi}_J], \boldsymbol{B} = [\boldsymbol{B}_1 \quad \cdots \quad \boldsymbol{B}_J], \qquad \boldsymbol{\Phi}_j \in C^{m \times N_j}, \boldsymbol{B}_j \in C^{m \times N_{C,j}} \qquad (12)$$

$$\widehat{\boldsymbol{\Phi}} = \begin{bmatrix} \widehat{\boldsymbol{\Phi}}_1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & \widehat{\boldsymbol{\Phi}}_J \end{bmatrix}, \qquad \widehat{\boldsymbol{\Phi}}_j \in C^{N_{C,j} \times N_j}$$

$$\arg\min_{\boldsymbol{B},\widehat{\boldsymbol{\Phi}}} \sum_{j=1}^{J} \left\| \boldsymbol{\Phi}_j - \boldsymbol{B}_j \widehat{\boldsymbol{\Phi}}_j \right\|_{\mathrm{F}}^2 = \left\{ \arg\min_{\mathbf{B}_j,\widehat{\boldsymbol{\Phi}}_j} \left\| \boldsymbol{\Phi}_j - \boldsymbol{B}_j \widehat{\boldsymbol{\Phi}}_j \right\|_F^2 \right\}_{j=1\cdots J} \qquad (13)$$

As seen in equation (13), for the block-diagonal matrix, the problem is separable in each sensor group. Also, since the multiplication of the final designed matrix by the left-hand matrix is not important, each problem can be solved as follows:

$$\arg\min_{\boldsymbol{B}_j,\widehat{\boldsymbol{\Phi}}_j} \left\| \boldsymbol{\Phi}_j - \boldsymbol{G}_j \right\|_F^2, \qquad \text{subject to } rank(\boldsymbol{G}_j) \leq N_{C,j} \qquad (14)$$

In this invention, for any full rank $\boldsymbol{\Phi}_j$ matrix, the solution of the above problem is provided. When the singular value decomposition of $\boldsymbol{\Phi}_j$ is expressed as $\boldsymbol{\Phi}_j = \boldsymbol{U}_j \boldsymbol{\Sigma}_j \boldsymbol{V}_j^{\mathrm{H}}$:

$$\boldsymbol{B} = \boldsymbol{U}_{j,N_{C,j}} \boldsymbol{\Sigma}_{j,N_{C,j}}, \qquad \widehat{\boldsymbol{\Phi}}_j = \boldsymbol{V}_{j,N_{C,j}}^{\mathrm{H}} \qquad (15)$$

Here, MATLAB notation is used again where we define $\boldsymbol{\Sigma}_{j,N_{C,j}} \equiv \boldsymbol{\Sigma}_j(1{:}\,N_{C,j}, 1{:}\,N_{C,j})$, $\boldsymbol{V}_{j,N_{C,j}}^{\mathrm{H}} \equiv \boldsymbol{V}_j^{\mathrm{H}}(1{:}\,N_{C,j}, :), \boldsymbol{U}_{j,N_{C,j}} \equiv \boldsymbol{U}_j(:, 1{:}\,N_{C,j})$ [20]. Only $\boldsymbol{V}_{j,N_{C,j}}^{\mathrm{H}}$ result is needed in this case. For this reason, it is sufficient to have only right singular vectors for the design. In this case, the hardware implementation of an RF system given in Figure 2 is simplified as shown in Figure 6.

Design of a Compressed Sensing Particle Filter for Generating Prior Information Compatible with a Compressed Sensing Measurement Matrix Design

In this section, how to design a tracking algorithm suitable for the proposed technique is explained. Tracking algorithms allow sequential state predictions in dynamic systems. While the tracking algorithms are being constructed, since the states are dynamic, the temporal relations with each other are also considered. While this relationship is provided by the "state equation", the measurements that have a noisy and indirect relationship with the desired states are also expressed with the "measurement equation". Using the knowledge of these two equations, a state estimate is conducted for each time step. The measurement equation for the scenario examined in the invention is given in equation (4). The Kalman filter is not an optimal tracking algorithm, as the relationship between the measurements $\boldsymbol{y}$ and directions of arrival (states) in equation (4) is not linear. Since it can work well even with non-linear models and with arbitrary noise distributions, and is a general tracking algorithm, a particle filter-based approach is presented in the invention. When the time variable is included in equation (4), the measurement equation in snapshot τ is as follows:

$$\boldsymbol{y}(\tau) = \boldsymbol{\Phi}(\tau)\boldsymbol{A}(\theta)\boldsymbol{s}(\tau) + \boldsymbol{\Phi}(\tau)\boldsymbol{n}(\tau)$$

Assuming that the velocities of the tracked targets do change abruptly, the state equation can be written as:

$$\boldsymbol{\theta}(\tau) = \boldsymbol{\theta}(\tau-1) + T\dot{\boldsymbol{\theta}}(\tau-1) + \boldsymbol{v}(\tau-1)$$

In this equation, $\boldsymbol{v}$ denotes the noise which is caused by the inability to estimate the angular velocity $\dot{\boldsymbol{\theta}}$ with exact accuracy. T is the time passed between consecutive snapshots. In other words, T denotes the time based between snapshots τ and $\tau-1$ and it depends on how fast the system can receive and process data. A classical particle filter application for the given measurement and state models, without using CS based techniques, is given below [21], [22]:

General Particle Filter Algorithm

Initialization (For $\tau = 0$):

I. For $n = 1,2,\dots,N$; the particles are sampled from the distribution $q(\boldsymbol{\theta}(0)|\boldsymbol{y}(0))$. (The choice of importance density function will be mentioned later)

II. The unnormalized weights are calculated:

$$\widetilde{w}^n(0) = \frac{p\big(\boldsymbol{y}(0)|\boldsymbol{\theta}^n(0)\big)p\big(\boldsymbol{\theta}^n(0)\big)}{q(\boldsymbol{\theta}(0)|\boldsymbol{y}(0))}.$$

III. The obtained weights are normalized: $w^n(0) = \frac{\widetilde{w}^n(0)}{\sum_{n=1}^{N}\widetilde{w}^n(\tau)}$

For $\tau = 1,2,\dots$:

I. For $n = 1,2,\dots,N$; the particles are sampled from the distribution $q\left(\boldsymbol{\theta}(\tau)|\boldsymbol{\theta}^n(\tau),\dot{\boldsymbol{\theta}}^n(\tau),\boldsymbol{y}(\tau)\right)$.

II. The unnormalized weights are calculated:

$$\widetilde{w}^n(\tau) = \widetilde{w}^n(\tau-1)\frac{p\big(\boldsymbol{y}(\tau)|\boldsymbol{\theta}^n(\tau)\big)p\left(\boldsymbol{\theta}^n(\tau)|\boldsymbol{\theta}^n(\tau-1),\dot{\boldsymbol{\theta}}^n(\tau-1)\right)}{q\left(\boldsymbol{\theta}(\tau)|\boldsymbol{\theta}^n(\tau),\dot{\boldsymbol{\theta}}^n(\tau),\boldsymbol{y}(\tau)\right)}.$$

III. The obtained weights are normalized: $w^n(\tau) = \widetilde{w}^n(\tau)/\sum_{n=1}^{N} \widetilde{w}^n(\tau)$.

IV. Optionally, the particles are resampled (The resampling procedure is explained below)

Typically, the importance density function in classical particle filter applications is selected as $q\left(\boldsymbol{\theta}(\tau)|\boldsymbol{\theta}^n(\tau), \dot{\boldsymbol{\theta}}^n(\tau), \boldsymbol{y}(\tau)\right) = p\left(\boldsymbol{\theta}^n(\tau)|\boldsymbol{\theta}^n(\tau-1), \dot{\boldsymbol{\theta}}^n(\tau-1)\right)$ [22]. As can be seen, no CS based reconstruction technique is used in the general particle filter application, and a time dependent measurement matrix is not designed. In the proposed algorithm, particle samples are generated using the output of the CS based reconstruction algorithm. It is assumed that the output of the CS based reconstruction algorithm is the noisy version of the real directions of arrival. Different particle samples are created with different implementations of this noise, whose exact characteristic is unknown. Assuming that the reconstruction algorithm used is unbiased, it can be approximated that the noise has a mean of $\mathbf{0}$ and the covariance matrix can be determined by the Cramer-Rao lower bound [14]. At this stage, the advantages of CS based techniques in reconstruction algorithms are utilized. In the next stages of the proposed algorithm, a CS based algorithm using both the state and measurement model is designed by continuing the particle filter approach, which also uses the temporal correlation between states. Finally, using the output of the particle filter, the $\boldsymbol{P}$ matrix is designed to generate the prior information for the next snapshot. Using this $\boldsymbol{P}$ matrix, the adaptive measurement matrix is designed in the next snapshot, the adaptive reconstruction algorithm is executed, and the algorithm is continued by creating new particles. The recommended CS based Particle Filter is given below, step by step:

Initialization (For $\tau = 0$):

Equation (10) is solved by writing $\boldsymbol{P} = \boldsymbol{I}_L$ and the measurement matrix $\boldsymbol{\Phi}(0)$ is obtained (measurement matrix design). By using equation (8) and $\hat{\boldsymbol{s}} = \boldsymbol{P}\hat{\bar{\boldsymbol{s}}}$, the DOA estimates at $\tau = 0$, i.e. $\boldsymbol{\theta}^*(0)$, are found. By using these estimates, $\boldsymbol{P}(0)$ matrix is formed (How the matrix $\boldsymbol{P}$ is formed will be explained in detail afterwards).

For $\tau = 1$:

For $\boldsymbol{P} = \boldsymbol{P}(0)$, equation (10) is solved and the measurement matrix $\boldsymbol{\Phi}(1)$ is obtained (adaptive measurement matrix design). By using equation (8) and $\hat{\boldsymbol{s}} = \boldsymbol{P}\hat{\bar{\boldsymbol{s}}}$, the DOA estimates at $\tau = 1$, i.e. $\boldsymbol{\theta}^*(1)$, are found (reweighted ℓ_1 minimization). By using these estimates, $\boldsymbol{P}(1)$ matrix is formed. The angular velocity estimates are found by writing

$\dot{\boldsymbol{\theta}}^*(1) = (\boldsymbol{\theta}^*(1) - \boldsymbol{\theta}^*(0))/T$. Let N denote the number of samples, then the particle weights are written as $w^n(1) = 1/N$ for $n = 1,2, \ldots, N$.

For $\tau = 2, 3, \ldots$:

I. An importance density function is formed for importance sampling. For that:
   a) Using $\boldsymbol{P}(\tau - 1)$ and the adaptive measurement matrix design methodology given in equation (10), $\boldsymbol{\Phi}(\tau)$ is formed (adaptive measurement matrix design).
   b) Equation (8) is solved using $\boldsymbol{\Phi}(\tau)$, and $\hat{\boldsymbol{s}} = \boldsymbol{P}\hat{\bar{\boldsymbol{s}}}$ is used to obtain $\widehat{\boldsymbol{\theta}}(\tau)$ estimates (reweighted ℓ_1 minimization).
   c) Let $\boldsymbol{\theta}(\tau)$ denote the real directions of arrival, the relationship between $\widehat{\boldsymbol{\theta}}(\tau)$ and $\boldsymbol{\theta}(\tau)$ is given below:

$$\boldsymbol{\theta}(\tau) = \widehat{\boldsymbol{\theta}}(\tau) + \boldsymbol{\eta}(\tau).$$

   In this equality, $\boldsymbol{\eta}$ denotes the noise in the estimation. The noise characteristics can be achieved by using Cramer-Rao lower bound obtained for equation (4) if $\mathrm{E}\{\widehat{\boldsymbol{\theta}}(\tau)\} = \mathrm{E}\{\boldsymbol{\theta}(\tau)\}$ is satisfied, i.e. the estimator is unbiased [14]. Depending on the characteristics of $\boldsymbol{\eta}$, for N different realizations of $\boldsymbol{\eta}$, N different samples are achieved as below for $n = 1,2, \ldots, N$:

$$\boldsymbol{\theta}^n(\tau) = \widehat{\boldsymbol{\theta}}(\tau) + \boldsymbol{\eta}(\tau).$$

   In that case, the probability distribution for importance sampling is written as $q\left(\boldsymbol{\theta}(\tau)|\boldsymbol{\theta}^n(\tau), \dot{\boldsymbol{\theta}}^n(\tau), \boldsymbol{y}(\tau)\right) = p\left(\boldsymbol{\theta}(\tau)\middle|\widehat{\boldsymbol{\theta}}(\tau)\right)$.

II. The unnormalized weights are obtained as below:

$$\widetilde{w}^n(\tau) = \widetilde{w}^n(\tau - 1)\frac{p(\boldsymbol{y}(\tau)|\boldsymbol{\theta}^n(\tau))p\left(\boldsymbol{\theta}^n(\tau)|\boldsymbol{\theta}^n(\tau-1), \dot{\boldsymbol{\theta}}^n(\tau-1)\right)}{p\left(\boldsymbol{\theta}^n(\tau)\middle|\widehat{\boldsymbol{\theta}}(\tau)\right)}.$$

III. The obtained weights are normalized: $w^n(\tau) = \widetilde{w}^n(\tau)/\sum_{n=1}^{N}\widetilde{w}^n(\tau)$.

IV. Effective number of samples is computed:

$$N_e = \frac{N}{1 + N^2\mathrm{var}(w^n(\tau))} \approx \frac{1}{\sum_{n=1}^{N} w^n(\tau)^2}.$$

V. If N_e is less than some threshold (e.g., $2N/3$), the particles are resampled. Resampling is performed as described below:
For $n = 1,2, \ldots, N$: the index $\gamma(n)$ is sampled with respect to $P(\gamma(n) = i) = w^i(\tau)$, $i = 1,2, \ldots N$, then $\boldsymbol{\theta}_{0:\tau}^n = \boldsymbol{\theta}_{0:\tau}^{\gamma(n)}$ and $w^n(\tau) = 1/N$ are updated.

VI. For each target ($k = 1,2, \dots, K$), the estimation is written as below:

$$\theta_k^*(\tau) = [w^1(\tau), w^2(\tau), \dots, w^N(\tau)]\left[\theta_k^1(\tau), \theta_k^2(\tau), \dots, \theta_k^N(\tau)\right]^{\mathrm{T}}.$$

The angular velocity estimate is found as:

$$\dot{\theta}_k^*(\tau) = \frac{\theta_k^*(\tau) - \theta_k^*(\tau - 1)}{T}$$

The expressions $\theta_k^*(\tau)$ and $\dot{\theta}_k^*(\tau)$ are sufficient to form the prior information matrix $\boldsymbol{P}$. $\boldsymbol{P}$ can be formed in different ways. For example, by using the information $\theta_k^*(\tau)$ and $\dot{\theta}_k^*(\tau)$, some particular angular regions are defined as focused regions. The diagonal entries of the matrix $\boldsymbol{P}$ are equal to 1 in those regions, and they are between 0 and 1 in other regions. For each target, the focused regions are different. If the focused regions are denoted by Ω_k:

$$\Omega_{\mathrm{k}}(\theta) = \begin{cases} 1, \text{if } \theta \in \left[\theta_k^*(\tau) + T\dot{\theta}_k^*(\tau) - \omega_k, \theta_k^*(\tau) + T\dot{\theta}_k^*(\tau) + \omega_k\right] \\ \alpha, \text{if } \theta \notin \left[\theta_k^*(\tau) + T\dot{\theta}_k^*(\tau) - \omega_k, \theta_k^*(\tau) + T\dot{\theta}_k^*(\tau) + \omega_k\right] \end{cases}.$$

Here $\alpha \in [0,1]$ and denotes how much importance is desired to be attached on focused regions. ω_k is a parameter depending on the estimation reliability, and depends on how much the target will deviate in terms of expected angle $\theta_k^*(\tau) + T\dot{\theta}_k^*(\tau)$. More generally, the diagonal entries of $\boldsymbol{P}$ can be estimated using a kernel density estimator. For example, similar to the approach used in [5], the convex combination of K Gaussian distributions whose means are determined as the expected angle of each target, i.e., $\theta_k^*(\tau) + T\dot{\theta}_k^*(\tau)$, and a uniform distribution can be used as the diagonal entries of $\boldsymbol{P}$.

With this given particle filter algorithm, all the elements of the system given in Figure 5, as an example application, are completed.

References

[1] J. Capon, "High resolution frequency-wavenumber spectrum analysis," *Proc. IEEE,* vol. 57, pp. 1408–1418, Aug. 1969.

[2] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagat.*, vol. 34, no. 3, pp. 276–280, Mar. 1986.

[3] E. J. Candes, and T. Tao, "Near-optimal signal recovery from random projections: Universal encoding strategies?," *IEEE Trans. Inform. Theory*, vol. 52, no. 12, pp. 5406–5425, Dec. 2006.

[4] M. Ibrahim, F. Roemer, and G. D. Galdo, "An adaptively focusing measurement design for compressed sensing based doa estimation," in *Proc. Eur. Signal Process. Conf.,* 2015, pp. 859–863.

[5] B. Kılıç, A. Güngör, M. Kalfa, and O. Arıkan, "Adaptive measurement matrix design in compressed sensing based direction of arrival estimation," in *Proc. Eur. Signal Process. Conf.,* 2020, pp. 1881–1885.

[6] E. J. Candes, M. B. Wakin, and S. P. Boyd, "Enhancing sparsity by reweighted ℓ^1 minimization," *J. Fourier Anal. Appl.*, vol. 14, pp. 877–906, 2008.

[7] M. Elad, "Optimized projections for compressed sensing," *IEEE Trans. Signal Process.*, vol. 55, no. 12, pp. 5695–5701, Dec. 2007.

[8] B. Li, L. Zhang, T. Kirubarajan, and S. Rajan, "Projection matrix design using prior information in compressive sensing," *Signal Processing*, vol. 135, pp. 36–47, 2017.

[9] M. Guo, Y. D. Zhang, and T. Chen, "Doa estimation using compressed sparse array," *IEEE Trans. on Signal Process.*, vol. 66, no. 15, pp. 4133–4146, Aug. 2018.

[10] R. Obermeier, and J. A. Martinez-Lorenzo, "Sensing matrix design via capacity maximization for block compressive sensing applications," *IEEE Trans. on Computational Imaging*, vol. 5, no. 1, pp. 27–36, Mar. 2019.

[11] T. Huang, Y. Liu, H. Meng, and X. Wang, "Adaptive compressed sensing via minimizing Cramer-Rao bound," *IEEE Signal Process. Letters*, vol. 21, no. 3, pp. 270–274, Mar. 2014.

[12] B. Özer, A. Lavrenko, S. Gezici, G. Del Galdo, and O. Arıkan, "Adaptive measurement matrix design for compressed doa estimation with sensor arrays," in *Proc. Asilomar Conf. Signals, Syst. Comput.*, 2015, pp. 1769–1773.

[13] M. Ibrahim, V. Ramireddy, A. Lavrenko, J. König, F. Römer, M. Landmann, M. Grossmann, G. D. Galdo, and R. S. Thoma, "Design and analysis of compressive

antenna arrays for direction of arrival estimation," *Signal Process.*, vol. 138, pp.35–47, Sep. 2017.

[14] C. R. Rao, C. R. Sastry, and B. Zhou, "Tracking the direction of arrival of multiple moving targets," *IEEE Trans. on Signal Process.*, vol. 42, no. 5, pp. 1133–1144, May 1994.

[15] N. Vaswani, "Kalman filtered compressed sensing," in *Proc. IEEE International Conf. on Image Processing,* 2008, pp. 893–896.

[16] S. Das, and N. Vaswani, "Particle filtered modified compressive sensing (PaFiMoCS) for tracking signal sequences," in *Proc. Asilomar*, 2010, pp. 354–358.

[17] E. Wang, J. Silva, and L. Carin, "Compressive particle filtering for target tracking," in *Proc. IEEE 15th Workshop on Statistical Signal Processing,* 2009, pp. 233–236.

[18] A. Carmi, P. Gurfil, and D. Kanevsky, "Methods for sparse signal recovery using Kalman filtering with embedded pseudo-measurement norms and quasi-norms," *IEEE Trans. on Signal Process.*, vol. 58, no. 4, pp. 2405–2409, Apr. 2010.

[19] B. Kılıç, M. Kalfa, and O. Arıkan, "Adaptive sensing matrix design in compressive sensing based direction of arrival estimation with hardware constraints," in *Proc. IEEE International Symposium on Antennas and Propagation and North American Radio Science Meeting,* 2020, pp. 149–150.

[20] D. Higham, and N. Higham, *MATLAB Guide*. Philadelphia: SIAM, 2005.

[21] M. Orton, and W. Fitzgerald, "A Bayesian approach to tracking multiple targets using sensor arrays and particle filters," *IEEE Trans. on Signal Process.*, vol. 50, no. 2, pp. 216–223, Feb. 2002.

[22] F. Gustafsson, "Particle filter theory and practice with positioning applications," *IEEE A&E Systems Magazine,* vol. 25, no. 7, pp. 53–81, July 2010.

[23] B. Kılıç, M. Kalfa, and O. Arıkan, "Prior based grid selection algorithm for compressed sensing based direction of arrival estimation methods," in *10th International Symposium on Phased Array Systems and Technology*, Waltham, MA USA, Oct. 2019.

CLAIMS

1. Compressed sensing based measurement matrix design methodology for adaptive direction of arrival estimation using sensor arrays characterized by the generation of an adaptive measurement matrix that is suitable for reconstruction and non-heuristic with reweighted ℓ_1-norm minimization.

2. The measurement matrix design method according to claim 1, wherein the grid points of measurement matrix are selected adaptively.

3. The measurement matrix design method according to claim 1 characterized in that use in the systems where the number of receivers is larger than the number of samplers.

4. A block-diagonal, measurement matrix compression method for compressed sensing based, adaptive direction of arrival estimation using sensor arrays characterized by comprising the following steps;
    - generation of a measurement matrix,
    - expressing the created measurement matrix as the multiplication of a block-diagonal matrix and an invertible matrix to achieve its projection suited for the grouped sensor subarray constraints.

5. A direction of arrival and velocity estimation method for compressed sensing based adaptive direction of arrival estimation using sensor arrays characterized by comprising the following steps;
    - the generation of different particles using different noise realizations with unknown characteristics, assuming that the direction of arrival outputs of compressed sensing based reconstruction algorithm are the noisy states of the real directions of arrival,
    - the estimation of direction of arrival and velocity, using the importance density function generated with the mentioned particles.

6. The direction of arrival and velocity estimation method according to claim 5, characterized by comprising the following steps;

- design of the matrix determining the prior information for the next snapshot by using the particle filter output,
- design of the adaptive measurement matrix using the mentioned matrix for the next snapshot,
- execution of the reconstruction algorithm and generation of new particles to continue the algorithm.

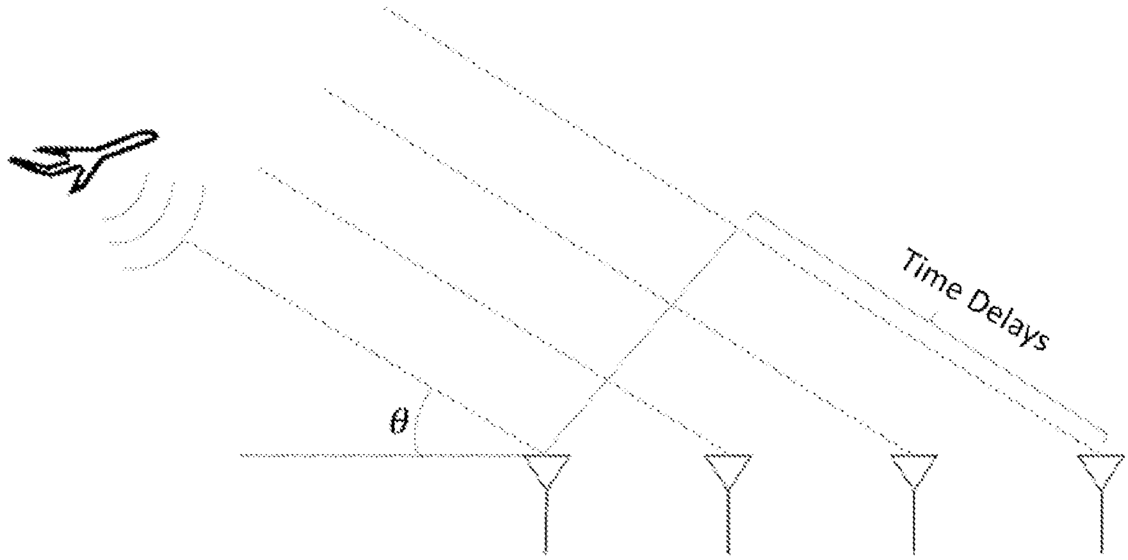

Figure 1

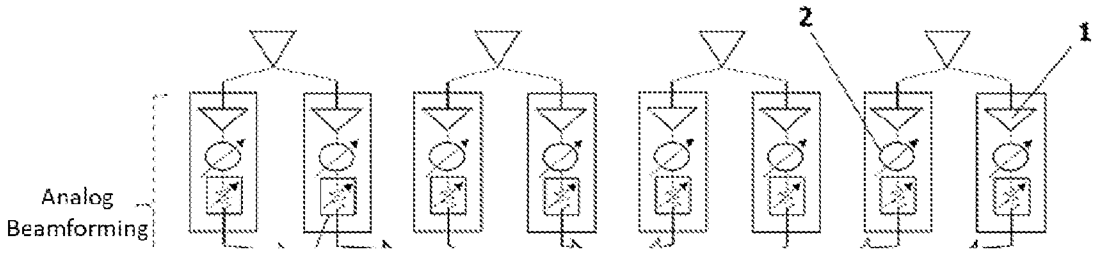

Figure 2

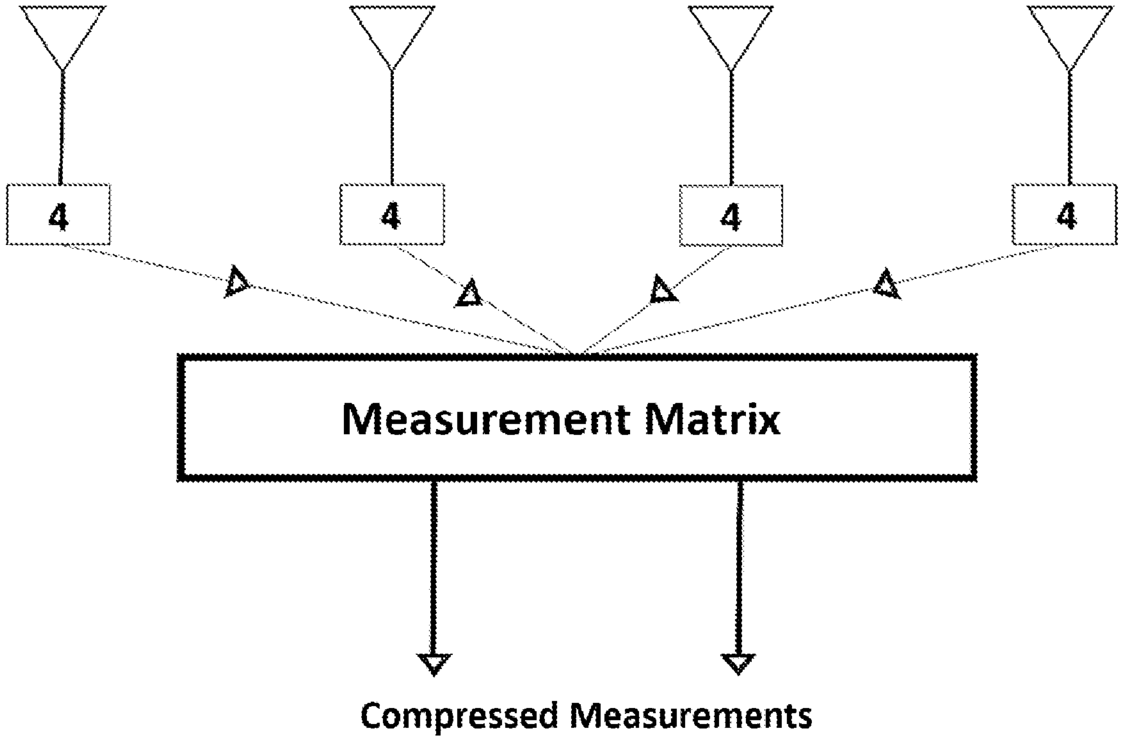

Figure 3

Figure 4

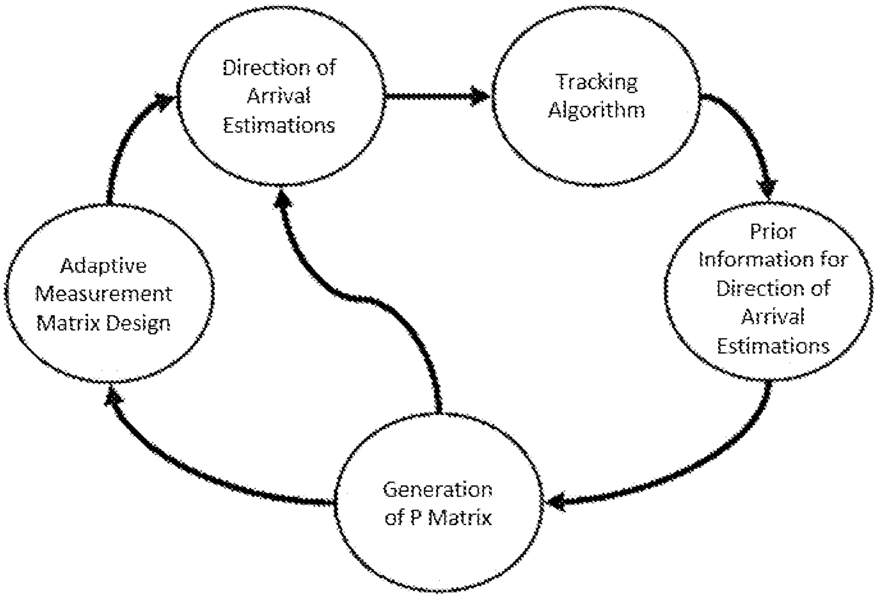

Figure 5

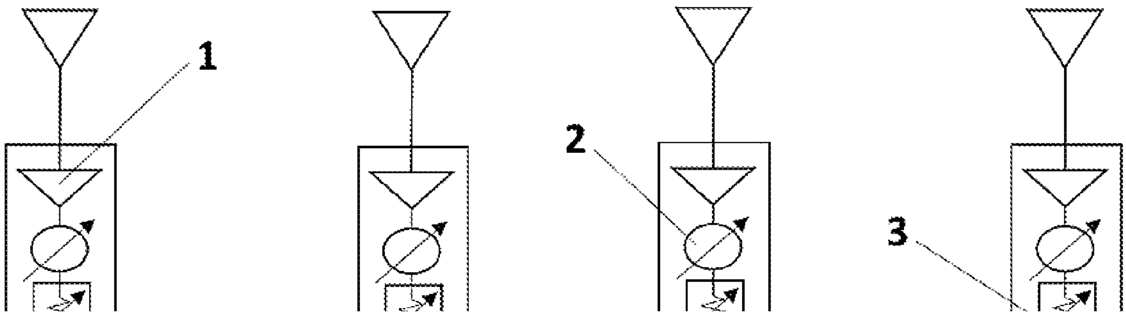

Figure 6